Reflection of longitudinal micro-rotational wave from a fixed micro-elastic surface

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ABSTRACT:

Reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic surface is studied and obtained the amplitude ratios of micro-rotation, transverse displacement and transverse micro-rotation waves. The result of micropolar theory is obtained as a particular case of it.

KEY WORDS: Reflection – micro-rotational wave – micro-elastic surface.

INTRODUCTION:

The theory of micromorphic materials was developed by Eringen[1]. This theory is simplified by Koh[2] by extending the concept of coincidence of the principal directions of the stress and strain of classical elasticity to theory of micromorphic materials and called it as theory of micro-isotropic, micro-elastic solids. It consists of 12 deformation fields namely, 3 macro-displacements, 3 micro-rotations and 6 micro-strains. Further it has 10 elastic constants. If an element ∆V + ∆S contains N discrete micro-material elements ∆V(α) + ∆S(α), (α=1,2,…, N), then the micro-rotational and micro-strains are the rotation and strain of micro elements ∆V(α) + ∆S(α) about the centre of mass of ∆V + ∆S.

Eringen[3] have investigated the problem of plane waves from the flat boundary of a micropolar elastic half space and obtained the amplitude ratios of different reflected waves. Reflection and refraction of a longitudinal micro-rotational wave at an interface between two micropolar elastic media in welded contact is investigated by Tomar and Gonga[4]. In the present work the amplitude ratios of micro-rotation waves and transverse displacement waves at a fixed micro-isotropic, micro-elastic surface are obtained.
BASIC EQUATIONS:

The micro-displacement in the micro-elastic continuum is denoted by $u_k$ and micro-deformation by $\phi_{mn}$. Further the macro-strain $\varepsilon_{km} = u_{(k,m)}$, the macro-elastic vector $r_k = \frac{1}{2} \varepsilon_{kmn}$ $u_{n,m}$, the micro-strain $\phi_{(mn)}$ and the micro-rotation vector $\phi_p = \frac{1}{2} \varepsilon_{pmn} \phi_{kn}$, where ( ) denotes the symmetric part, comma indicates differentiation with respect to the coordinate $x_k$.

The stress measures are the asymmetric stress (macro stress) $t_{km}$, the relative stress (micro-stress) $\sigma_{km}$ and stress moment $t_{kmn}$. The couple stress tensor $m_{kl}$ is defined by $m_{kl} = \varepsilon_{pmn} t_{kmn}$.

The constitutive equations for micro-isotropic, micro-elastic medium are given by [2,5].

\[
t_{km} = A_1 \varepsilon_{pp} \delta_{km} + 2A_2 \varepsilon_{km} \tag{1}
\]

\[
t_{[km]} = \sigma_{[km]} = 2A_3 \varepsilon_{pmn} \left(r_p + \phi_p\right) \tag{2}
\]

\[
\sigma_{[km]} = -A_4 \varepsilon_{pp} \delta_{km} - 2A_5 \phi_{[km]} \tag{3}
\]

\[
t_{k(mn)} = B_1 \phi_{pp,k} \delta_{mn} + 2B_2 \phi_{(mn),k} \tag{4}
\]

\[
m_{il} = -2(B_3 \phi_{il,k} + B_4 \phi_{l1} + B_5 \phi_{p,p} \delta_{il}) \tag{5}
\]

where [ ] denotes the anti-symmetric part and

\[
A_1 = \lambda + \sigma_1, \quad B_1 = \tau_3
\]

\[
A_2 = \mu + \sigma_2, \quad 2B_2 = \tau_7 + \tau_{10}
\]

\[
A_3 = \sigma_5, \quad B_3 = 2\tau_4 + 2\tau_6 + \tau_7 - \tau_{10}
\]

\[
A_4 = -\sigma_1, \quad B_4 = -2\tau_4
\]

\[
A_5 = -\sigma_2, \quad B_5 = -2\tau_9
\]

where $\lambda, \mu, \sigma_1, \sigma_2, \sigma_5, \tau_3, \tau_4, \tau_6, \tau_7, \tau_9$ and $\tau_{10}$ are elastic moduli.

The displacement equations of motion for micro-isotropic, micro-elastic body occupying a region $R$ are given by

\[
(A_1 + A_2 - A_3)u_{p,pm} + (A_2 + A_3)u_{m,pp} + 2A_3 \varepsilon_{pmn} \phi_{p,k} + \rho f_m = \rho \frac{\partial^2 u_m}{\partial t^2} \tag{7}
\]
\[
B_{ij} \phi_{pp,kk} \delta_{ij} + 2B_{2ij} \phi_{(ij),kk} - A_{4ij} \phi_{pp} \delta_{ij} - 2A_{5ij} \phi_{(ij)} + \rho f_{(ij)} = \frac{1}{2} \rho j \frac{\partial^2 \phi_{(ij)}}{\partial t^2}
\]

(8)

\[
2B_{3j} \phi_{p,mn} + 2(B_4 + B_5) \phi_{m,np} - 4A_3 \left( r_p + \phi_p \right) - \rho l_p = \rho j \frac{\partial^2 \phi_p}{\partial t^2}
\]

(9)

where \( \rho \) is the average mass density, \( f_m \) is the body force per unit mass, \( f_{(ij)} \) is the symmetric body moment per unit mass, \( j \) is the micro-inertia, \( l_p \) is the body couple per unit mass.

**Reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic solid**

K. Somaiah and K. Sambaiah[6] have shown that there are 12 waves propagate in an infinite micro-isotropic, micro-elastic solid with six different velocities.

a) A longitudinal displacement wave propagating with speed \( v_1 \) where

\[
v_1^2 = \frac{A_1 + 2A_3}{\rho}
\]

(10)

b) A longitudinal micro-rotation wave with speed \( v_2 \) where

\[
v_2^2 = \frac{2(B_3 + B_4 + B_5)}{\rho j \left( 1 - \frac{\omega_1^2}{\omega^2} \right)}
\]

(11)

with a cut-off frequency

\[
\omega_1^2 = \frac{4A_1}{\rho j}
\]

(12)

c) Two sets of coupled transverse displacement and transverse micro-rotation waves with speeds \( v_3 \) and \( v_4 \) where

\[
v_3^2 = \frac{1}{2 \left( 1 - \frac{2\omega_0^2}{\omega^2} \right)} \left[ c_4^2 + c_2^2 \left( 1 - \frac{2\omega_0^2}{\omega^2} \right) + c_3^2 \left( 1 - \frac{3\omega_0^2}{\omega^2} \right) \right]
\]

\[
+ \left[ c_4^2 - c_2^2 - c_3^2 + 2 \left( c_2^2 + \frac{3}{2} c_3^2 \right) \frac{\omega_0^2}{\omega^2} - 4c_3^2 \frac{\omega_0^2}{\omega^2} \right]^{1/2}
\]

(13)

\[
v_4^2 = \frac{1}{2 \left( 1 - \frac{2\omega_0^2}{\omega^2} \right)} \left[ c_4^2 + c_2^2 \left( 1 - \frac{2\omega_0^2}{\omega^2} \right) + c_3^2 \left( 1 - \frac{3\omega_0^2}{\omega^2} \right) \right]
\]
\[- \left[ c_i^2 - c_j^2 - c_k^2 + 2 \left( c_i^2 + \frac{3}{2} c_j^2 \right) \frac{\omega_0^2}{\omega^2} - 4 c_i^2 c_j^2 \frac{\omega_0^2}{\omega^2} \right]^{1/2} \]  

(14)

with \( \sqrt{2} \omega_0 \) as cut-off frequency .

d) Six waves corresponding to micro-strains and they propagate with two distinct velocities.

We investigate the reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic surface. Eringen[3] shown that the incident and reflected waves propagate in the same plane. We select this plane to be the \( z = 0 \) plane.

Thus we take

\[ h' = (u, 0, 0) \]

(15)

\[ \phi' = (0, \phi_2, \phi_3) \]

(16)

where \( u, \phi_2, \phi_3 \) are functions of \( y, z \) and \( t \). The boundary conditions are \( u = 0, \phi_2 = 0, \phi_3 = 0 \) at \( z = 0 \).

We decompose the vector \( h' \) and \( \phi' \) into scalar and vector potentials as

\[ h' = \nabla G + \nabla \times H', \quad \nabla \cdot H' = 0 \]

(17)

\[ \phi' = \nabla R + \nabla \times S', \quad \nabla \cdot S' = 0 \]

(18)

Substituting (17), (18) into (7) and (8), we get that these equations are satisfied if

\[ (c_i^2 + c_j^2) \nabla^2 G = \mathcal{G} \]

(19)

\[ (c_i^2 + c_j^2) \nabla^2 R - 2 \omega_0^2 R = \mathcal{G} \]

(20)

\[ (c_i^2 + c_j^2) \nabla^2 H + c_i^2 \nabla \times S = \mathcal{H} \]

(21)

\[ c_i^2 \nabla^2 S - 2 \omega_0^2 S - \omega_0^2 \nabla \times H = \mathcal{S} \]

(22)

where

\[ c_i^2 = A_i + 2A_i - 2A_3 \]

\[ c_i^2 = - A_i \]

\[ \phi' = \nabla^2 G + \nabla \times H', \quad \nabla \cdot H' = 0 \]

(17)

\[ \phi' = \nabla R + \nabla \times S', \quad \nabla \cdot S' = 0 \]

(18)

Consider an incident longitudinal micro-rotation wave travelling with speed \( v_2 \) in the direction \( v_1 \) making an angle \( \theta_1 \) with the plane boundary give rise to

a) A reflected longitudinal micro-rotation wave travelling with speed \( v_2 \) in the direction \( v_2' \) and making an angle \( \theta_2 \) with the fixed surface.

b) A set of reflected waves travelling with speed \( v_3 \) in the direction \( v_3 \) and making an angle \( \theta_3 \) with the surface.
c) A similar set of reflected waves travelling with speed $v_4$ in the direction $\vec{v}_4$ and making an angle $\theta_4$ with the fixed surface. The geometry of the problem is shown in fig.1

In this case the appropriate potentials are given by

$$H_p = \left[ A_{py} j^{-\frac{\omega}{\sin \theta_p}} A_{py} k \right] \exp[ik \{ \cos \theta_p y + \sin \theta_p z \} - i \omega t]$$

$$S_p = i \beta_{Ap} \left( \frac{A_{py}}{\sin \theta_p} \right) \exp[ik \{ \cos \theta_p y + \sin \theta_p z \} - i \omega t]$$

where $p = 3,4; a_1, a_2$ are the amplitudes of incident and reflected longitudinal micro-rotation waves respectively; $A_{py}$ are the amplitudes of reflected coupled waves.

$$\beta_{Ap} = \left[ \begin{array}{c} \frac{i \omega^2}{k_p} \\ \frac{v_p^2 - 2 \frac{\omega^2}{k_p^2} c_4^2} {k_p} \end{array} \right] \beta_{Bp}$$

$$\omega_i = k_p v_i, \quad (i = 2,3,4)$$

$\omega_i$ is angular frequency and $k_p$ is the wave number.

In view of (17) and (18), the boundary conditions become

$$u = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = 0$$

$$\phi_z = \frac{\partial R}{\partial y} + \frac{\partial S}{\partial z} = 0$$
\[ \phi_3 = \frac{\partial R}{\partial y} - \frac{\partial S}{\partial z} = 0 \]

Eringen\[7\] has shown that
\[ \frac{\cos \theta_2}{\nu_2} = \frac{\cos \theta_3}{\nu_3} = \frac{\cos \theta_4}{\nu_4} \text{ and } \theta_2 = \theta_1 \] (27)

and \( \omega_2 = \omega_3 = \omega_4 = \omega \) (say) (28)

Substituting (24) into (26) and using (27) and (28) we obtain
\[ \sin \theta_4 z_2 + \sin \theta_3 z_3 = 0 \] (29)

\[ \cos \theta_2 z_1 + \frac{\beta_{A3}}{k_2 \beta_{B3}} z_2 + \frac{\beta_{A4}}{k_2 \beta_{B4}} z_3 = -\cos \theta_1 \] (30)

\[ \sin \theta_2 z_1 - \frac{\cos \theta_3}{\sin \theta_3} \frac{\beta_{A3}}{k_2 \beta_{B3}} z_2 - \frac{\cos \theta_4}{\sin \theta_4} \frac{\beta_{A4}}{B_{B4} k_2} z_3 = \sin \theta_1 \] (31)

\[ z_1 = \frac{a_2}{a_1}, \quad z_2 = \frac{k_2 A_{3y}}{a_1}, \quad z_4 = \frac{k_2 A_{4y}}{a_1} \] (32)

where

Solving the equations (30) to (32) we obtain the amplitude ratios and they are given by
\[ z_1 = \frac{1}{\Delta} \left[ \frac{\beta_{A3}}{k_2} \frac{1}{\beta_{B3}} \cos(\theta_1 + \theta_3) - \frac{\beta_{A4}}{k_2} \frac{1}{\beta_{B4}} \cos(\theta_1 + \theta_4) \right] \] (33)

\[ z_2 = \frac{1}{\Delta} \sin \theta_2 \sin(\theta_1 + \theta_2) \] (34)

\[ z_3 = \frac{1}{\Delta} \sin \theta_4 \sin(\theta_1 + \theta_2) \] (35)

where \( \Delta = \frac{\beta_{A4}}{\beta_{B4}} \frac{1}{k_2} \cos(\theta_2 - \theta_4) - \frac{\beta_{A3}}{\beta_{B3}} \frac{1}{k_2} \cos(\theta_2 - \theta_3) \)
REFERENCES


