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A numerical analysis of MHD mixed convective-radiative flow and heat transfer of couple stress fluid in a porous expanding or contracting walls with convective boundary condition

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Abstract

This article deals with the effects of radiation on unsteady mixed convection flow of couple stress fluid in a porous medium with expanding or contracting walls. We assume symmetric suction or injection all along the uniformly expanding or contracting walls maintained at different but constant temperatures. The governing equations are reduced to non linear ordinary differential equations using similarity transformations, then solved numerically by the quasilinearization technique. The graphs for velocity components and temperature distribution are presented and discussed in detail for different values of the fluid and geometric parameters.

Keywords: MHD; Couple stress fluid; Mixed convection; Radiation; Convective boundary condition.

1. Introduction

There is an increasing interest in the analysis of flow of fluids through porous walls. The relevance of this study is in fields of engineering and science including petroleum industries, ground water transportation, chemical reactions in surface catalysis, coalescence, chromatography, adsorption, soil absorption, food preservation, blood flow etc,. Berman (1953) analyzed the laminar flow in channels with porous walls. Later the same problem for high Reynolds numbers was extended by Sellars (1955). Terril and Shrestha (1965) investigated laminar flow through parallel and uniformly porous walls of different permeability. Wang et al. (2001) investigated the flow distribution and pressure drop in a channel with porous walls theoretically. Aydin and Kaya (2005) studied the laminar boundary layer flow over a porous flat plate with injection or suction. Stokes (1966) discussed the simplest generalization of the classical viscous fluid theory that sustains couple stress and body couples as couple stress fluid theory. The characterizing feature of such fluid is that the stress tensor is not symmetric and classical Newtonian theory fails to predict their behavior. Laminar flow of a non Newtonian fluid in channels with wall suction or injection has been studied by Kamish (2006). Reddy et al. (2013) investigated analytically the asymmetric laminar flow between expanding or contracting walls with chemical reaction. Xin-Hui et al. (2011) analyzed the flow of viscoelastic fluid through a porous channel with expanding or contracting walls. Devakar and Iyengar (2008) studied the strokes problem for an incompressible couple stress fluid and solved analytically using Laplace transforms. Viscous flow inside an impermeable tube of contracting cross section has been examined by Uchida and Akoi (1977). Srinivasacharya et al. (2009) studied the flow and heat transfer of couple stress fluid in a porous channel with expanding or contracting wall in which the effect of various parameters on velocity and temperature has been studied. The effects of Hall and ion slip on the steady incompressible MHD flow of a micropolar fluid in rectangular duct considered by Srinivasacharya and Mekonnen (2008). Umavathi et al. (2009) studied the flow of immiscible viscous and couple stress fluid through a vertical channel. MHD poiseuille flow in porous medium has been studied by Prasad and Prasad (2012). Mahmoud and Meghed (2013) considered an incompressible laminar flow of mixed convective and radiative non-Newtonian fluid over a permeable vertical plate embedded in a porous medium. Vedavathi et al. (2015) has been investigated an unsteady laminar convective-radiative flow past a vertical plate. The mixed convection flow of micropolar fluid over a vertical plate with chemical reaction was examined by Pal and Talukdar (2012) and an analytical approximate solution obtained by using the perturbation method. An incompressible laminar MHD nanofluid over a stretching sheet with convective boundary condition has been examined by Kalidas et al.(2015) and obtained a numerical solution by Runge–Kutta–Fehlberg method. Sheikh Irfanullah Khan et al.(2015) has been investigated the flow and temperature distribution over a static flat plate and a moving flat plate with convective boundary condition and the problem is analysed numerically by sixth order Runge–Kutta method. Xin-hui et al. (2010) considered unsteady incompressible laminar flow of a viscous fluid in a porous channel with expanding or contracting walls in the presence of transverse magnetic field and the reduced governing equations are solved by perturbation technique. Bujurke and Pai (1998) investigated an unsteady flow of viscous fluid in a contracting or expanding pipe and the problem was solved using series method. Majdalani et al. (2002) studied the two dimensional laminar incompressible viscous flow with expanding or contracting walls at weak permeability and solved in both numerically and analytically. An analytical solution for the magnetohydrodynamic pulsatile flow through a porous medium is examined by Kumar and Prasad (2014). Bhatnagar et al. (1994) studied the steady incompressible flow of a viscoelastic fluid in a porous cylindrical annulus and obtained the numerical solution by using the quasilinearization technique. The convective viscous fluid flow in a vertical channel investigated by Huang (1978). Hymavathi and Shankar (2009) applied the quasilinearization method to solve the visco-elastic fluid flow and heat transfer under the influence of magnetic field.

In this paper, we considered the radiation effect on an incompressible MHD mixed convective heat transfer of couple stress fluid flow through porous expanding or contracting walss with convective boundary condition. The flow field equations are solved numerically using the quasilinearization method and the effects of various fluid and geometric parameters are presented in the form of graphs.

Nomenclature

t	Time
a	Distance between parallel plates
V_1	Suction/injection velocity at the lower plate
Pr	Prandtl number, $\frac{\mu c}{k}$
h	Coefficient of convection
Re	Reynolds number, $\frac{\rho V_1 a}{\mu}$
$\overline{J}$	Current density
$\overline{B}$	Total magnetic field

$\bar{b}$	Induced magnetic field
B_0	Magnetic flux density
$\bar{E}$	Electric field
Ha	Hartmann number, $B_0 a \sqrt{\frac{\sigma}{\mu}}$.
D^{-1}	Inverse Darcy parameter, $\frac{a^2}{k_1}$
p	Fluid pressure
$\bar{Q}$	Velocity vector
c	Specific heat at constant temperature
E	Eckert number, $\frac{\mu V_1}{\rho ac(T_2 - T_1)}$
k	Thermal conductivity
k_1	Permeability Parameter
u	Velocity component in x-direction
v	Velocity component in y-direction
T	Temperature
T_1	Temperature of the lower plate
T_2	Temperature of the upper plate
T^*	Dimensionless temperature, $\frac{T - T_1}{(T_2 - T_1)}$
D	Rate of deformation tensor
Gr	Thermal Grashof number, $\frac{\rho g \beta_T (T_2 - T_1) a^3}{\upsilon^2}$
σ_1	Stefan-Boltzman constant
k_3	Mean absorption coefficient

Rd	Radiation parameter, $\frac{16\sigma_1 T_1^3}{3k_3 k}$
T_m	Mean temperature
k_T	Thermal-diffusion ratio
c_s	Concentration susceptibility
$\hat{i}, \hat{j}, \hat{k}$	Unit vectors

Greek Letters

λ	Dimensionless y coordinate, $\frac{y}{a}$
ζ	Dimensionless axial variable, $\frac{x}{a}$
ρ	Fluid density
μ	Fluid viscosity
μ'	Magnetic permeability
σ	Electric conductivity
α	Couple stress parameter, $\sqrt{\frac{\eta}{\mu a^2}}$
β_T	Coefficient of thermal expansion
β	Wall expansion ratio, $\frac{a\dot{a}}{\upsilon}$
γ	Convection parameter, $\frac{k\,a}{h}$

2. Mathematical formulation of the problem

Consider an incompressible couple stress fluid flow through expanding or contracting walls with large aspect ratio of height 'a'. Hence, their separation is a function of time a(t). The lower and upper walls are assumed to have equal permeability. Assume that the fluid is injected or aspirated uniformly and orthogonally through the channel walls at an absolute velocity V_1. One end of the rectangular channel (x = 0) is closed by a solid membrane that is allowed to stretch with channel expansions or contractions. At the other end, the channel is fully open. The influence of the opening at this end can be neglected by assuming semi-infinite length despite of its finite body length. The region inside the parallel walls is subjected to a constant external magnetic field of strength B_0, perpendicular to both the walls. The induced magnetic and electric fields produced by the motion of the electrically conducting fluid are negligible and no external electric field is applied.

The flow of an incompressible couple stress fluid in the absence of body force and body couple, in the presence of dissipation and Joule heating is governed by the following equations,

$$\nabla.\bar{q} = 0 \tag{1}$$

$$\rho\left[\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{q}\right] = -\nabla P + \mu\nabla^2\bar{q} - \eta\nabla^4\bar{q} - \frac{\mu}{k_1}\bar{q} + \bar{J}\times\bar{B} + \overline{F_B} \tag{2}$$

$$\rho c\left[\frac{\partial T}{\partial t} + (\bar{q}.\nabla)T\right] = 2\mu D:D + 4\eta(\nabla\bar{\omega}:\nabla\bar{\omega}^T) + 4\eta'(\nabla\bar{\omega}:\nabla\bar{\omega}) + \frac{\bar{J}^2}{\sigma} + \frac{\mu}{k_1}\bar{q}^2 - \nabla.q_r \tag{3}$$

The assumption of large aspect ratio (the height is smaller than the width) enables us to treat the problem as a case of two-dimensional flow. We choose the velocity vector as $\bar{q} = u\hat{i} + v\hat{j}$.

The boundary conditions imposed on the velocity profile and temperature are

u=0, v=V_1, $\nabla\times\bar{q}$=0, T=T_2 at y=a(t)

u=0, v=0, $\nabla\times\bar{q}$=0, $-k\dfrac{\partial T}{\partial y} = h(T-T_1)$, at y=0 (4)

Where $\overline{F_B}$ is the buoyancy force and it is defined as $(\rho g\beta_T(T-T_1))\hat{i}$.

Using Rosseland approximation, the radiative heat flux q_r is defined by

$$q_r = -\frac{4\sigma_1}{3k_3}\nabla T^4 \tag{5}$$

Where σ_1 and k_3 are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. The Taylor's series expansion of T^4 about T_1 is

$$T^4 = T_1^4 + 4T_1^3 (T - T_1) + 6T_1^2 (T - T_1)^2 + ... \tag{6}$$

Neglecting the higher order terms beyond the first degree in T-T_1, we have

$$T^4 \cong T_1^4 + 4T_1^3 (T - T_1) \tag{7}$$

The force stress tensor τ and the couple stress tensor M that arises in the theory of couple stress fluids are given by

$$\tau = (-p + \lambda_1 \text{ div } \bar{Q})\, I + \mu\,[\text{ grad } \bar{Q} + (\text{grad } \bar{Q})^T] + ½\, I \text{ x } [\text{div } M + \rho C] \tag{8}$$

and

$$M = mI + 2\eta \text{ grad(curl } \bar{Q}) + 2\eta^{/} (\text{grad (curl } \bar{Q}))^T \tag{9}$$

Where m is 1/3rd Trace of M and ρC is the body couple tensor. The quantity λ_1 is the material constant and $\eta^{/}$ is the constant associated with couple stresses. The dimensions of the material constant λ_1 is that of viscosity where as the dimensions of η and $\eta^{/}$ are those of momentum. These material constants are considered by the inequalities,

$$\mu \geq 0, 3\lambda_1 + 2\mu \geq 0, \eta \geq 0, \eta^{/} \leq \eta \tag{10}$$

Neglecting the displacement currents, the Maxwell equations and the generalized Ohm's law are

$$\nabla . \bar{B} = 0, \qquad \nabla \times \bar{B} = \mu' \bar{J}, \qquad \nabla \times \bar{E} = \frac{\partial \bar{B}}{\partial t},$$

$$\bar{J} = \sigma(\bar{E} + \bar{q} \times \bar{B}) \tag{11}$$

Where $\bar{B} = B_0 \hat{k} + \bar{b}$, $\bar{b}$ is induced magnetic field and μ′ is magnetic permeability.

Let the induced magnetic field be negligible compared to the applied magnetic field so that magnetic Reynolds number is small, the electric field is zero and magnetic permeability is constant throughout the flow field.

At the wall, it is assumed that the flow velocity V_1 is independent of position. Following Berman (1953),

$$u = \frac{\upsilon x}{a^2} F^{/}(\lambda, t), \; v = -\frac{\upsilon}{a} F(\lambda, t) \tag{12}$$

where λ = y/a .

Substituting Eq. (12) in to Eq. (2) and eliminating pressure from the resulting equations, we get

$$\alpha^2 f^{V/} - (f^{/V} + 3\beta f^{//} + \beta\lambda f^{///} - \text{Re}\, f^{/} f^{//} + \text{Re}\, f f^{///} - (Ha^2 + D^{-1}) f^{//}$$

$$-\frac{Ec\ Gr}{\zeta\, \text{Re}}\left(\phi_1^{/} + \zeta^2 \phi_1^{/}\right)) = 0 \tag{13}$$

Srinivasacharya et al. (2009) suggests that the form of temperature may be taken as

$$T = T_1 + \frac{\upsilon V_1}{\text{ac}}\left[\phi_1(\lambda) + \frac{x^2}{a^2}\phi_2(\lambda)\right] \tag{14}$$

Substituting (14) in (3), and equating the coefficients of $\frac{x^2}{a^2}$ and the terms without $\frac{x^2}{a^2}$ on both sides of the equation, we get

$$\phi_1^{//} = -\frac{\text{Pr}}{(1+Rd)}(\beta\phi_1 + \beta\lambda\phi_1^{/}\ \text{Re}\, f\phi_1^{/}) - \frac{\text{Re}\,\text{Pr}}{(1+Rd)}(4 f^{/2} + \alpha^2 f^{//2} + (Ha^2 + D^{-1}) f^2)$$
$$-2\phi_2 \tag{15}$$

$$\phi_2^{//} = \frac{\text{Re}\,\text{Pr}}{(1+Rd)}\left(-\frac{3\beta\phi_2}{\text{Re}} - \frac{\beta\lambda\phi_2^{/}}{\text{Re}} + 2 f^{/}\phi_2 - f\phi_2^{/} - f^{//2} - \alpha^2 f^{///2} - (Ha^2 + D^{-1}) f^{/2}\right) = 0 \tag{16}$$

Where prime denotes the differentiation with respect to λ and $f(\lambda) = \frac{F}{\text{Re}}$.

The dimensionless form of temperature and concentration from (14) can be written as

$$T = \frac{T - T_1}{(T_2 - T_1)} = Ec(\phi_1 + \zeta^2\,\phi_2) \tag{17}$$

Where $Ec = \dfrac{\mu V_1}{\rho ac(T_2 - T_1)}$ is the Eckert number.

The boundary conditions (13) in terms of f, ϕ_1 *and* ϕ_2 are

$$f(0)=0, \quad f(1)=1,$$

$$f^{/}(0)=0, \quad f^{/}(1)=0,$$

$$f^{//}(0)=0, \quad f^{//}(1)=0,$$

$$\phi_1^{/}(0)=-\gamma\phi_1(0), \quad \phi_1(1)=1/Ec$$

$$\phi_2^{/}(0)=-\gamma\phi_2(0), \quad \phi_2(1)=0 \tag{18}$$

3. Solution of the problem

The nonlinear equations (13), (15) and (16) are converted into the following system of first order differential equations by the substitution

$$y_1 = f,\ y_2 = f^{/},\ y_3 = f^{//},\ y_4 = f^{///},\ y_5 = f^{/V}, y_6 = f^{V}, y_7 = \phi_1, y_8 = \phi_1^{/}\ y_9 = \phi_2,\ y_{10} = \phi_2^{/}.$$

Then we get following system of non linear differential equations

$$\frac{dy_1}{d\lambda} = y_2, \frac{dy_2}{d\lambda} = y_3, \frac{dy_3}{d\lambda} = y_4, \frac{dy_6}{d\lambda} = y_5, \frac{dy_5}{d\lambda} = y_6,$$

$$\frac{dy_6}{d\lambda} = \frac{1}{\alpha^2}(y_5 + 3\beta y_3 + \beta\lambda y_4 - \mathrm{Re}\, y_2 y_3 + \mathrm{Re}\, y_1 y_4 - (Ha^2 + D^{-1}) y_3 - \frac{Ec\ Gr}{\zeta\,\mathrm{Re}}\left(y_8 + \zeta^{2} y_{10}\right)),$$

$$\frac{dy_7}{d\lambda} = y_8,$$

$$\frac{dy_8}{d\lambda} = -\frac{\mathrm{Pr}}{(1+Rd)}(\beta y_9 + \beta\lambda y_{10}) - \frac{\mathrm{Re}\,\mathrm{Pr}}{(1+Rd)}(4y_2^2 + \alpha^2 y_3^2 + (Ha^2 + D^{-1}) y_1^2 + y_1 y_{10}) - 2y_9$$

$$\frac{dy_9}{d\lambda} = y_{10},$$

$$\frac{dy_{10}}{d\lambda} = \frac{\mathrm{Pr}\,\mathrm{Re}}{(1+Rd)}(\frac{-3\beta y_7}{\mathrm{Re}} - \frac{\beta\lambda y_8}{\mathrm{Re}} + 2y_2 y_7 - y_1 y_8 - y_3^2 - \alpha^2 y_4^2 - (Ha^2 + D^{-1}) y_2^2) \tag{19}$$

The boundary conditions in terms of y's are

$y_1(0)=0$, $y_1(1)=1$

$y_2(0)=0$, $y_2(1)=0$

$y_3(0)=0$, $y_3(1)=0$

$y_8(0)=-\gamma\ y_7(0)$, $y_7(1)=1/Ec$

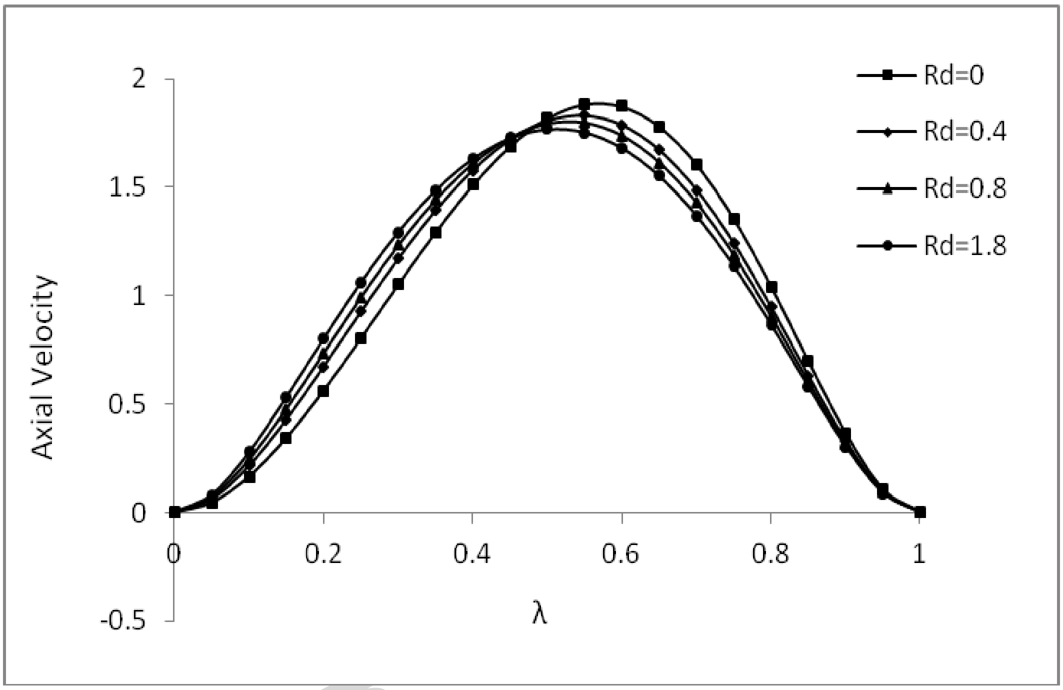
$y_{10}(0)=-\gamma\ y_9(0)$, $y_9(1)=0$ (20)

The system of equations (19) is solved numerically subject to the boundary conditions (20).

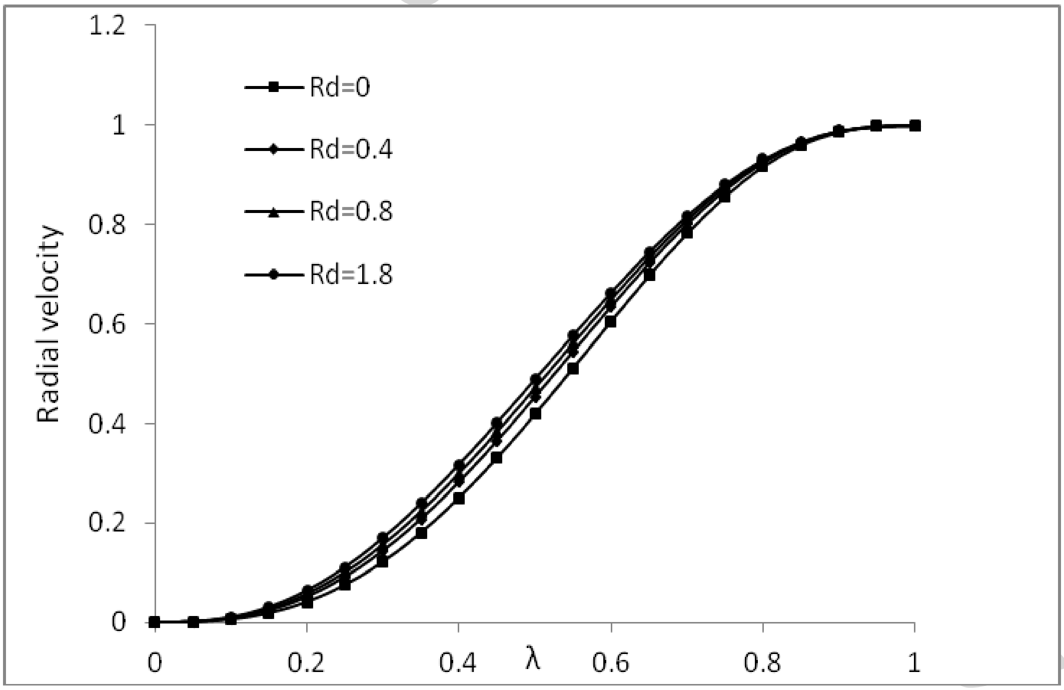
5. Results and Discussions

The influence of various fluid and geometric parameters such as permeation Reynolds number Re, couple stress parameter α, wall expansion ratio β, Hartmann number Ha, inverse Darcy parameter D^{-1}, Prandtl number Pr, radiation parameter Rd and convection parameter γ on axial velocity, radial velocity and temperature distribution has been studied and presented in the form of graphs.

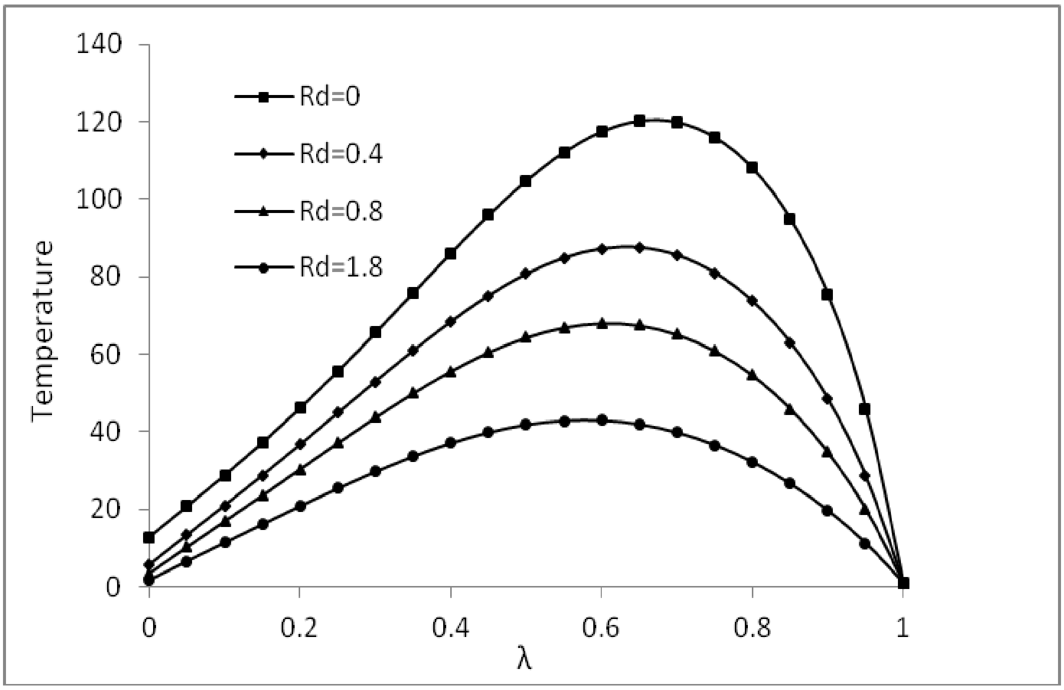
The effect of Rd on non dimensional velocity components and temperature are depicted in the Fig. 1. It is observed that when Rd increases the axial velocity also increases for $0< \lambda< 0.5$, then decreases and the radial velocity increases towards the upper wall, whereas the temperature distribution is decreased. Fig. 2 describes the effect of γ on velocity components and temperature distribution. It is noticed that the velocity components follow the similar trend of Rd whereas the temperature follows the opposite trend. Fig. 3 reveals the Pr effect on velocity components and temperature distribution. When Pr increases the axial decreases till $\lambda = 0.5$ and then increases towards the upper wall and the radial velocity decreases. Also, it can be clearly observed that the temporal maximum is attained at an early stage for lower Pr. This is expected, because a larger Pr results in a larger surface heat transfer and hence a larger temperature gradient. The behavior of velocity components and temperature distribution for Ha is depicted in Fig. 4**.** The profile of axial velocity is symmetric and the axial velocity attains maximum at $\lambda = 0.5$ and the radial velocity increases till $\lambda = 0.5$ then decreases. However, the temperature distribution increases towards the upper wall with Ha. It is because of the fact that the application of a transverse magnetic field normal to the flow direction gives rise to drag force acting in a direction opposite to flow. This force causes a reduction in both the fluid velocity and an increase in the fluid temperature. The effect of D^{-1} on axial velocity, radial velocity and temperature distribution is shown in Fig.5. Peak axial velocity is attained at the λ = 0.5. However, the radial velocity and temperature follow the similar trend of Ha.



1 (a)



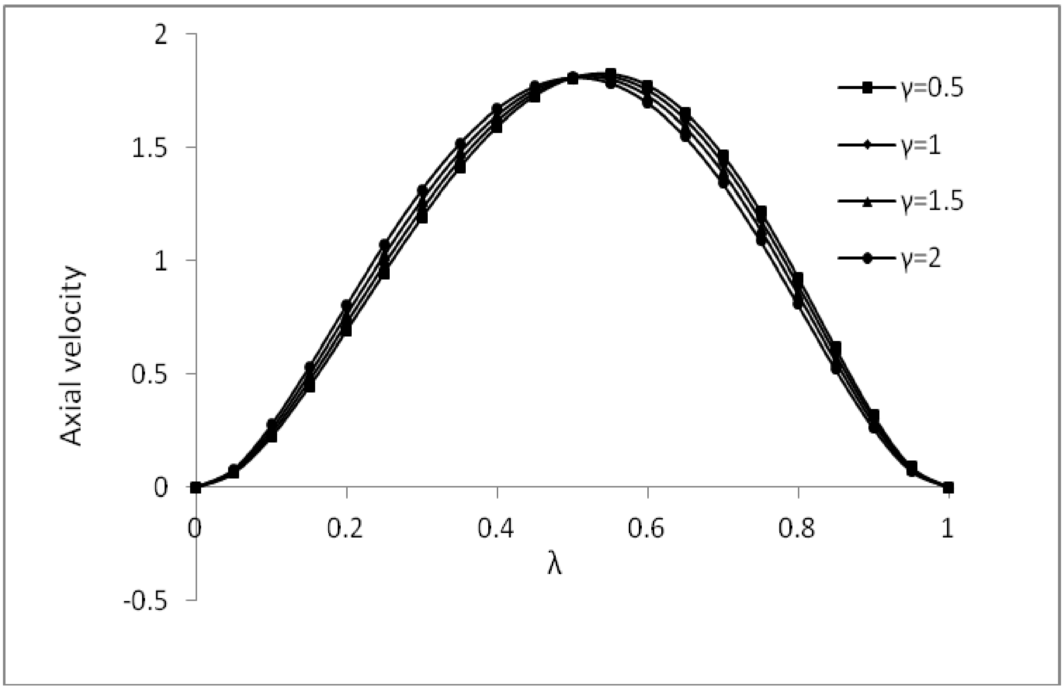
1 (b)



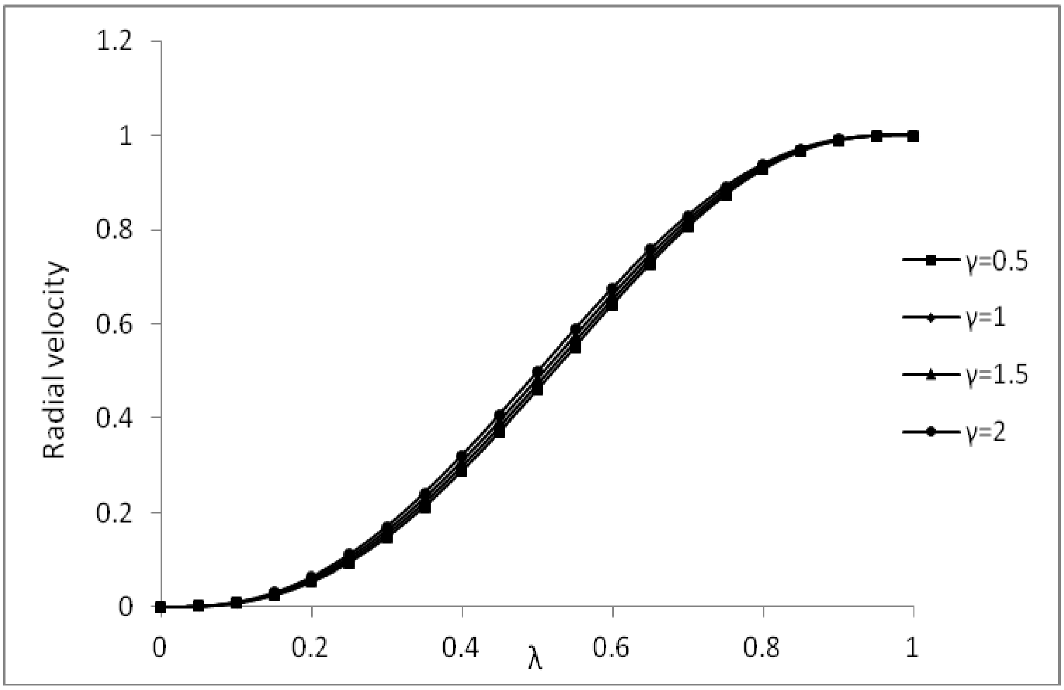
1 (c)

Fig 1. Effect of Rd on (a) Axial velocity, (b) Radial velocity and (c) Temperature for

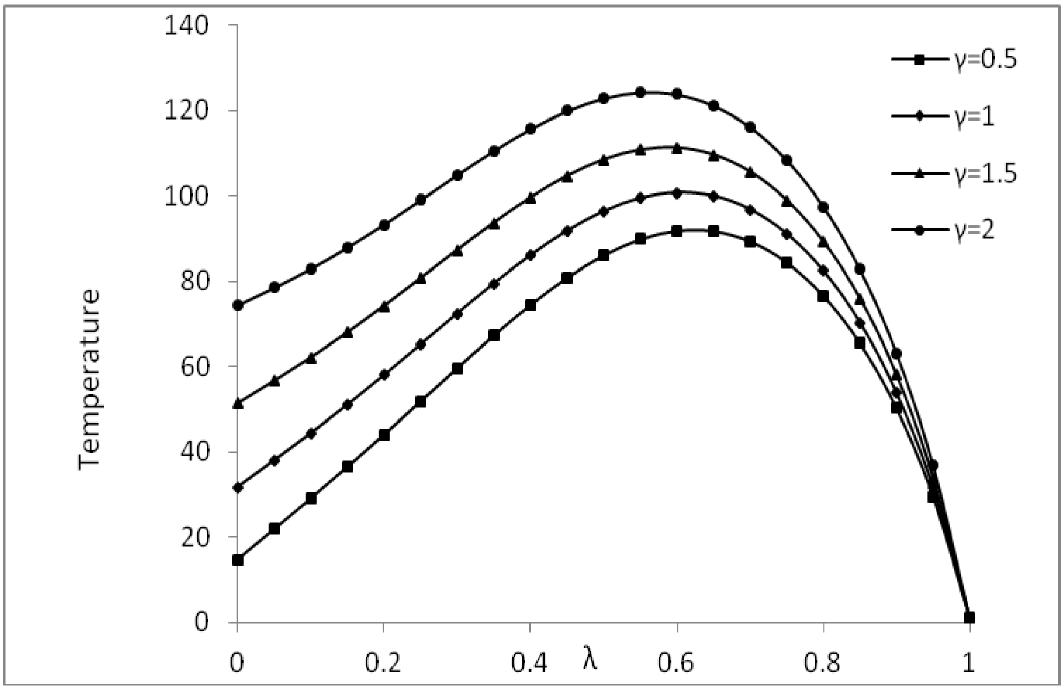
Gr=4, Ec=1, Pr=2, Re=2, β=2, α=0.2, Ha=10, D^{-1}=10, γ=0.2



2 (a)

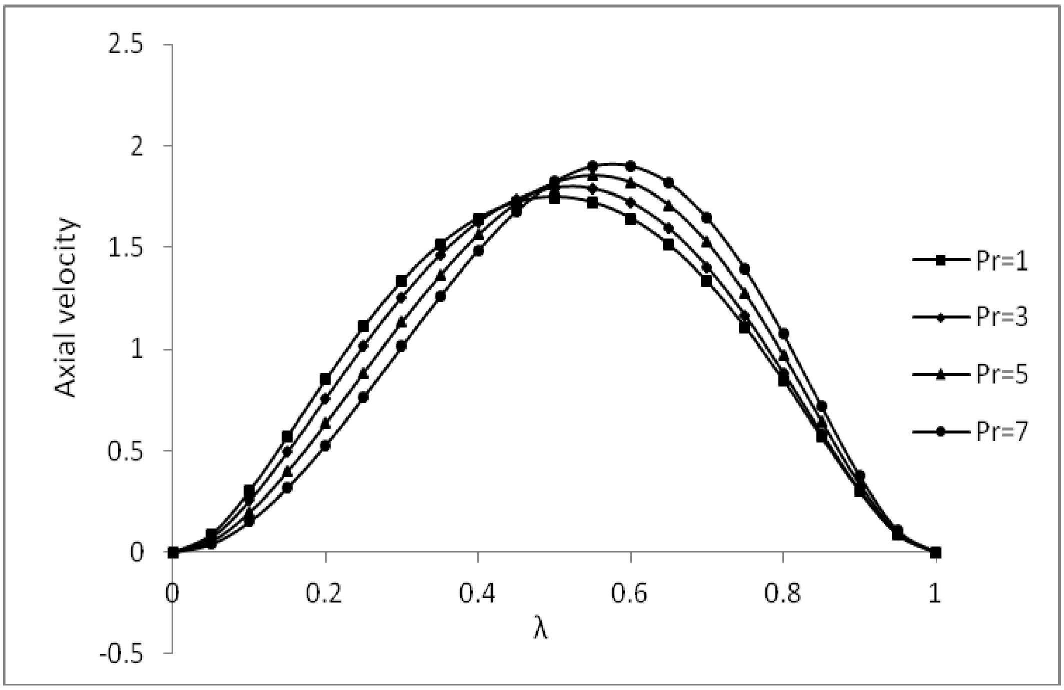


2 (b)

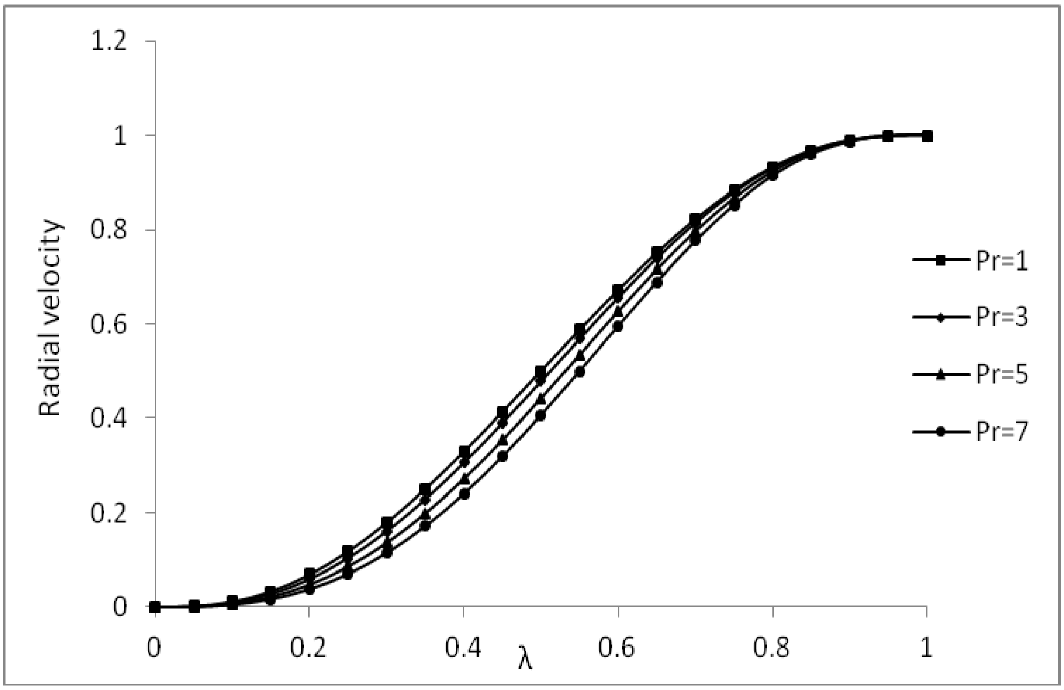


2 (c)

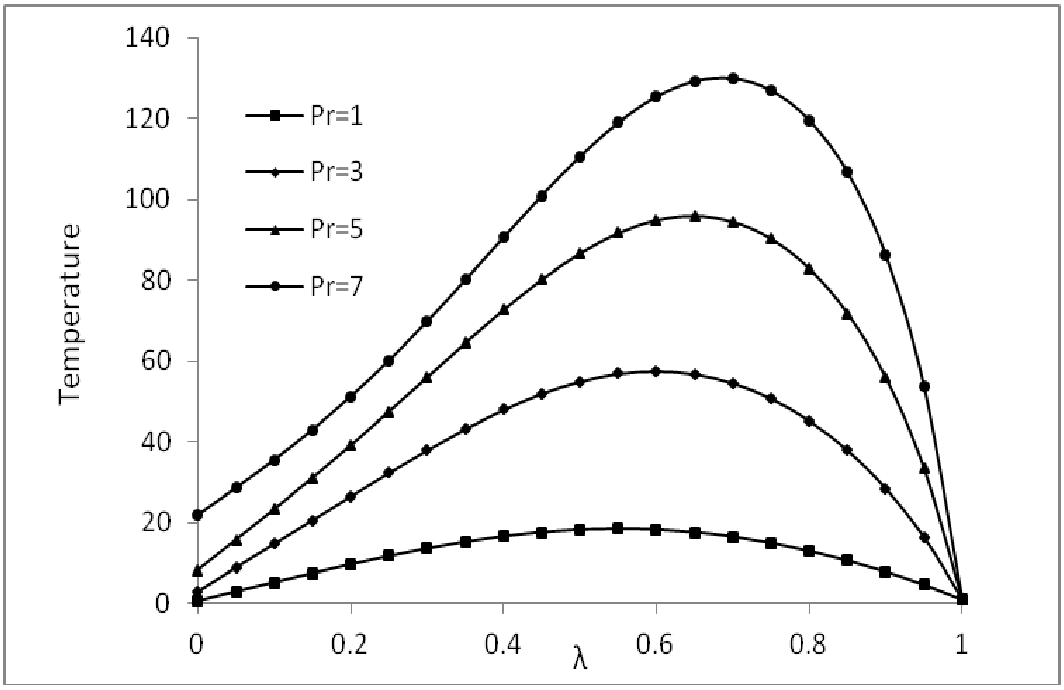
Fig 2. Effect of γ on (a) Axial velocity, (b) Radial velocity and (c) Temperature for Gr=4, Ec=1, Pr=2, Re=2, β=2, α=0.2, Ha=10, D^{-1}=10, Rd=0.4


2.5
2
1.5
1
0.5
0
-0.5
Axial velocity
0
0.2
0.4
0.6
0.8
1
λ
Pr=1
Pr=3
Pr=5
Pr=7

3 (a)

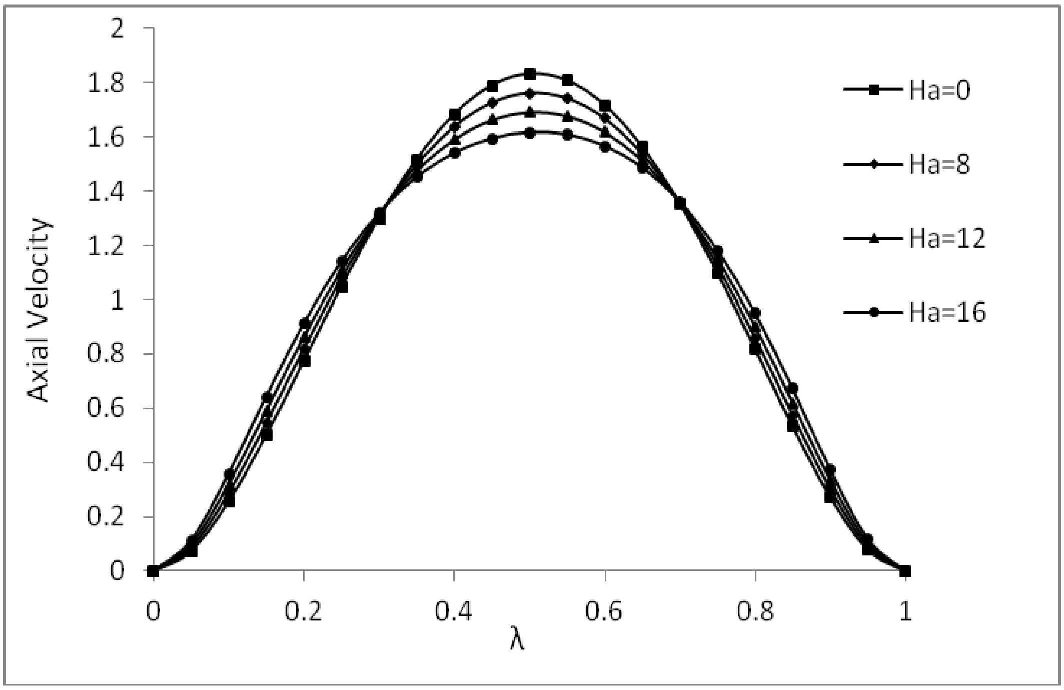

1.2
1
0.8
0.6
0.4
0.2
0
Radial velocity
0
0.2
0.4
0.6
0.8
1
λ
Pr=1
Pr=3
Pr=5
Pr=7

3 (b)

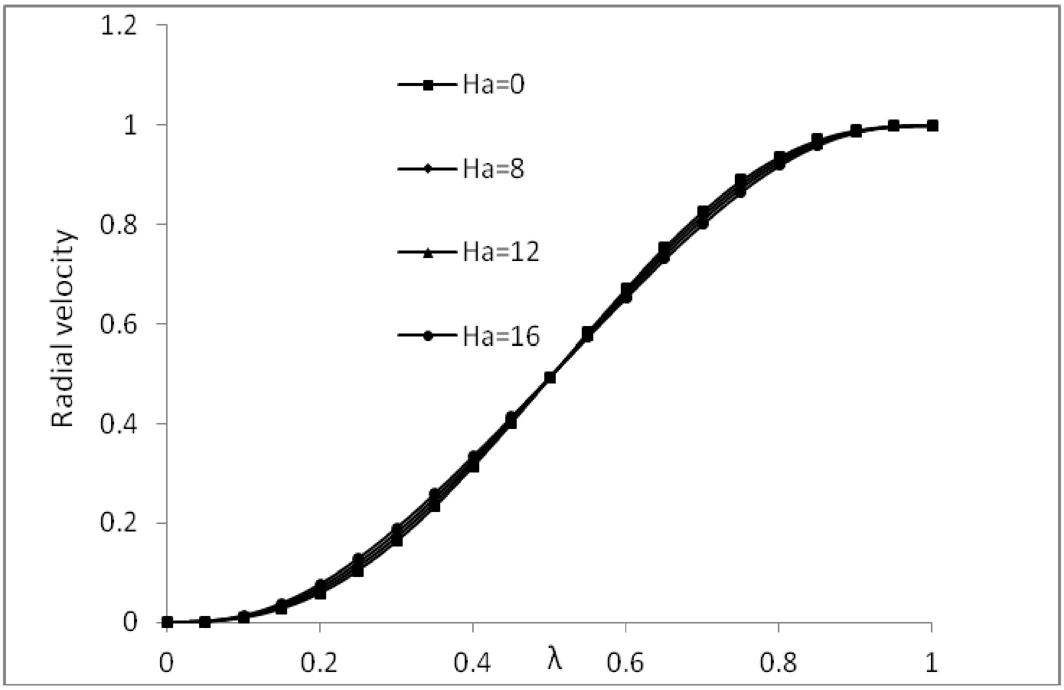


3 (c)

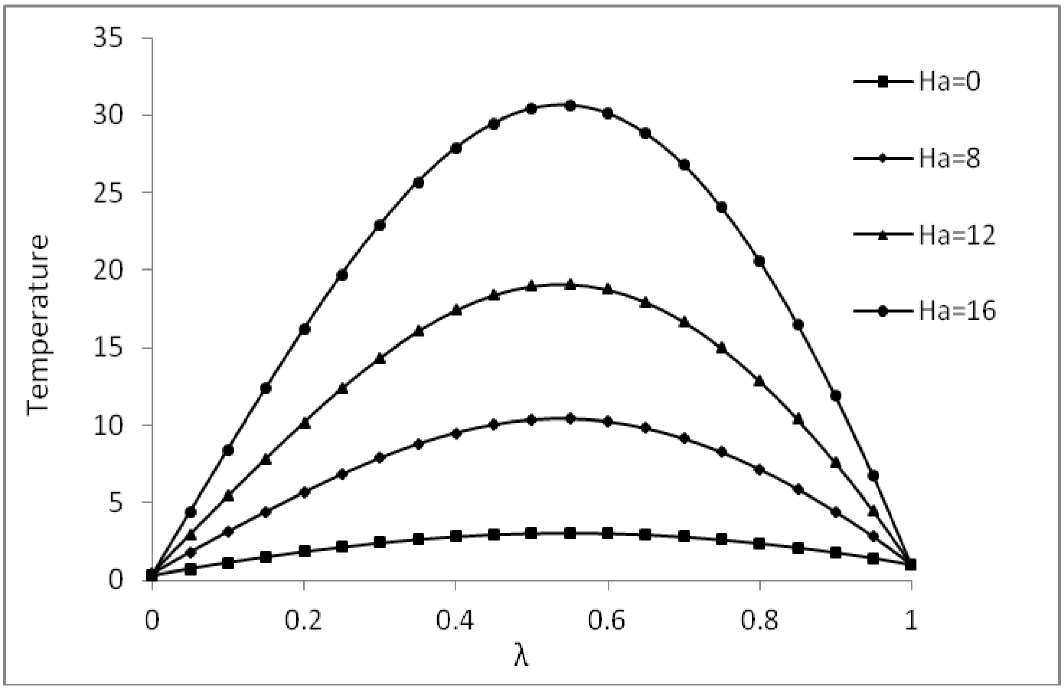
Fig 3. Effect of Pr on (a) Axial velocity, (b) Radial velocity and (c) Temperature for Gr=4, Ec=1, γ=0.2, Re=2, β=2, α=0.2, Ha=10, D^{-1}=2, Rd=2



4 (a)

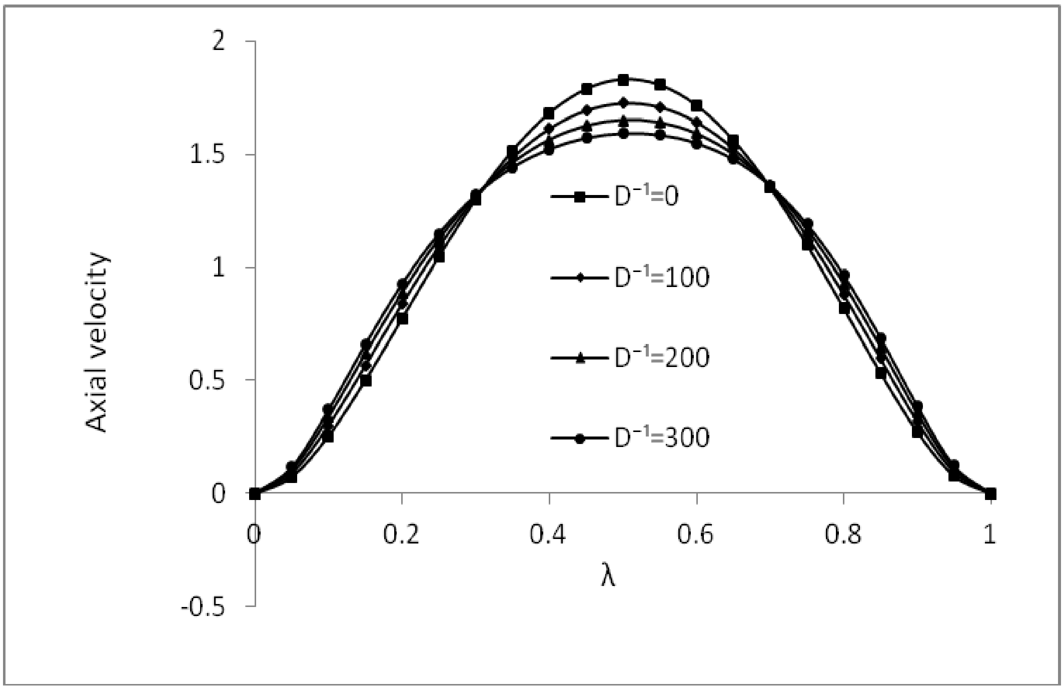


4 (b)

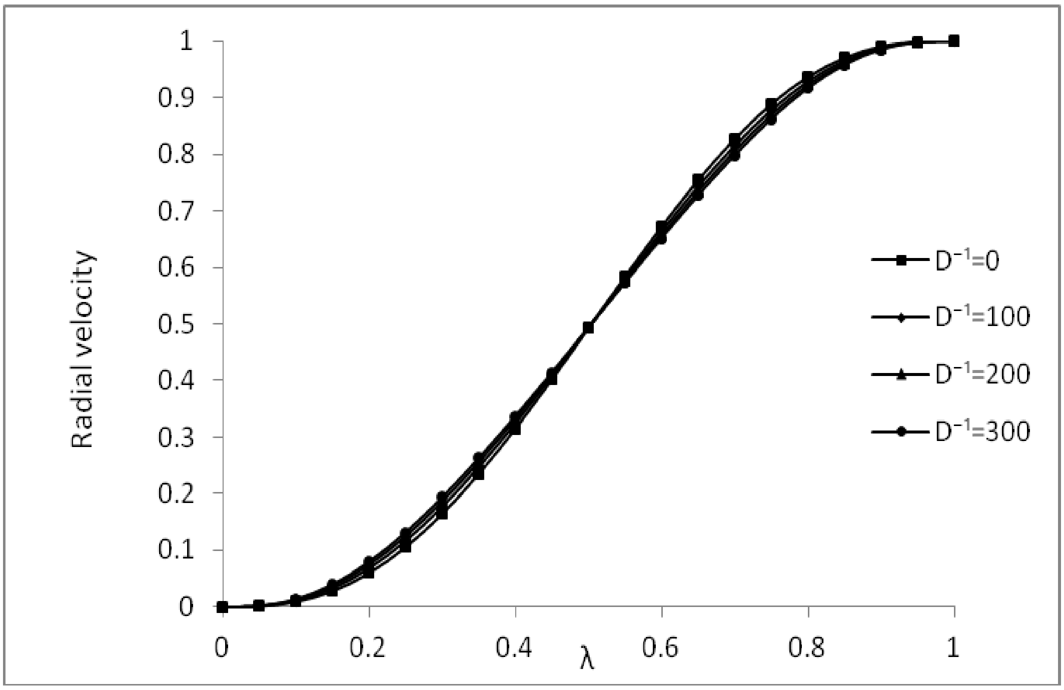


4 (c)

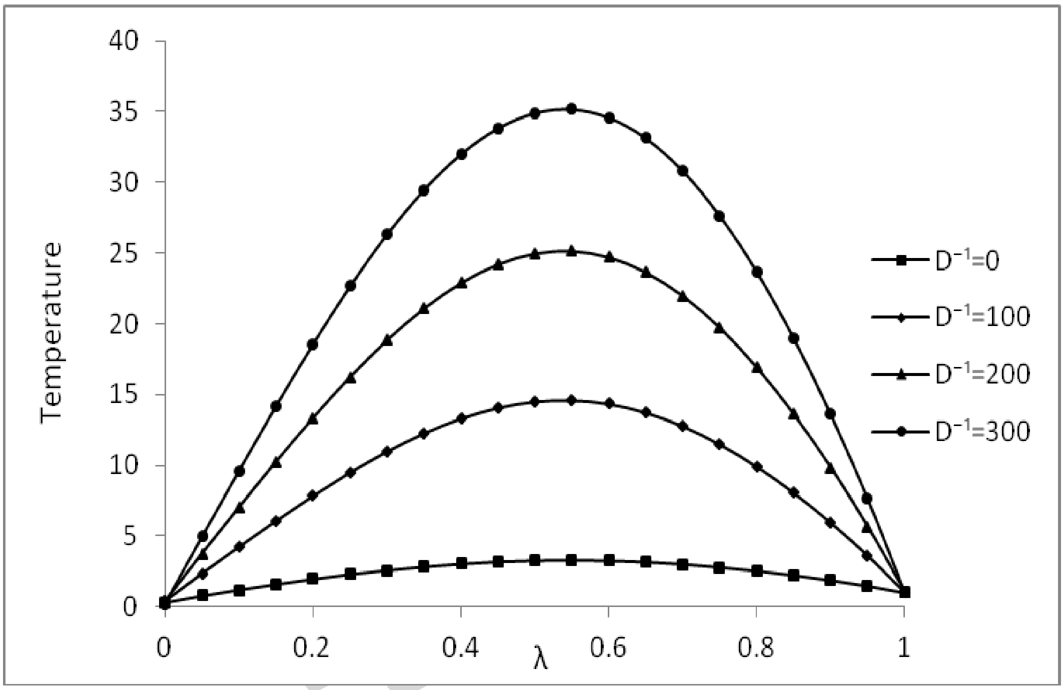
Fig 4. Effect of Ha on (a) Axial velocity, (b) Radial velocity and (c) Temperature for Gr=4, Ec=1, Pr=0.71, Re=2, β=0.2, α=0.2, γ=0.2, D^{-1}=2, Rd=2


Axial velocity
λ
2
1.5
1
0.5
0
-0.5
0
0.2
0.4
0.6
0.8
1
D⁻¹=0
D⁻¹=100
D⁻¹=200
D⁻¹=300

5 (a)


Radial velocity
λ
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
D⁻¹=0
D⁻¹=100
D⁻¹=200
D⁻¹=300

5 (b)



5 (c)

Fig 5. Effect of D^{-1} on (a) Axial velocity, (b) Radial velocity and (c) Temperature for Gr=4, Ec=1, Pr=0.71, Re=2, β=0.2, α=0.2, γ=0.2, Ha=2, Rd=2

6. Conclusions

The two dimensional incompressible, laminar, MHD mixed convection flow and heat transfer of radiating couple stress fluid through porous expanding or contracting walls with convective boundary conditions is considered. The reduced governing equations are solved numerically by quasilinearization method. Fluid injection or suction at the surface is considered, which has been found to have a considerable influence on the flow mechanism. From the results we noticed that the temperature is enhanced with Pr, Ha, D^{-1} and γ whereas it decreases with increasing Rd. The Rd and γ exhibit the similar effect on velocity components whereas the opposite effect is found in the case of Pr.

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