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EFFECTS OF MHD ON MOVING VERTICAL PLATE IN THE PRESENCE OF THERMAL RADIATION AND CHEMICAL REACTION OF FIRST ORDER

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Abstract

An exact solution of MHD stokes problem for the flow of an electrically conducting incompressible , viscous fluid past an impulsively started infinite vertical plate in the presence of uniform temperature and mass diffusion is presented here, taking into account the homogeneous chemical reaction of first order and the effect of thermal radiation. The temperature near the plate is made raised to T_w and the concentration level near the plate is also raised to C'_w. The fluid is considered is a gray, absorbing emitting radiation but a non-scattering medium. The dimensionless governing equations are solved using Laplace-Transform technique. The velocity profiles are studied for different physical parameters in the form of graphs. It is observed that the velocity decreases with increasing the value of the chemical reaction parameter or magnetic field parameter. But this effect is reversed with respect to time.

Key words :gray, radiation, magnetic field, chemical reaction, vertical plate.
Mathematics Subject Classification : 76R10

1. Introduction

The Effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre and Young [2] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. APelblat [1] studied analytical solution for mass with a chemical reaction of first order. Das *et al.* [4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment, Nuclear power plants,gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry are examples of such engineering applications. England and Emery[5] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar [7] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[6]. In all above studies, the stationary vertical plate is considered. Das {\em et al}[3] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique.

MHD plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar *et al* [8]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar {\em et al} [9]. The dimensionless governing equations were solved using Laplace transform technique.

It is proposed to study thermal radiation and chemical reaction effects on flow past an impulsively started infinite vertical plate with uniform temperature and mass diffusion in the presence of transverse applied magnetic field. The effect of velocity, temperature and concentration for different magnetic field parameter, chemical reaction, radiation parameter and time are studied graphically. The governing equations are solved by the Laplace transform technique and the solutions are in terms of exponential and complementary error function.

2. Governing Equations

Radiation effects on unsteady MHD flow past an impulsively started infinite vertical plate with variable temperature and mass diffusion, in the presence of chemical reaction of first order. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration. At time t'>0, the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0 in a fluid, in the presence of thermal radiation. At the same time,
the temperature of the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesqs' approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u' \tag{1}$$

$$\rho\, C_p\, \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l C' \tag{3}$$

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate if proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.
With the following initial and boundary conditions:

$$\begin{aligned} t' \le 0: &\quad u' = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad for\ all \quad y \\ t' > 0: &\quad u' = u_0, \quad T = T_w, \quad C' = C'_w \quad at \quad y = 0 \\ &\quad u' = 0, \quad T \to T_\infty, \quad C' \to C'_\infty \quad as \quad y \to \infty \end{aligned} \tag{4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\,\sigma\left(T_\infty^4 - T^4\right) \tag{5}$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\, C_p\,\frac{\partial T}{\partial t'} = k\,\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3\,(T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$\begin{aligned} &u = \frac{u'}{u_0},\ t = \frac{t' u_0^2}{\nu},\ Y = \frac{y u_0}{\nu},\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ &Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3},\ \omega = \frac{\omega'\nu}{u_0^2}, \\ &R = \frac{16a^*\nu^2\sigma T_\infty^3}{k u_0^2},\ Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D},\ M = \frac{\sigma B_0^2\nu}{\rho u_0^2},\ K = \frac{\nu K_l}{u_0^2} \end{aligned} \tag{8}$$

in equations (1) to (4), leads to

$$\frac{\partial u}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 u}{\partial Y^2} - M\,u \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\,\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - K\,C \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{aligned} &\quad u = 0, \quad \theta = 0, \quad C = 0, \quad for\ all \quad Y, t \le 0 \\ t > 0: &\quad u = 1, \quad \theta = 1, \quad C = 1, \quad at \quad Y = 0 \\ &\quad u \to 0, \quad \theta \to 0, \quad C \to 0 \quad as \quad Y \to \infty \end{aligned} \tag{12}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydromagnetic flow field in the presence of thermal radiation.

3. Method of solution

The dimensionless governing equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{at})\right] \quad (13)$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \quad (14)$$

$$\begin{aligned}
u = &\left(\frac{1}{2} + d + e\right)\left[\exp\left(2\eta\sqrt{Mt}\right) erfc\left(\eta + \sqrt{Mt}\right) + \exp\left(-2\eta\sqrt{Mt}\right) erfc\left(\eta - \sqrt{Mt}\right)\right] \\
&- d\exp(bt)\Big[\exp\left(-2\eta\sqrt{(M+b)t}\right) erfc\left(\eta - \sqrt{(M+b)t}\right) \\
&\qquad + \exp\left(2\eta\sqrt{(M+b)t}\right) erfc\left(\eta + \sqrt{(M+b)t}\right)\Big] \\
&- e\exp(ct)\Big[\exp\left(-2\eta\sqrt{(M+c)t}\right) erfc\left(\eta - \sqrt{(M+c)t}\right) \\
&\qquad + \exp\left(2\eta\sqrt{(M+c)t}\right) erfc\left(\eta + \sqrt{(M+c)t}\right)\Big] \\
&- d\left[\exp\left(2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right] \\
&- d\exp(bt)\Big[\exp\left(-2\eta\sqrt{\Pr(a+b)t}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{(a+b)t}\right) \\
&\qquad + \exp\left(2\eta\sqrt{\Pr(a+b)t}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{(a+b)t}\right)\Big] \\
&- e\left[\exp\left(2\eta\sqrt{KtSc}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{KtSc}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \\
&+ e\exp(ct)\Big[\exp\left(-2\eta\sqrt{Sc(K+c)t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{(K+c)t}\right) \\
&\qquad + \exp\left(2\eta\sqrt{Sc(K+c)t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{(K+c)t}\right)\Big]
\end{aligned} \quad (15)$$

where, $a = \frac{R}{Pr}, b = \frac{M-R}{Pr-1}, c = \frac{M-K\,Sc}{Sc-1}, d = \frac{Gr}{2b(1-Pr)}$ and $e = \frac{Gc}{2c(1-Sc)}$.
where $\eta = Y/2\sqrt{t}$ and erfc is called complementary error function.

4. Results and Dicussion

The numerical values of the velocity, temperature and concentration fields are computed for different parameters like Magnetic field parameter, chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number and Pr=0.71. The purpose of the calculations given here is to assess the effects of the parameters M, K, R, Gr, Gc and Sc upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

Figure 1 illustrates the effect of the velocity for different values of the reaction parameter (K=2,5,10), M=2, R=4, Gr=5=Gc, Sc=0.6, Pr=0.71 and t=0.2. The trend shows that the velocity increases with decreasing chemical reaction parameter. It is observed that the velocity suppress in the presence of chemical reaction. The velocity profiles for different thermal radiation parameter(R=3,5,10), Gr=Gc=5, K=2, M=2, Sc=0.6, Pr=0.7 and t=0.2 are presented in figure 2. It is clear that the velocity increases with decreasing thermal radiation parameter.

Figure 3 demonstrates the effect of the velocity profiles for different values of the magnetic field parameter (M=0,1,2),Sc=0.6, K=2, R=4, Gr=Gc=5, Pr=0.71 and time t=0.2. It is observed that the velocity increases with decreasing magnetic field parameter. The effect of velocity for different values of time (t=0.2,0.4,0.6), M=2, R=4, K=2, Gr=Gc=5, Sc=0.6 are presented in figure 4. for air (Pr=0.71) at time t=0.2. It is observed that the velocity increases with increasing values of time t.

5. Conclusion

Theoretical solution of thermal radiation effects on flow past an impulsively started infinite vertical plate with uniform plate temperature and mass diffusion in the presence of transverse applied magnetic field is considered. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of the velocity studied for different physical parameters like radiation parameter, magnetic field parameter, chemical reaction parameter and time are studied graphically. The study concludes that the velocity decreases with increasing chemical reaction parameter or magnetic field parameter. But the trend is just reversed with respect to time. It is also observed that the velocity increases with decreasing values of radiation parameter.

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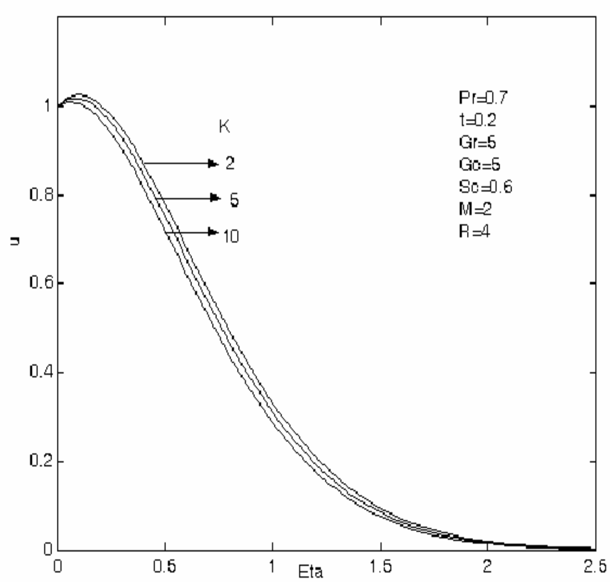

Fig.1. Velocity profiles for different K

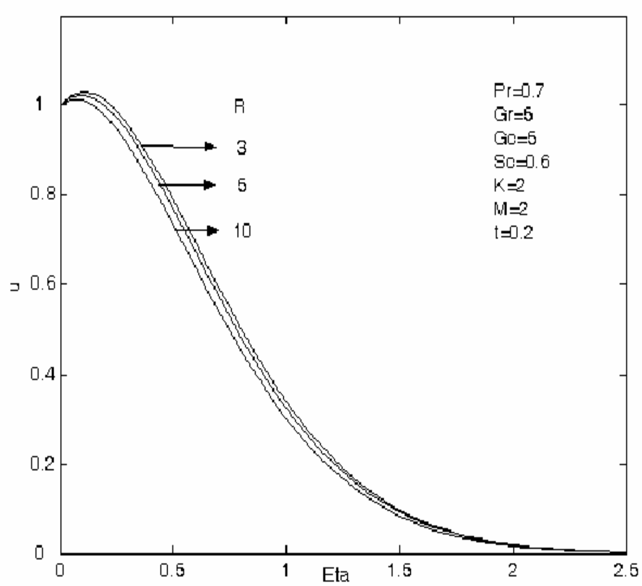

Fig.2.Velocity profiles for different R

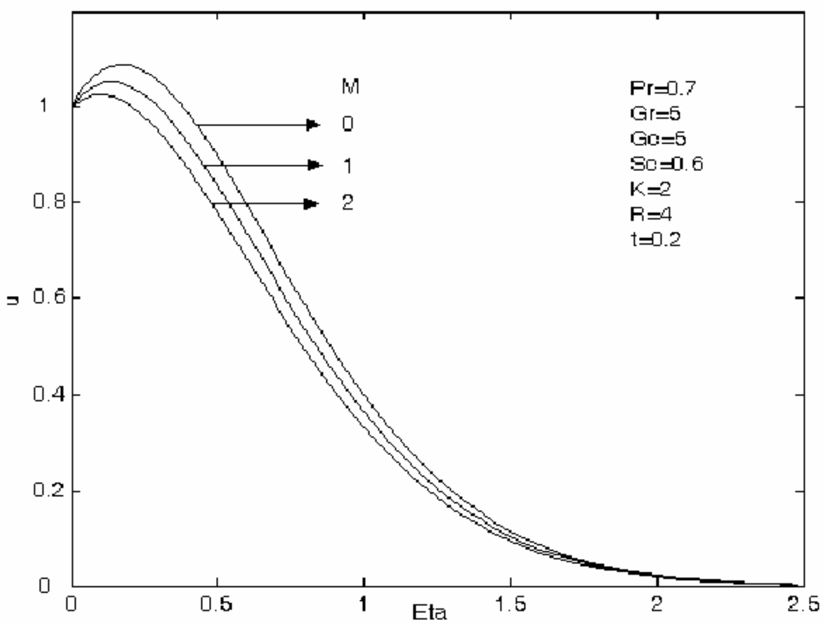

Fig.3.Velocity profiles for different M

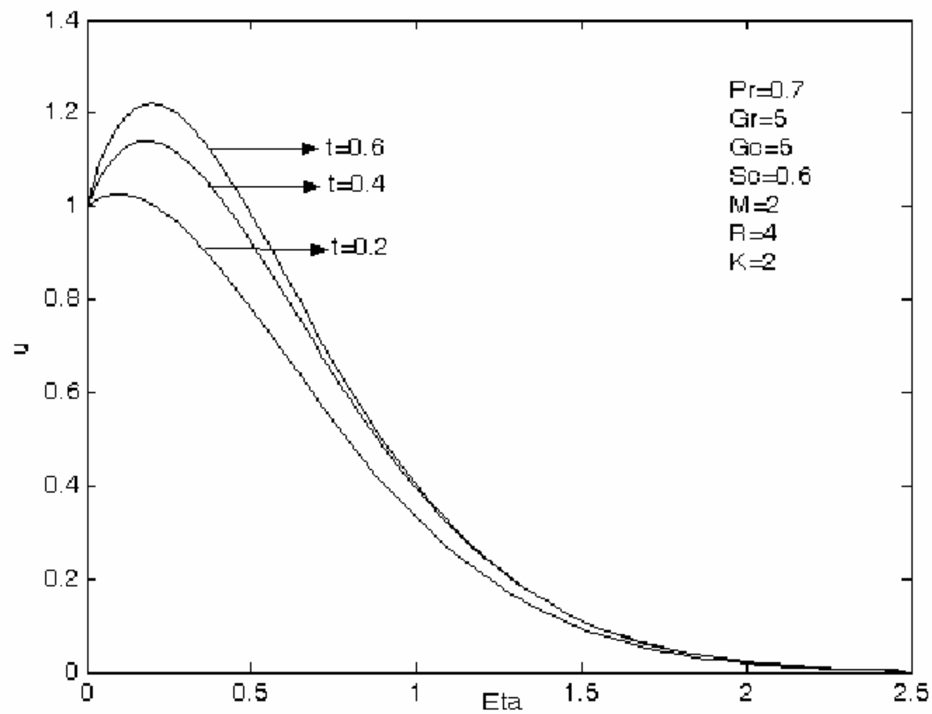

Fig.4. Velocity profiles for different t