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ON THE STABILITY OF A FOUR SPECIES: A PREY-PREDATOR-HOST-COMMENSAL-SYN ECO-SYSTEM-I (FULLY WASHED OUT STATE)

B. Hari Prasad[1], N.Ch. Pattabhi Ramacharyulu[2]

[1] Department. of Mathematics, Chaitanya Degree & P.G. College (Autonomous), Hanamkonda, Warangal A.P, India. Email :sumathi_prasad73@yahoo.com

[2] Former Faculty, Department. of Mathematics , NIT Warangal, India.

Abstract

This paper deals with an investigation on A Four Species Syn-Ecological System (Fully Washed Out State). The System comprises of a prey (S_1), a predator (S_2) that survives upon S_1, two hosts S_3 and S_4 for which S_1, S_2 are commensal respectively i.e., S_3 and S_4 benefit S_1 and S_2 respectively, without getting effected either positively or adversely. Further S_3 and S_4 are neutral. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all, there are sixteen equilibrium points. Criteria for the asymptotic stability of one of the sixteen equilibrium points : the fully washed out state is established. The system would be stable if all the characteristic roots are negative, in case they are real, and have negative real parts, in case they are complex. The linearized equations for the perturbations over the equilibrium point are analyzed to establish the criteria for stability and the trajectories illustrated.

1. Introduction

Mathematical modeling of Eco-System was initiated in 1925 by Lotka [8] and in 1931 by Volterra[12]. The general concepts of modeling have been presented in the treatises of Meyer[9], Kushing[5], Kapur J.N. [3,4] and several others. The ecological interactions can be broadly classified as Prey-Predator, Commensalism, Competition, Neutralism, Mutualism and so on. N.C. Srinivas [11] studied competitive eco-systems of two species and three species with limited and unlimited resources. Later Lakshminarayan [6], Lakshminarayan and Pattabhi Ramacharyulu [7] studied Pre-Preadtor ecological models with partial cover for the Prey and alternate food for the predator. Recently, Archana Reddy [1] and Bhaskara Rama Sharma [2] investigated diverse problems related to two species competitive systems with time delay, employing analytical and numerical techniques. Further Phani Kumar, Seshagiri Rao and Pattabhi Ramacharyulu [10] studied the stability of a Host-A flourishing commensal species pair with limited resources.

The present investigation is devoted to an analytical study of a four species (S_1,S_2,S_3,S_4) Prey-Predator-Host-Commensal-Syn Eco-System. The System comprises of a prey (S_1), a predator (S_2) that survives upon S_1, two hosts S_3 and S_4 for which S_1, S_2 are commensal respectively i.e., S_3 and S_4 benefit S_1 and S_2 respectively, without getting effected either positively or adversely. Further S_3 and S_4 are neutral. Fig.1 shows the Schematic Sketch of the system under investigation. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all the sixteen equilibrium points of the system are identified and the stability analysis is carried out only for the fully washed out state. The linearized perturbed equations over the equilibrium states are solved and the trajectories illustrated.

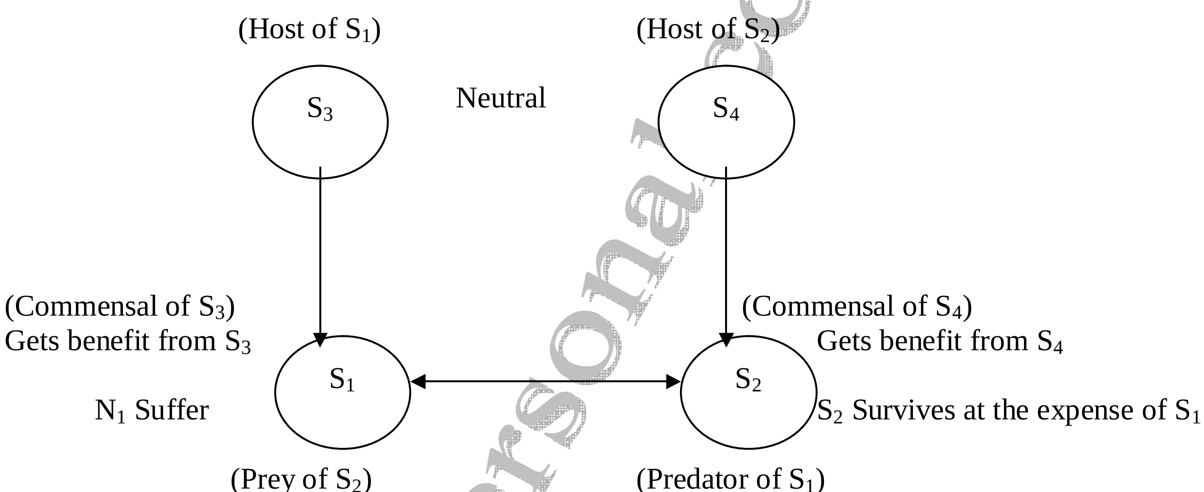


Fig. 1 Schematic Sketch of the Syn Eco - System

2. BASIC EQUATIONS:

Notation Adopted:

$N_1(t)$: The Population of the Prey (S_1)

$N_2(t)$: The Population of the Predator (S_2)

$N_3(t)$: The Population of the Host (S_3) of the Prey (S_1)

$N_4(t)$: The Population of the Host (S_4) of the Predator (S_2)

t : Time instant

a_1,a_2,a_3,a_4 : Natural growth rates of S_1, S_2, S_3, S_4

$a_{11},a_{22},a_{33},a_{44}$: Self inhibition coefficients of S_1, S_2, S_3, S_4

a_{12},a_{21} : Interaction (Prey-Predator) coefficients of S_1 due to S_2 and S_2 due to S_1

a_{13} : Coefficient for commensal for S_1 due to the Host S_3

a_{24} : Coefficient for commensal for S_2 due to the Host S_4

$\frac{a_1}{a_{11}}, \frac{a_2}{a_{22}}, \frac{a_3}{a_{33}}, \frac{a_4}{a_{44}}$: Carrying capacities of S_1, S_2, S_3, S_4

Further the variables N_1, N_2, N_3, N_4 are non-negative and the model parameters a_1, a_2, a_3, a_4; a_{11}, a_{22}, a_{33}, a_{44}; a_{12}, a_{21}, a_{13}, a_{24} are assumed to be non-negative constants.

The model equations for the growth rates of S_1, S_2, S_3, S_4 are

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^2 - a_{12}N_1N_2 + a_{13}N_1N_3 \qquad \ldots. \quad (2.1)$$

$$\frac{dN_2}{dt} = a_2N_2 - a_{22}N_2^2 + a_{21}N_1N_2 + a_{24}N_2N_4 \qquad \ldots. \quad (2.2)$$

$$\frac{dN_3}{dt} = a_3 N_3 - a_{33} N_3^{\ 2} \qquad \ldots. \quad (2.3)$$

$$\frac{dN_4}{dt} = a_4 N_4 - a_{44} N_4^{\ 2} \qquad \ldots. \quad (2.4)$$

3 EQUILIBRIUM STATES:

The system under investigation has sixteen equilibrium states defined by

$$\frac{dN_i}{dt} = 0,\ i = 1,2,3,4 \qquad \ldots\ldots\ldots\ldots \ (3.1)$$

are given in the following table.

S.No.	Equilibrium States	Equilibrium Point
1	Fully Washed out state	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=0$
2	Only the Host (S_4)of S_2 survives	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$
3	Only the Host (S_3)of S_1 survives	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$
4	Only the Predator S_2 survives	$\overline{N_1}=0, \overline{N_2}=\frac{a_2}{a_{22}}, \overline{N_3}=0, \overline{N_4}=0$
5	Only the Prey S_1 survives	$\overline{N_1}=\frac{a_1}{a_{11}}, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=0$
6	Prey (S_1) and Predator (S_2) washed out	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=\frac{a_4}{a_{44}}$
7	Prey (S_1) and Host (S_3) of S_1 washed out	$\overline{N_1}=0, \overline{N_2}=\frac{a_2 a_{44}+a_4 a_{24}}{a_{22}a_{44}}, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$
8	Prey (S_1) and Host (S_4) of S_2 washed out	$\overline{N_1}=0, \overline{N_2}=\frac{a_2}{a_{22}}, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$
9	Predator (S_2) and Host (S_3) of S_1 washed out	$\overline{N_1}=\frac{a_1}{a_{11}}, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$
10	Predator (S_2) and Host (S_4) of S_2 washed out	$\overline{N_1}=\frac{a_1 a_{33}+a_3 a_{13}}{a_{11}a_{13}}, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$
11	Prey (S_1) and Predator (S_2)survives	$\overline{N_1}=\frac{a_1 a_{22}-a_2 a_{12}}{a_{11}a_{22}+a_{12}a_{21}}, \overline{N_2}=\frac{a_1 a_{21}+a_2 a_{11}}{a_{11}a_{22}+a_{12}a_{21}}, \overline{N_3}=0, \overline{N_4}=0$
12	Only the Prey (S_1) washed out	$\overline{N_1}=0, \overline{N_2}=\frac{a_2 a_{44}+a_4 a_{24}}{a_{22}a_{44}}, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=\frac{a_4}{a_{44}}$
13	Only the predator (S_2) washed out	$\overline{N_1}=\frac{a_1 a_{23}+a_3 a_{13}}{a_{11}a_{13}}, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=\frac{a_4}{a_{44}}$

14	Only the Host (S_3) of S_1 washed out	$\overline{N_1}=\frac{\delta_2}{\delta_1},\overline{N_2}=\frac{\delta_3}{\delta_1},\overline{N_3}=0,\overline{N_4}=\frac{a_4}{a_{44}}$ *where* $\delta_1=a_{44}(a_{11}a_{22}+a_{12}a_{21})>0$ $\delta_2=a_1a_{22}a_{44}-a_{12}(a_2a_{44}+a_4a_{24})$ $\delta_3=a_1a_{21}a_{44}-a_{11}(a_2a_{44}+a_4a_{24})$
15	Only the Host (S_4) of S_2 washed out	$\overline{N_1}=\frac{\sigma_2}{\sigma_1},\overline{N_2}=\frac{\sigma_3}{\sigma_1},\overline{N_3}=\frac{a_3}{a_{33}},\overline{N_4}=0$ *where* $\sigma_1=a_{33}(a_{11}a_{22}+a_{12}a_{21})>0$ $\sigma_2=a_{22}(a_1a_{33}+a_3a_{13})-a_2a_{12}a_{33}$ $\sigma_3=a_{21}(a_1a_{33}+a_3a_{13})+a_2a_{11}a_{33}>0$
16	The co-existent state (or) Normal steady state	$\overline{N_1}=\frac{a_{22}a_{44}\psi_1-a_{12}a_{33}\psi_2}{\psi_3},\overline{N_2}=\frac{a_{21}a_{44}\psi_1+a_{11}a_{33}\psi_2}{\psi_3},$ $\overline{N_3}=\frac{a_3}{a_{33}},\overline{N_4}=\frac{a_4}{a_{44}}$ *where* $\psi_1=a_1a_{33}+a_3a_{13}>0$ $\psi_2=a_2a_{44}+a_4a_{24}>0$ $\psi_3=a_{33}a_{44}(a_{11}a_{22}+a_{12}a_{21})>0$

The present paper deals with the fully washed out state only. The stability of the other equilibrium states will be presented in the forth coming communications.

4. STABILITY OF THE FULLY WASHED OUT EQUILIBRIUM STATE: (Sl.No. 1 in the above table)

To discuss the stability of equilibrium point $\overline{N_1}=0,\overline{N_2}=0,\overline{N_3}=0,\overline{N_4}=0$

Let us consider small deviations $u_1(t)$, $u_2(t)$, $u_3(t)$, $u_4(t)$ from the steady state i.e.,

$$N_i(t)=\overline{N_i}+u_i(t), i=1,2,3,4 \qquad \ldots\ldots \quad 4.1$$

where $u_i(t)$ is a small perturbations in the species S_i.

Substituting (4.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of u_1, u_2, u_3, u_4

We get

$$\frac{du_1}{dt}=a_1u_1 \quad \ldots\ldots\ 4.2 \qquad\qquad \frac{du_2}{dt}=a_2u_2 \quad \ldots\ldots\ 4.3$$

$$\frac{du_3}{dt}=a_3u_3 \quad \ldots\ldots\ 4.4 \qquad\qquad \frac{du_4}{dt}=a_4u_4 \quad \ldots\ldots\ 4.5$$

The characteristic equation of which is

$(\lambda - a_1)(\lambda - a_2)(\lambda - a_3)(\lambda - a_4) = 0$ ………… 4.6

the roots a_1, a_2, a_3, a_4 of which are all positive.

Hence the Fully Washed Out State is **unstable**.

The solutions of the equations (4.2), (4.3), (4.4), (4.5) are

$u_1 = u_{10} e^{a_1 t}$ ………… 4.7 $\qquad$ $u_2 = u_{20} e^{a_2 t}$ ………… 4.8

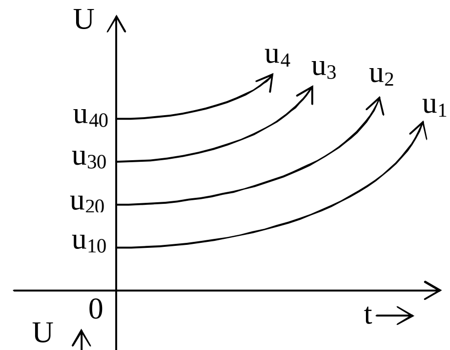
$u_3 = u_{30} e^{a_3 t}$ … 4.9 $\qquad$ $u_4 = u_{40} e^{a_4 t}$ …………..4.10

where u_{10}, u_{20}, u_{30}, u_{40} are the initial values of u_1, u_2, u_3, u_4 respectively.

There would arise in all 576 cases depending upon the ordering of the magnitudes of the growth rates a_1, a_2, a_3, a_4 and the initial values of the perturbations $u_{10}(t)$, $u_{20}(t)$, $u_{30}(t)$, $u_{40}(t)$ of the species S_1, S_2, S_3, S_4. Of these 576 situations some typical variations are illustrated through respective solution curves that would facilitate to make some reasonable observations.

<u>Case (i) :</u> If $u_{10}<u_{20}<u_{30}<u_{40}$, $a_1<a_2<a_3<a_4$

In this case prey (S_1) has the least natural birth rate and the host (S_4) of S_2 dominates the prey (S_1), predator (S_2), host (S_3) of S_1 in natural growth rate as well as in its population strength.



<u>Case (ii):</u> If $u_{10}<u_{20}<u_{30}<u_{40}$, $a_2<a_1<a_3<a_4$

In this case predator (S_2) has the least natural birth rate. Initially the predator (S_2) dominates over the prey (S_1)till the time instant $t_{21}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$ and there-after the prey (S_1) dominated the predator(S_2). The time t_{21}^* may be called the dominance time of the predator (S_2) over the prey (S_1).

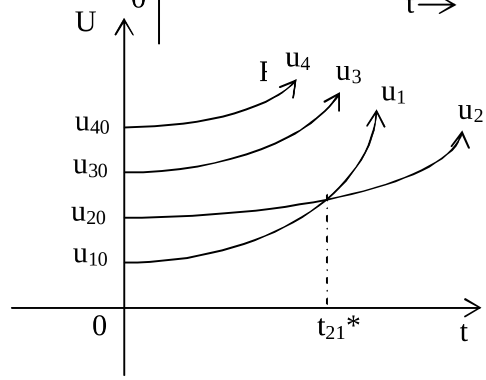


Fig. 4.2

<u>Case (iii):</u> If $u_{10}<u_{30}<u_{40}<u_{20}$, $a_2<a_1<a_3<a_4$

In this case predator (S_2) has the least natural birth rate. Initially the predator (S_2) dominates over the host (S_4) of N_2, host (S_3) of S_1 and prey (N_1) till the time instant $t_{24}^*, t_{23}^*, t_{21}^*$ respectively and there after the dominance is reversed

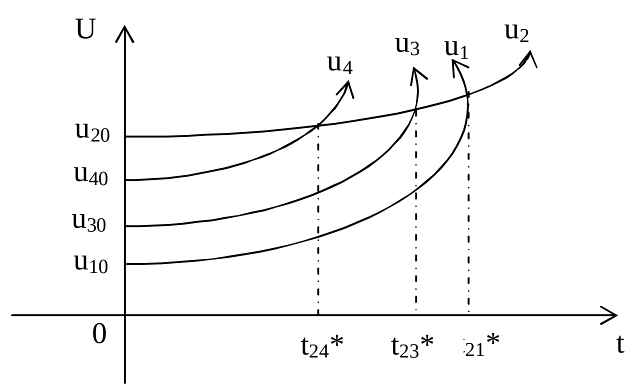


Fig. 4.3

Here

$$t_{24}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}}), t_{23}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{30}}{u_{20}}), t_{21}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$$

Case (iv) : If $u_{20}<u_{40}<u_{30}<u_{10}$, $a_3<a_1<a_2<a_4$

In this case the host (S_3) of S_1 has the least natural birth rate. Initially the host (S_3) of S_1 dominates over the host (S_4)of S_2, the predator till the times instant t^*_{34}, t^*_{32} respectively. Thereafter the dominance is reversed.

Here $t_{34}^* = \frac{1}{a_3 - a_4}\log(\frac{u_{40}}{u_{30}}); t_{32}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{30}}{u_{20}})$

Also the prey (S_1) dominates over the host (S_4) of S_2, Predator till the time instant t^*_{14}, t^*_{12} respectively and thereafter the dominance is reversed.

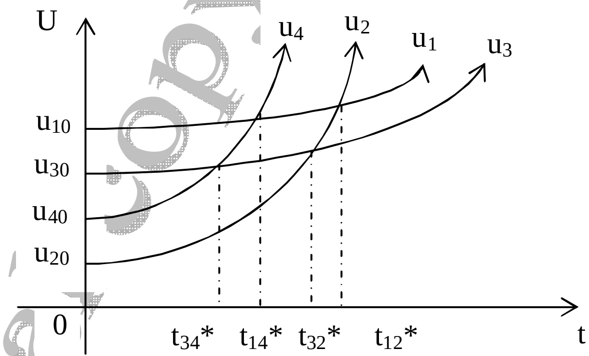


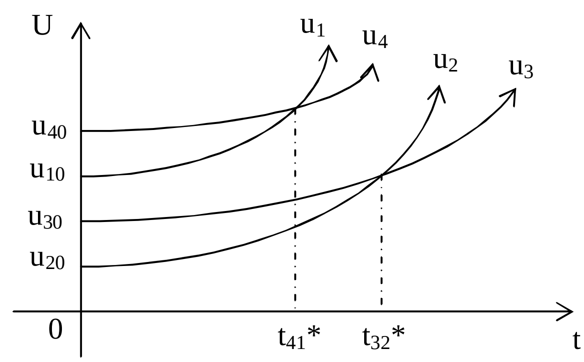
Fig. 4.4

Here $t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}}); t_{12}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$

Case (v): If $u_{20}<u_{30}<u_{10}<u_{40}$, $a_3<a_2<a_4<a_1$

In this case the Host (S_3) of S_1 has the least natural birth rate. Initially it is dominates over the predator (S_2) till the time instant $t_{32}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{30}}{u_{20}})$ and thereafter the dominance is reversed.

Also the host (S_4) of S_2 dominates over the prey (S_1) till the time instant $t_{41}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}})$ and thereafter the dominance is reversed.



Case (vi): If $u_{20}<u_{30}<u_{40}<u_{10}$, $a_1<a_4<a_3<a_2$

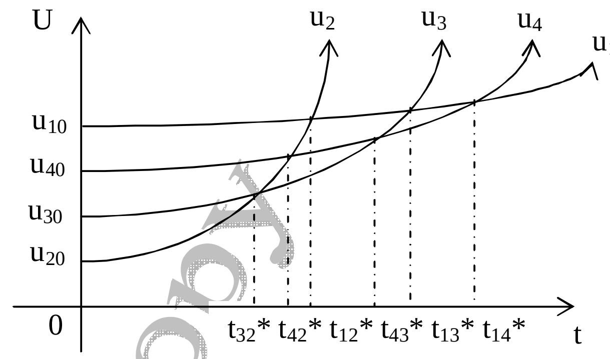


Fig. 4.6

In this case the prey (S_1) has the least natural birth rate. Initially the prey (S_1)dominates over its host, the host (S_4) of S_2 and Predator (S_2) till the time instant t^*_{13}, t^*_{14}, t^*_{12} respectively and thereafter the dominance is reversed.

Also the host (S_4) of S_2 dominates over the host (S_3) of S_1, and the predator (S_2) till the time instant t^*_{43}, t^*_{42} and thereafter the dominance is reversed.

Similarly, the host (S_3) of S_1 dominates over the predator (S_2) till the time instant t^*_{32} and the dominance gets reversed thereafter.

Here

$$t_{13}^* = \frac{1}{a_1 - a_3}\log(\frac{u_{30}}{u_{10}}); t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}}); t_{12}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$$

$$t_{43}^* = \frac{1}{a_3 - a_4}\log(\frac{u_{40}}{u_{30}}); t_{42}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}}); t_{32}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{30}}{u_{20}})$$

Case (vii): If $u_{30}<u_{20}<u_{10}<u_{40}$, $a_3<a_4<a_1<a_2$

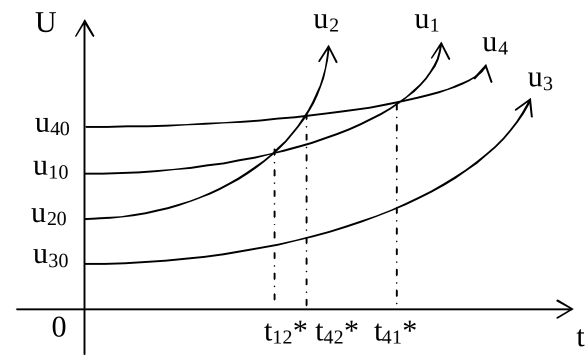


Fig. 4.7

In this case the host (S_3) of S_1 has the least natural birth rate. Initially the host (S_4) of S_2 dominates over both the prey (S_1) and predator (S_2) till the time instant t^*_{41}, t^*_{42} respectively and thereafter the dominance is reversed.

Also the Prey(S_1) dominates over the Predator (S_2) upto the time instant t^*_{12} and the dominance gets reversed after

Here $t_{41}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{30}}); t_{42}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}})$

and $t_{12}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$

Case (viii): If $u_{30}<u_{10}<u_{20}<u_{40}$, $a_2<a_1<a_4<a_3$

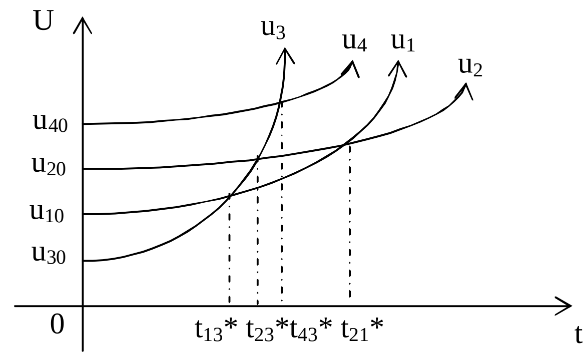


Fig. 4.8

In this case the predator (S_2) has the least natural birth rate. Initially the predator (S_2) dominates over the prey (S), host (S_3) of S_1 till the time instant t^*_{21}, t^*_{23} respectively and thereafter the dominance is reversed.

Also the prey (S_1) dominates over its host till the time instant t^*_{12} and thereafter the dominance is reversed. Similarly the host (S_4) of N_2 the dominates over the host (S_3) of S_1 till the time instant t^*_{43} the dominance gets reversed after.

Here

$$t_{21}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}}); t_{23}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{30}}{u_{20}})$$

$$t_{13}^* = \frac{1}{a_1 - a_3}\log(\frac{u_{30}}{u_{10}}); t_{43}^* = \frac{1}{a_3 - a_4}\log(\frac{u_{40}}{u_{30}})$$

<u>Case (ix):</u> If $u_{30}<u_{20}<u_{40}<u_{10}$, $a_1<a_4<a_3<a_2$

In this case the prey (S_1) has the least natural birth rate. Initially the prey (S_1) dominates over its host, predator (S_2), host (S_4) of S_2 till the time instant t^*_{13}, t^*_{12}, t^*_{14} respectively and there after the dominance is reversed. Also the host (S_4) of S_2 dominates over the predator (S_2) and the host (S_3)of N_1 till the times instant t^*_{42} and t^*_{43} and thereafter the dominance is reversed.

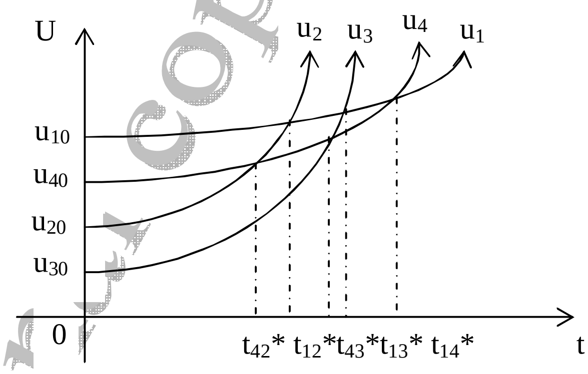


Fig. 4.9

Here

$$t_{13}^* = \frac{1}{a_1 - a_3}\log(\frac{u_{30}}{u_{10}}); t_{12}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}}); t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_4}{u_{10}})$$

and $$t_{42}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}}); t_{43}^* = \frac{1}{a_3 - a_4}\log(\frac{u_{40}}{u_{30}})$$

<u>Case (x):</u> If $u_{40}<u_{10}<u_{30}<u_{20}$, $a_1<a_2<a_4<a_3$

In this case the prey (S_1) has the least natural birth rate. Initially the prey(S_1) dominates over the host (S_4)of S_2 till the time instant t^*_{14} and thereafter the dominance is reversed.
Also the predator (S_2) dominates over its host, host (S_3)of S_1 till the time instants t^*_{24}, t^*_{23} respectively and thereafter the dominance is reversed.

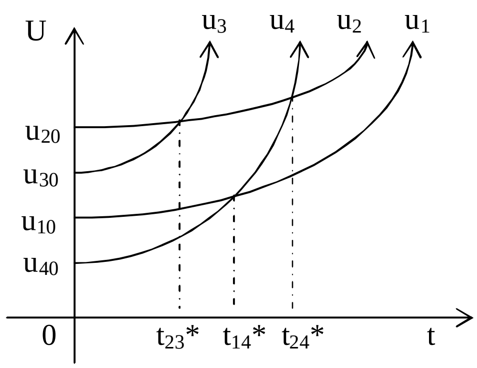


Fig. 4.10

Here $$t_{14}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{40}}{u_{10}}); t_{24}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}})$$

and $$t_{23}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{30}}{u_{20}})$$

Case (xi): If $u_{40}<u_{10}<u_{20}<u_{30},\ a_2<a_3<a_1<a_4$

In this case the predator (S2) has the least natural birth rate. Initially the predator (S_2) dominates over its host, prey(S_1) till the time instant t^*_{24}, t^*_{21} respectively and thereafter the dominance is reversed.

Also the host (S_3) of S_1 dominates over the prey(S_1), host (S_4) of S_2 till the time instant t^*_{31}, t^*_{34} respectively and thereafter the dominance is reversed. Similarly the prey(S_1) dominates over the host (S_4) of S_2 till the time instant t^*_{14} and the dominance gets reversed after.

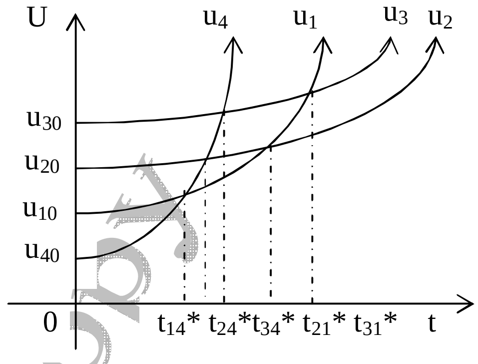


Fig. 4.11

Here

$$t_{24}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}}); t_{21}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$$

$$t_{31}^* = \frac{1}{a_1 - a_3}\log(\frac{u_{30}}{u_{10}}); t_{34}^* = \frac{1}{a_3 - a_4}\log(\frac{u_{40}}{u_{30}})$$

and $$t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}})$$

Case (xii): If $u_{40}<u_{20}<u_{30}<u_{10},\ a_4<a_1<a_2<a_3$

In this case the host (S_4) of S_2 has the least natural birth rate. Initially the prey(S_1) dominates over its host, predator (S_2) till the time instant t^*_{13}, t^*_{12} respectively and thereafter the dominance is reversed.

Here $$t_{13}^* = \frac{1}{a_1 - a_3}\log(\frac{u_{30}}{u_{10}}); t_{12}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$$

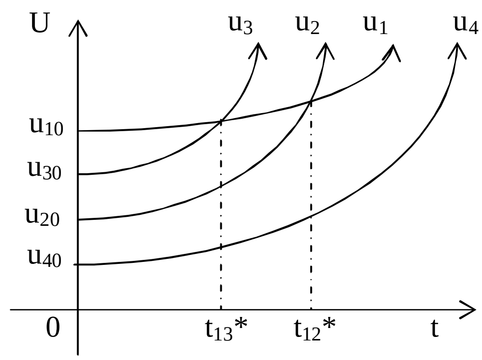


Fig. 4.12

5. Trajectories of Perturbations :

The trajectories in u_1-u_2, u_1-u_3, u_1-u_4, u_2-u_3, u_2-u_4, u_3-u_4 planes are

$$(\frac{u_1}{u_{10}})^{a_2} = (\frac{u_2}{u_{20}})^{a_1}, (\frac{u_1}{u_{10}})^{a_3} = (\frac{u_3}{u_{30}})^{a_1}, (\frac{u_1}{u_{10}})^{a_4} = (\frac{u_4}{u_{40}})^{a_1},$$

$$(\frac{u_2}{u_{20}})^{a_3} = (\frac{u_3}{u_{30}})^{a_2}, (\frac{u_2}{u_{20}})^{a_4} = (\frac{u_4}{u_{40}})^{a_2}, (\frac{u_3}{u_{30}})^{a_4} = (\frac{u_4}{u_{40}})^{a_3}$$ respectively and these are illustrated in the following figures from 5.1 to 5.6

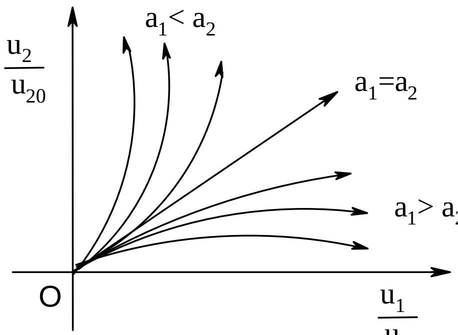


Fig 5.1

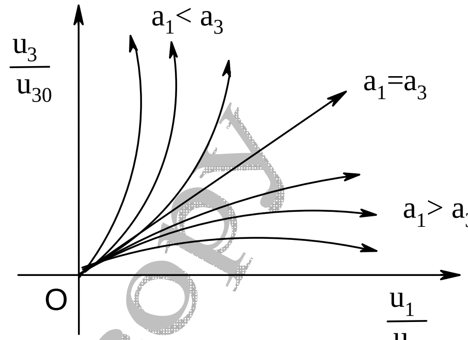


Fig 5.2

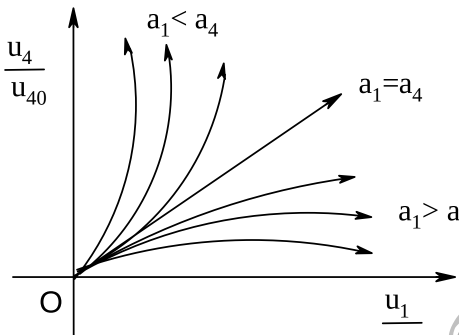


Fig 5.3

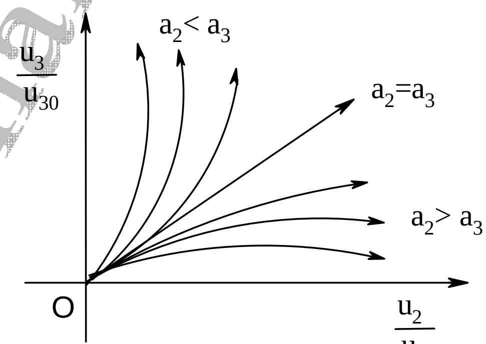


Fig 5.4

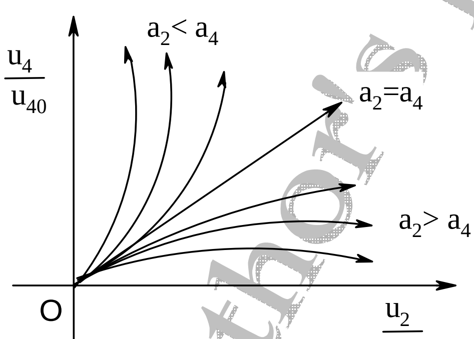


Fig 5.5

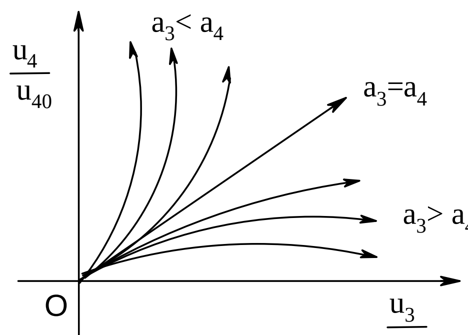


Fig 5.6

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