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MHD Flow of Newtonian Fluid through Straight Circular Tube with Porous Lining

Dr.K.Shivashanker
Assistant Professor, Dept. of Mathematics, Kakatiya Institute of Technology & Science
Warangal – 506015, Telangana, INDIA. E-mail: sskache@gmial.com

Abstract: In the present paper the flow of an incompressible viscous fluid through a straight circular tube is considered. The innerside of the circular cylindrical tube is coated with porous lining of thickness δ, radius of the tube is taken to be 'b' and the clear region (excluding the porous lining) will be of radius 'a'. The problem becomes the two-layered flow one in porous region and other in clear region. This has analogy with blood flow, which considered being two layered. A Magnetic field is applied in the clear region perpendicular to the flow of the fluid. The porous region is free from magnetic field. The effect of magnetic field, porous lining on the flow of the fluid is investigated.

Key words: *Incompressible viscous fluid, Two layered, Permeability, Porous lining.*

1. Introduction:

The study of flow through porous medium assumed importance because of its interesting applications in the diverse field of science, Engineering and Technology. The practical applications are in the percolation of water through soil., extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter-disciplinary fields such as bio-medical engineering etc. The lung alveolar is an example that finds application in an animal body. The classical Darcy's law Musakat [4] states that the pressure gradient pushes the fluid against the body forces excreted by the medium which can be expressed as $\vec{V} = -\left(\frac{K}{\mu}\right)\nabla P$.

The law gives good results in the situations when the flow is uni-directional or the flow is at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to it's viscous nature and produces distortions in the velocity field. In the case of highly porous media such as fiber glass, papus of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by the Beavers and Joseph [1], Saffman [9] and others. A generalized Darcy's law proposed by Brinkman [2] is given by

$$0 = -\nabla P - \left(\frac{\mu}{k}\right)\vec{V} + \mu\nabla^2\vec{V}$$

Where μ and K are coefficients of viscosity of the fluid and permeability of the porous medium.

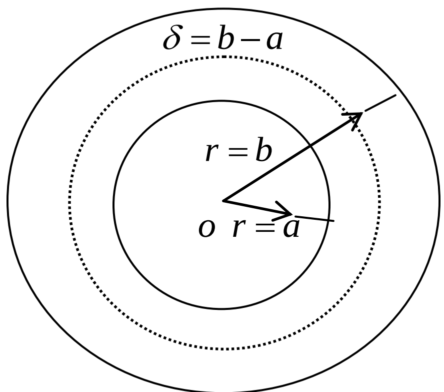
The generalized equation of momentum for the flow through the porous medium is

$$\rho\left[\frac{\partial\vec{V}}{\partial t} + \left(\vec{V}.\nabla\right)\vec{V}\right] = -\nabla P + \mu\nabla^2\vec{V} - \left(\frac{\mu}{K}\right)\vec{V}$$

The classical Darcy's law helps in studying flows through porous medium. In the case of highly porous medium such as papus of dandelion etc. The Darcy's law fails to explain the flow near the surface in the absence of pressure gradient. The non-Darcian approach is employed to study the problem of flow through highly porous medium by several investigators. Narasimhacharyulu and Pattabhi Ramacharyulu [5, 6] Narsimhacharyulu [7] and Singh [8] etc. Studied the flow employing Brinkman law [2] for the flow through highly porous medium.

Earlier the flow of Newtonian fluid between two parallel plates with porous lining is examined by V.Narsimhacharyulu & VenkatrRaman [9]. In the present study, the flow of an incompressible viscous fluid is considered through a straight circular tube of infinite length. The tube is coated with porous lining of thickness δ. The flow will be two layered one region is porous region other region is clear region. Fluid flows through the tube in two layers. The magnetic field is applied in the clear region, the porous region is made free from magnetic field. Such type of flows find applications in the inter-disciplinary fields such as bio-medical engineering etc. the flow of the blood is one such application. The blood may be represented as a Newtonian fluid and the flow of the blood is two layered light foot [3] and Shukla et al [10]. The effect of the coefficient of the porous medium and the effect of the thickness of the porous lining on the physical quantities at the fluid flow are discussed.

2. Formulation of the problem:



Consider the flow of an incompressible viscous fluid through a infinite straight circular tube of radius ‘b’. The inner region of the tube is coated with porous lining of thickness δ. There exists two regions, one clear region of radius $r = b - \delta = a$ and porous region of radius $\delta = b - a$.

A magnetic field is applied in the clear region perpendicular to the flow of the fluid. The porous region is free from magnetic field. (Such situation occurs in practical problems of blood flow where plasma region will be away from magnetic field).

A cylindrical polar coordinate system (r, θ, z) is taken such that z is the axis of the tube, r radius of the tube, θ is azimuthal angle. The velocity is independent of z and θ because of infinite length of the tube and symmetric of the flow. Therefore velocity depends on $'r'$ only.

The velocities of the fluid in the two regions is given by

$\left[0, 0, w_c(r)\right]$ in clear region, $0 < r < a$ (2.1)

and $\left[0, 0, w_p(r)\right]$ in porous region, $a < r < b$ (2.2)

Where w_c is velocity in clear medium, w_p is velocity in the porous medium.

The equation of continuity

$$\nabla . \vec{V} = 0 \tag{2.3}$$

is satisfied by the choice of velocity.

The constant pressure gradient acting in the clear region is assumed to induce flow in porous region.

$\frac{\P p}{\P r} = G$ in the clear region (2.4)

$\frac{\P p}{\P r} = 0$ in the porous region (2.5)

The equation of motion in the two regions

$$0 = -\frac{\partial p}{\partial z} + \mu \nabla^2 w_c ;\ o < r < a \tag{2.6}$$

$$0 = \mu \nabla^2 w_p - \frac{\mu}{k} w_p ;\ a < r < b \tag{2.7}$$

Together with the Boundary conditions

$$w_c = \text{finite at } r = 0 \tag{2.8}$$

$$w_c = w_p \text{ at } r = a \tag{2.9}$$

$$\frac{\partial w_c}{\partial r} = \frac{\partial w_p}{\partial r} \text{ at } r = a \tag{2.10}$$

$$w_p = 0 \text{ at } r = b \tag{2.11}$$

The equation of motion in the two regions will be

$$\frac{d^2 w_c}{dr^2} + \frac{1}{r}\frac{dw_c}{dr} = \frac{-G}{m} - M^2 w_c \tag{2.12}$$

Where $M^2 = \frac{m s_e^2 b_0^2}{m}$

$$\frac{d^2 w_p}{dr^2} + \frac{1}{r}\frac{dw_p}{dr} = -\frac{1}{k} w_p \tag{2.13}$$

Where the velocities in the two regions will be given by

$$w_c\left(r\right)=A_1I_0\left(Mr\right)+B_1K_0\left(Mr\right)-\frac{Gr^2}{4m} \tag{2.14}$$

$$w_p\left(r\right)=A_2I_0\left(\frac{r}{\sqrt{k}}\right)+B_2K_0\left(\frac{r}{\sqrt{k}}\right) \tag{2.15}$$

Where A_1, B_1, A_2, B_2 are constants.

Applying boundary conditions

We obtain velocity field, in the clear zone w_c

$$w_c\left(r\right)=\frac{G\left(a^2-r^2\right)}{4m}+\frac{Ga\sqrt{K}}{2m}\left[\frac{K_0\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)-I_0\left(\frac{a}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right)+K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)}\right]$$

$$+\frac{Ga\sqrt{M}}{2m}\left[\frac{K_0(aM)I_0(bM)-I_0(aM)K_0(bM)}{K_0(bM)I_1(aM)+K_1(aM)I_0(bM)}\right] \tag{2.16}$$

Velocity in the porous region w_p

$$w_p\left(r\right)=\frac{Ga\sqrt{K}}{2\mu}\left[\frac{K_0\left(\frac{r}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)-I_0\left(\frac{r}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)+K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right)}\right] \tag{2.17}$$

Flow rate in clear zone

$$Q_c=2\pi\int_0^a rw_c\,dr$$

$$Q_c = \frac{Gpa^4}{8m} + \frac{Gpa^3\sqrt{K}}{2m}\left[\frac{K_0\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right) - I_0\left(\frac{a}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right) + K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)}\right]$$

$$+\frac{Gpa^3\sqrt{M}}{2m}\left[\frac{K_0(aM)I_0(bM) - I_0(aM)K_0(bM)}{K_0(bM)I_1(aM) + K_1(aM)I_0(bM)}\right] \tag{2.18}$$

Flow rate in porous region

$$Q_p = 2\pi\int_a^b rw_p \; dr$$

$$Q_p = \frac{G\pi a^2 k}{\mu}\left[1 - \frac{\sqrt{k}}{a}\frac{1}{\left\{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right) + K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)\right\}}\right] \tag{2.19}$$

(from Watson [11] page 80, we have $I_0(z)k_1(z) + I_1(z)k_0(z) = \frac{1}{Z}$)

Deductions :

Flow of the fluid in the absence of magnetic field

Velocity in the clear region w_c

$$w_c(r) = \frac{G\left(a^2 - r^2\right)}{4\mu} + \frac{Ga\sqrt{K}}{2\mu}\left[\frac{K_0\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right) - I_0\left(\frac{a}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right) + K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)}\right] \tag{2.20}$$

Velocity in the porous region w_p

$$w_p(r)=\frac{Ga\sqrt{K}}{2\mu}\left[\frac{K_0\left(\frac{r}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)-I_0\left(\frac{r}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)+K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right)}\right] \tag{2.21}$$

Flow rate in clear zone

$$Q_c=2\pi\int_0^a rw_c\ dr$$

$$Q_c=\frac{G\pi a^4}{8\mu}+\frac{G\pi a^3\sqrt{K}}{2\mu}\left[\frac{K_0\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)-I_0\left(\frac{a}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right)+K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)}\right] \tag{2.22}$$

Flow rate in porous region

$$Q_p=2\pi\int_a^b rw_p\ dr$$

$$Q_p=\frac{G\pi a^2 k}{\mu}\left[1-\frac{\sqrt{k}}{a}\frac{1}{\left\{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right)+K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)\right\}}\right] \tag{2.23}$$

(from Watson [11] page 80, we have $I_0(z)k_1(z)+I_1(z)k_0(z)=\frac{1}{Z}$)

CASE – A :

If the permeability is large.

$$w_c(r)=\frac{G\left(a^2-r^2\right)}{4\mu}+\frac{Ga^2}{2\mu}\left(1+\frac{a^2}{2k}\right)\log\left(\frac{b}{a}\right)$$

$$=\frac{a^2G}{4\mu}\left[1-\left(\frac{r}{a}\right)^2+2\left(1+\frac{a^2}{2k}\right)\log\left(1+\frac{\delta}{a}\right)\right] \tag{2.24}$$

$$w_p(r)=\frac{Gr}{2\mu}\left(1+\frac{r^2}{2k}\right)\log\left(\frac{b}{r}\right)$$

$$= \frac{Gr}{2\mu}\left(1+\frac{r^2}{2k}\right)\log\left(1+\frac{\delta}{a}\right) \tag{2.25}$$

$$Q_c = \frac{G\pi a^4}{8\mu} + \frac{G\pi a^4}{4\mu}\left[2+\frac{a^2}{k}\right]\log\left(\frac{b}{a}\right)$$

$$= \frac{G\pi a^4}{8\mu}\left[1+2\left(2+\frac{a^2}{k}\right)\log\left(1+\frac{\delta}{a}\right)\right] \tag{2.26}$$

$$Q_p = \frac{\pi G}{9\mu}\left[\left(b^3-a^3\right)-3a^3\log\left(\frac{b}{a}\right)\right] + \frac{\pi G}{50\mu k}\left[\left(b^5-a^5\right)-5a^5\log\left(\frac{b}{a}\right)\right]$$

$$= \frac{\pi b a^3}{\mu}\left\{\frac{a^2}{50k}\left(\left(1+\frac{\delta}{a}\right)^5-\log\left(1+\frac{\delta}{a}\right)^5-1\right)+\frac{1}{9}\left(\left(1+\frac{\delta}{a}\right)^3-\log\left(1+\frac{\delta}{a}\right)^3-1\right)\right\} \tag{2.27}$$

CASE – B :

If the permeability is very small.

$$w_c(r) = \frac{G\left(a^2-r^2\right)}{4\mu} + \frac{32}{67}\frac{Gak}{\mu}\frac{\sinh\left(\frac{b-a}{\sqrt{k}}\right)}{\cosh\left(\frac{a-b}{\sqrt{k}}\right)} \tag{2.28}$$

$$w_p(r) = \frac{32Ga^{3/2}\sqrt{k}}{67\mu}\frac{\sinh\left(\frac{b-r}{\sqrt{k}}\right)}{\cosh\left(\frac{a-b}{\sqrt{k}}\right)} \tag{2.29}$$

$$Q_c = \frac{G\pi a^4}{8\mu} + \frac{32}{67}G\pi a^3 k \tag{2.30}$$

$$Q_p = \frac{64G\pi a^{3/2}k}{67\mu}\left[a+k\tanh\left(\frac{b-a}{\sqrt{k}}\right)-b\sec h\left(\frac{b-a}{\sqrt{k}}\right)\right] \tag{2.31}$$

Results and Discussions:

The flow of Newtonian fluid through a straight circular tube, the inner side of which is having porous lining of the thickness δ is examined. The fluid flow becomes two layered flow one in porous region and other in clear region. The magnetic field is applied in the clear region, the porous region is made free from the magnetic field. Such type of flows find applications in the inter-disciplinary fields such as bio-medical engineering etc. This has analogy with blood flow which is considered to be two layered.

From Fig.1 it is observed that as thickness of the porous lining (δ) increases the velocity (W_c) in the clear region is increasing. Therefore the effect of the porous lining is to increase the velocity of the fluid in clear region. The velocity profile of (W_c) becomes more parabolic with increasing δ.

From Fig.2, it is observed that as thickness of the porous lining (δ) increases the velocity (W_p) in the porous region is increasing.

From Fig.3, it is observed that as thickness of the porous lining (δ) increases, the flow rate in the clear region $(Q_c)$is increasing. Further it is observed that as permeability of the porous lining increases flow rate in the clear region (Q_c) is decreasing.

From Fig.4, it is observed that as thickness of the porous lining (δ) increases, the flow rate in the porous region $(Q_p)$is decreasing. Further it is observed that as permeability of the porous lining increases flow rate in the porous region $(Q_p)$is decreasing.When the fluid is in clear region, and flows towards porous region then velocity profile of $(W_c)$is decreasing. Further it is observed that when the fluid is in porous region and flows towards clear region then velocity profile of (W_p) is increasing.

Increase in magnetic parameter effects the velocity in two regions.

As magnetic parameter increases the velocity of the fluid in both regions is increasing.

Velocity Profile of w_c for different δ

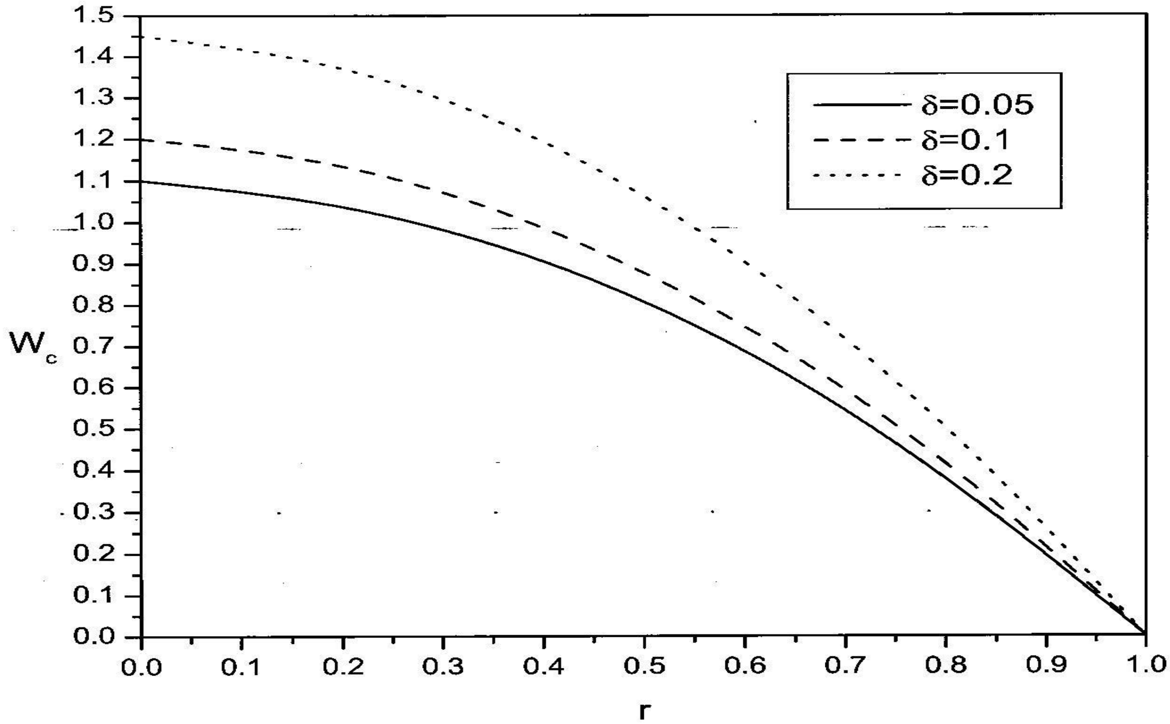


Fig. 1

Velocity Profile of w_p for different δ

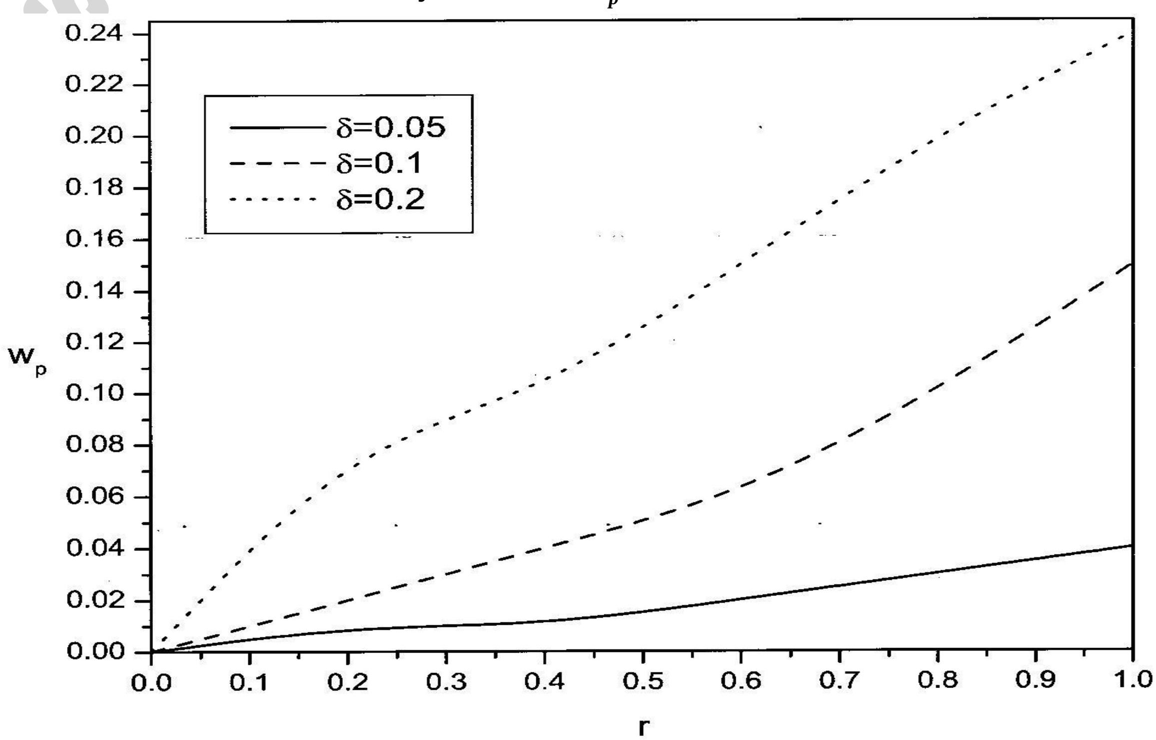


Fig. 2

Flow rate Q_c against thickness of porous lining δ

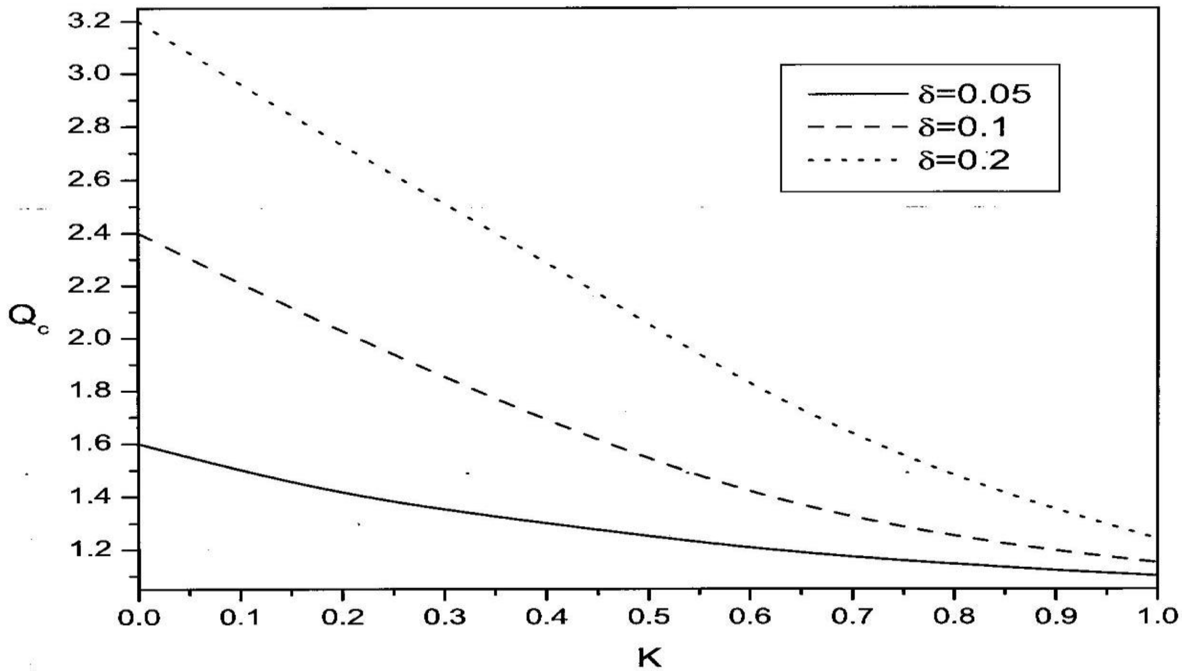


Fig. 3

Flow rate Q_p against thickness of porous lining δ

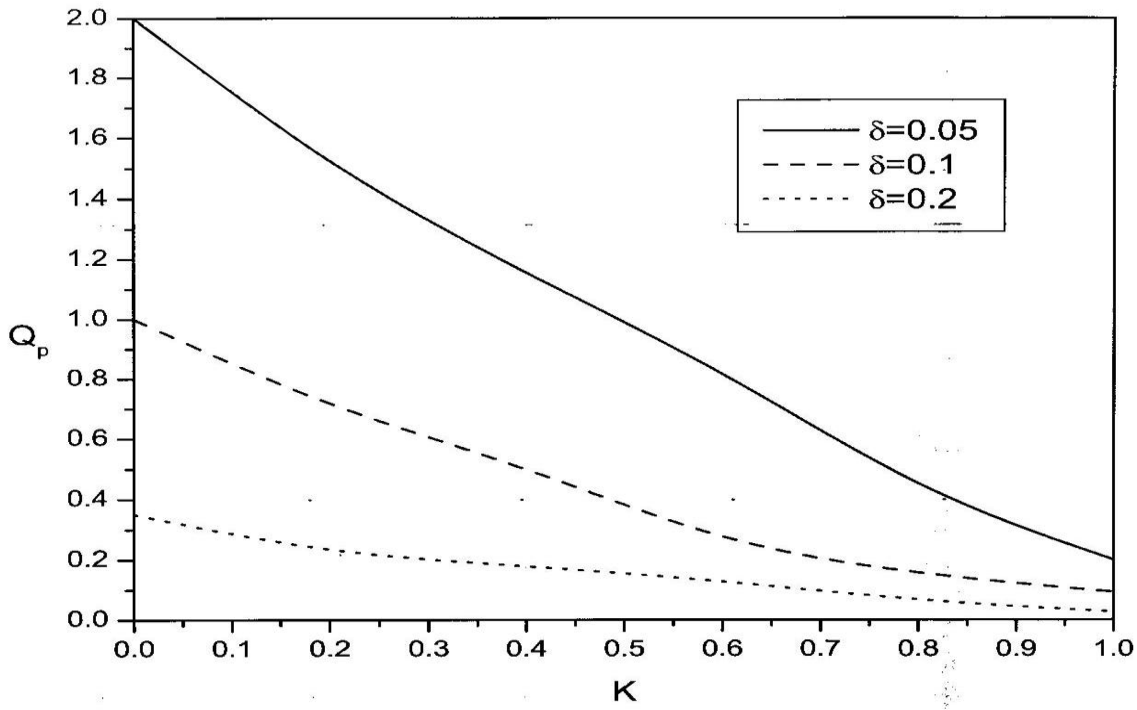


Fig. 4

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