ISSN 2249 5460 Available online at www.internationaleJournals.com

INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES, TECHNOLOGY AND HUMANITIES

www.internationalejournals.com

International eJournals

International Journal of Mathematical Sciences, Technology and Humanities 96 (2013) 1063 – 1069

Image Interpolation Using High-Resolution Cubic Spline Curves

Ranjeet Kumar Roy and Tarun Gulati
Department of Electronics & Communication Engineering,
Maharishi Markandeshwar University, Mullana (Ambala), INDIA
Email: ranjeetroy208@gmail.com & gulati_tarun@rediffmail.com

ABSTRACT

Image processing for low resolution digital images (e.g. low resolution camera and computed tomography (CT) scan medical images) is very challenging problems. It is because of the errors due to quantization and sampling. Over the last several years; significant improvements have been made in this area; however, it is still very challenging. Therefore, this paper focuses on investigating the effect of interpolation functions on using high-resolution cubic spline functions. For this purpose, ideally, an ideal low-pass filter is preferred; however, it is difficult to realize in practice. Therefore, four interpolation functions (nearest neighbor, linear, cubic B-spline and high-resolution cubic spline ($-2 \le a \le 0$). From the results, it is found that cubic B-spline and high-resolution cubic spline have a better frequency response than nearest neighbor and linear interpolation functions.

Keywords: Pixel, Quantization, Sampling, & Interpolation.

I. INTRODUCTION:

This paper focuses on low resolution digital images such as images taken by low resolution camera and CT scan medical images. The captured images are usually processed by digital image processors which render an image in a two-dimensional grid of pixels; characterized by a discrete horizontal and vertical quantization resolution. This finite resolution, especially for low resolution images, often results in visual artifacts, known as "aliasing" artifacts. These are very common in low resolution images and usually these aliasing artifacts either appear as zigzag edges called jaggies or produce blurring effects. Another type of aliasing artifacts is variation of color of pixels over a small number of pixels (termed pixel region) [1-4]. This type of aliasing artifacts produces noisy or flickering shading. A typical example of these artifacts is shown in Figure 1. These artifacts can be reduced by increasing the resolution of an image. This can be done using image interpolation, which is generally referred as a process of estimating a set of unknown pixels from a set of known pixels in an image. Image interpolation is in use for last many years, with the key interpolation techniques include linear, nearest neighbour & B-spline interpolation techniques [5].

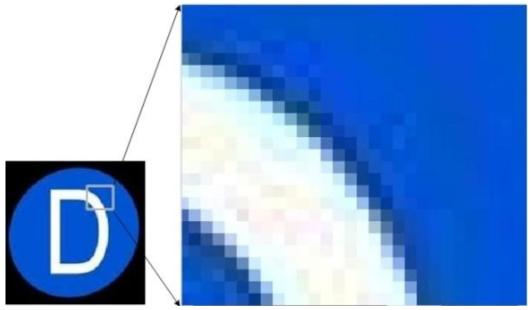


Figure 1: Typical example showing the effects of aliasing on the sharp corners, when zooming operation is performed.

II. INTRODUCING INTERPOLATION

By definition, the term interpolation arises from the concept of re-sampling [5]. The term *re-sampling* refers to the process of transforming a discrete image that is defined at one set of coordinate locations to a new set of coordinate points. In particularly, the process of interpolation refers to finding the information for undefined pixels or missed pixels in an image based on the information provided by given pixels, so that the interpolated image is as close to the actual one as possible. The given information usually includes information related to coordinates, color, gray level or density; with the image having any dimensions. The key interpolation methods include *sinc*, nearest neighbor, linear, cubic B-spline and high resolution cubic spline. A review of these techniques is presented below.

I) Ideal filter

The ideal interpolating function can be defined as a function with constant one value in the pass band and zero value in stop band in frequency domain [6]. It is shown in Figure 2 and is usually reported difficult to realize in practice. A closely ideal interpolating function is *sinc* function, which has infinite length in space domain and can be mathematically expressed as:

$$h(x) = sinc(x) \tag{1}$$

The *sinc* function is sinusoidal in behavior with each next sinusoidal peak having decreasing maximum amplitude; when starting from the zero axis and continued toward infinity in both directions. This feature makes the interpolation usually not practical; as it is difficult to convolve signal with such infinite function. Intuitively, one solution is to truncate the *sinc* function to a shorter length. However, truncating the *sinc* function in space domain will make the frequency domain representation no longer a perfect rectangle. The response in pass band will not be flat like before and there will be some irregular behavior over the stop band, as can be seen in Figure 2.

II) Nearest-Neighbor

The nearest-neighbor interpolation function has a rectangular shape in space domain as shown in Figure 2. It can be mathematically expressed as:

$$h(x) = \begin{cases} 1 & |x| < 0.5 \\ 0 & elsewhere \end{cases} \tag{2}$$

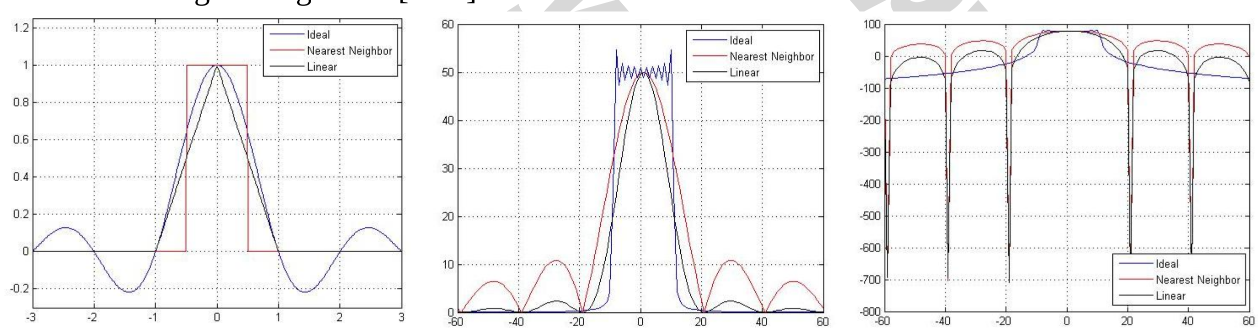
The nearest-neighbor method is usually reported as the most efficient from the computation point of view [7,8]; but, at the cost of poor quality as can be observed from its frequency domain. It is because the Fourier Transform of a square pulse is equivalent to a *sinc* function; with its gain in pass band falls off quickly. In addition, it has prominent side lobes as illustrated in the logarithmical scale. These side-lobes usually result in blurring and aliasing effects in the interpolated image. When applying nearest-neighbor algorithm for image interpolation, the value of the new pixel is made the same as that of the closest existing pixel.

III) Linear

The linear interpolation function has a triangular shape as shown in Figure 2. Its mathematical representation is:

$$h(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & elsewhere \end{cases} \tag{3}$$

Comparing to the nearest-neighbor function, this function is closer to the ideal square shape function so that more energy can be passed through. The side lobes in the stop band are also much smaller, though still considerable. Therefore, the performance of linear interpolation is reported better than the nearest-neighbor interpolation. However in frequency domain, this method is still attenuating the high frequency components and is aliasing data around the cutoff frequencies. This interpolation method has been reported to work better for image reduction, rather than image enlargement [9-11].



(a) (b) (c)

Figure 2: The comparison of ideal, nearest neighbor and linear interpolation functions.

IV) B-splines

From the above made discussion, both nearest-neighbor and linear interpolation have a significant deviation from the ideal rectangular shape in the pass band and have side lobes in the stop band. Basis spline (B-spline) interpolations have, therefore, been introduced. The concept of splines and their mathematical representations were first described by Schoenberg in 1946. By definition, splines can be referred as piecewise polynomials with pieces that are smoothly connected together. These can be derived by several self-convolutions of a so called basis function [6-11]. Mathematically, B-spline of degree *n* can be expressed as:

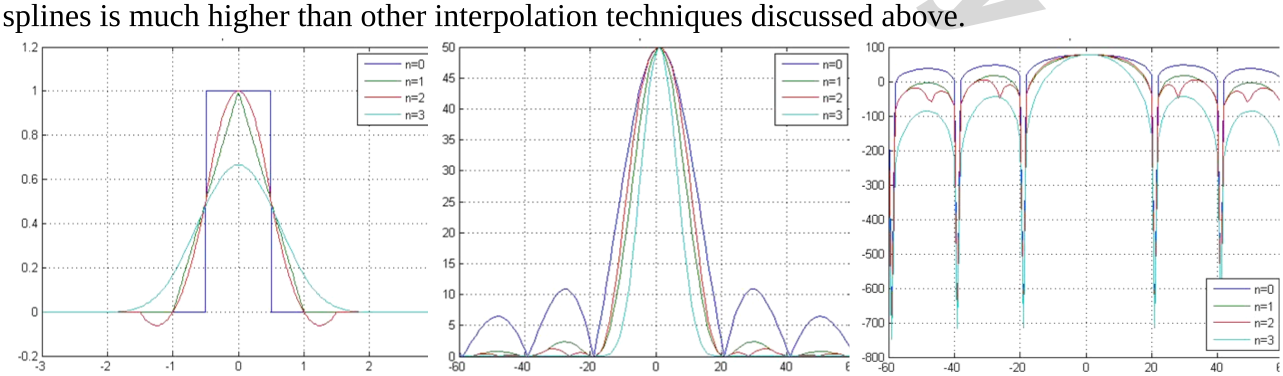
$$\beta^n(x) = \sum_{k=0}^{n+1} \frac{(-1)^k\,(n+1)}{(n+1-k)!\,k!}\left(\frac{n+1}{2} + x - k\right)^n \qquad (4)$$

Where, $\forall x \in R, \forall n \in N_{*}$, and $(X)_+^n = (\max(0,x))^n \;\; n > 0$. The B-spline function for n=0, 1, 2, 3 are mentioned in Table 1 and their corresponding functions in time and frequency domain are shown in Figure 3.

Table 1: B-spline functions for n=0,1, 2, 3.

n=0	***n=2***
$\boldsymbol{\beta^0(x)} = \begin{cases} \mathbf{1} & \lvert x\rvert < 0.5 \\ \mathbf{0.5} & \lvert x\rvert = \mathbf{0.5} \\ \mathbf{0} & \lvert x\rvert > 0.5 \end{cases}$	$\beta^2(x) = \begin{cases} -2\lvert x\rvert^2 + 1 & \lvert x\rvert \le 0.5 \\ \lvert x\rvert^2 - 2.5\lvert x\rvert + 1.5 & 0.5 < \lvert x\rvert \le 1.5 \\ 0 & elsewhere \end{cases}$
n=1	***n=3***
$\boldsymbol{\beta^1(x)} = \begin{cases} \mathbf{1 - \lvert x\rvert} & \mathbf{0 \le \lvert x\rvert < 1} \\ \mathbf{0} & \boldsymbol{elsewhere} \end{cases}$	$\beta^3((x)) = \begin{cases} (2/3) - 0.5\lvert x\rvert^2(2 - \lvert x\rvert) & 0 \le \lvert x\rvert < 1 \\ (1/6) * (2 - \lvert x\rvert)^3 & 1 \le \lvert x\rvert < 2 \\ 0 & 2 \le \lvert x\rvert \end{cases}$

It can be seen from Figures 2 and 3 that cubic spline function has better response in both pass band and stop band comparing with the nearest-neighbor and linear functions. However, the function is positive over the whole interval in the space domain which translates to have more deviation from the constant gain within the pass band in frequency domain. This will smooth more than is necessary below the cut-off frequency. In addition, the complexity of the cubic splines is much higher than other interpolation techniques discussed above.



(a) (b) (c)

Figure 3: The time and frequency domain cubic B-spline interpolation techniques for n=0, 1, 2 and 3.

IV) High resolution cubic splines

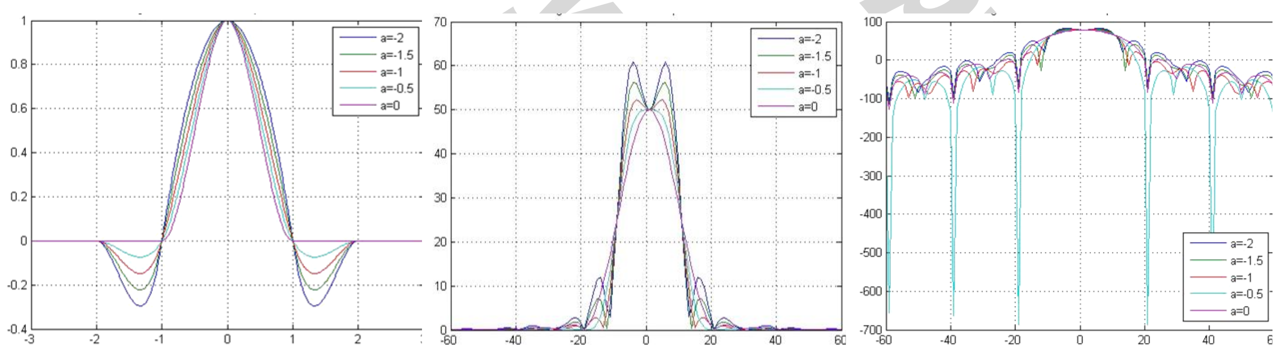
From Figures 2 & 3, the cubic spline has better response in both pass band and stop band comparing with the nearest-neighbor and linear functions. However, the function is positive over the whole interval (Figure 3) in the space domain which will smooth more than is necessary below the cut-off frequency. Therefore, the cubic B-spline function needs to be modified to have negative values in the space domain [10, 11]. This is called high resolution cubic spline function. To derive such function, the general cubic B-spline function is expressed as follows:

$$\beta^3(x) = \begin{cases} a_{30}x^3 + a_{20}x^2 + a_{10}x + a_{00} & x_1 \le x < x_2 \\ a_{31}x^3 + a_{21}x^2 + a_{11}x + a_{01} & x_2 \le x < x_3 \end{cases} \quad (5)$$

The above function is symmetrical about zero within the interval (x_1, x_2). To solve the above, the same constraints of cubic spline function are used, i.e., $a_{31} = a$, $\beta(x_1) = 1$, $\beta(x_2) = 0$, $\beta(x_3) = 0$, $\beta'(x_1) = 0$, $\beta'(x_3) = 0$, $\beta(x_2)_- = \beta(x_2)_+$ and $\beta(x_2)'_- = \beta(x_2)'_+$, to have:

$$\beta^3(x) = \begin{cases} (a+2)x^3 - (a+3)x^2 + 1 & x_1 \le x < x_2 \\ ax^3 - 5ax^2 + 8ax - 4a & x_2 \le x < x_3 \end{cases} \quad (6)$$

The constant *a* will be negative, in order to have the function positive in the interval (x_1, x_2) and negative in the interval (x_2, x_3). These high resolution cubic functions for different values of constant *a* are shown in Figure 4. It can be observed from Figure 4 that the same provides a better high-frequency performance than the cubic B-spline described above. The value of *a* is taken over the interval (-2,0). When the value of *a* is increased from -2 to 0, then the frequency response matches more closely to the ideal rectangular function in the pass band and the transition between the pass band and stop band gets more sharper. In addition, the amplitude of the side band is also decreased. Because this function has a better high-frequency performance than the cubic B-spline; therefore, it is termed high-resolution cubic spline interpolating function.



(a) (b) (c)

Figure 4: The response of high resolution cubic B-spline interpolation function for *a* between -2 & 0.

III. RESULTS AND DISCUSSION

In order to investigate the characteristics of the nearest neighbor, bilinear, cubic spline and high resolution cubic spline functions with *a* between -2 to 0 (inclusive), the same are implemented in Matlab. Their response in time domain and frequency domain in one dimensions is computed. In frequency domain, the response of these functions is evaluated on both linear and logarithmic scale. The linear scale is used to show the pass band performance; whereas, the logarithmic scale shows the stop band performance. From Figures 2 to 4, it is found that with respect to ideal interpolation function, the high resolution cubic spline functions have shown the best response in the pass band. It is observed that for the parameter *a*=-0.5, the response is flat at the intermediate frequencies. However, when the value of *a* is further decreased to -2, then the cut off frequency is traded for a more rapid transition between the pass band and stop band. In comparison to ideal and linear functions, the nearest neighbor function has a better response in the pass band; however, it offers more attenuation even at very low frequencies. Both the bilinear and cubic B-spline interpolation function have comparatively poor response in the pass band. However, the high resolution cubic splines have good response in the stop band similar to the cubic B-spline. The linear and nearest neighbor interpolating functions have poor stop band performances; which means that resampling after interpolation with either of these two functions will result in a large amount of aliasing. it is advisable to use cubic high resolution interpolation with a=-0.5, as it has wider response in the pass band and images interpolated using this function results better in visual appearance than others.

IV. CONCLUSION

From the analytical functions and their graphical representations in time and frequency domain (linear and logarithmic scale), the nearest neighbor function does moderately well in the pass band; however, has comparatively higher side lobes in the stop band. The bilinear interpolation function performs comparatively better in the stop band; however, at the expense of considerable amount of smoothing in the pass band. The cubic B-spline has better stop band performance than nearest neighbor and linear; however, it has wider pass band. The high resolution cubic splines have the best combination of both pass band and stop band performance. However, this is observed for *a*=0 and -0.5. In situations, when *a* is further decreased towards -2, then the function has a significant degradation in its response in both pass band and stop band. It is observed that the visual results of high-resolution cubic spline interpolation with *a*=-0.5 are better as compared to nearest neighbor, bilinear and cubic spline functions. The linear interpolating algorithm, extends over two interpixel distances; whereas, the high resolution cubic splines extends over four interpixel distances.

REFERENCES

[1] P. Thvenaz, T. Blu & M. Unser, "Image Interpolation and Resampling," Handbook of Medical Imaging, Processing and Analysis, I.N. Bankman (eds.), Academic Press, San Diego CA, USA, pp. 393-420, 2000.

[2] R. W. Schafer & L. R. Rabiner, "A digital signal processing approach to interpolation," Proc. IEEE, Volume 61, pp. 692–702, 1973.

[3] M. Hanumantharaju, M. Ravishankar, D. R. Rameshbabu & S. Ramachandran, "Color Image Enhancement using Multiscale Retinex with Modified Color Restoration Technique," Second International Conference on Emerging Applications of Information Technology (ICEAIT), pp. 93-97, 2011.

[4] T. Acharya & P. S. Tsai, "Computational Foundations of Image Interpolation Algorithms," ACM Ubiquity Volume 8, 2007.

[5] T. M. Lehmann, C. Gonner & K. Spitzer, "Interpolation Methods in Medical Image Processing," IEEE Transactions on Medical Imaging, Volume 18(11), pp. 1049-1075, 1999.

[6] J. Anthony Parker, R. V. Kenyon & D. E. Troxel, "Comparison of Interpolating Methods for Image Resampling," IEEE Transactions on Medical Imaging, Volume 2(1), pp. 31-39, 1983.

[7] E. Maeland, "On the comparison of the interpolation methods," IEEE Trans. Med. Imag., Volume 7(3), pp. 213-217, 1988.

[8] M. F. Fahmy, T. K. Abdel Hameed & G. F. Fahmy, "A Fast B-spline Based Algorithm for Image zooming and Compression," 24th National Radio Science Conference (NRSC), Egypt, 2007.

[9] H. S. Hou & H. C. Andrews, "Cubic Splines for Image Interpolation and Digital Filtering," IEEE Trans. Acoust., Speech, Signal Processing, Volume 26, pp. 508-517, 1978.

[10] S. E. Reichenbach & S. K. Park, "Two-parameter cubic convolution for image reconstruction", Proc. SPIE, Volume 1199, pp.833-840, 1989.

[11] M. Unser, "A Perfect Fit for Signal/Image Processing: Splines," Proceedings of the SPIE International Symposium on Medical Imaging: Image Processing (MI'02), 4684, Part I, pp. 225-236, San Diego CA, 2002.

[12] R. G. Keys, "Cubic convolution interpolation for digital image processing," IEEE Trans. Acoust., Speech, Sig. Proc., Volume 29(6), pp.1153-1160, 1981.

[13] R. Scroop, J. Eskridge & G. W. Britz, "Paradoxical Cerebral Arterial Embolization of Cement during Intraoperative Vertebroplasty: Case Report", American Journal of Neuroradiology, ISSN: 1936-959X

AUTHOR

Ranjeet Kumar Roy is currently pursuing Masters degree program in Electronics & Communication Engineering in Department of Electronics & Communication Engineering, of MM university, Mullana, Ambala, INDIA.

Tarun Gulati did B.Tech degree in Electronics & Instrumentation from Kurukshetra University, Kurukshetra, in 2001 and M.Tech degree from NITTTR, Chandigarh in 2008. He is working as an Asstt. Professor in MM University, Mullana, India since 2002 and is presently doing PhD from the same university in image processing.