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A Dynamical Problem of Generalized Thermo elasticity with Pulse Type Heat Flux

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ABSTRACT

In this paper, we have solved the problem of thermoelasticity concerning to a semi-infinite medium, when the boundary of the medium is subjected to pulse type heat flux. The results are evaluated numerically for various values of the parameters and are represented graphically. The results obtained here are more general, which reveal the inherent wave nature of the heat transfer process.

Key words: Generalized thermoelasticity, Heat flux, Hyperbolic Heat conduction equation, Unit Step function, Thermal Stresses.

Mathematical Subject classification: 34 B 40.

1 INTRODUCTION:

The thermal stress distribution pertaining to the problem of an elastic half space whose plane boundary is subjected to sudden heating was determined by Danilovskaya, V.I [1] by considering the effects of inertial terms. Tsui, Y.T [8] determined the thermal stress distribution in the half space when the half space is exposed to step temperature and velocity. Further work in this direction was carried out by Mura, T [2], Nowacki, W [5,6,7], Murthy.D.R and Murthy.M.G.K [3] and by others.

In this paper, we have discussed the problem of thermoelasticity concerning to a Semi – infinite medium, when the boundary of the medium is subjected to pulse type heat flux.

2 Formulation and Solution of the Problem:

The problem choosen here is that of a Semi – infinite medium $X \geq 0$, initially at temperature T_0, the boundary $X = 0$ of the medium is subjected to pulse type heat flux.

The governing hyperbolic one dimensional heat conduction equation for Semi – infinite medium in the absence of heat sources is given by

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{C^2}\frac{\partial^2 T}{\partial t^2} - \frac{1}{h}\frac{\partial T}{\partial t} = 0 \quad ------ (1)$$

Here, h is thermal diffusivity and is given by $h = \frac{k}{\rho C_V}$. K, ρ *and* C_V are thermal conductivity, density and specific heat at constant volume respectively. C is the finite speed of heat propagation.

When the boundary $X = 0$ of the medium, which is initially at T_0 temperature, is subjected to pulse type heat flux, we have the following initial and boundary conditions.

$$T(X, 0) = T_0, \; X > 0$$

$$\frac{\partial T(X, 0)}{\partial t} = 0$$

$$\lim_{X \to \infty} T(X, t) = T_0, \; t > 0$$ (Regularity Boundary Condition)

$$-K\frac{\partial T(0, t)}{\partial X} = q \qquad ------- (2)$$

Where,

$$q = \begin{cases} 0, & for \;\; t \le 0 \\ Q, & for \;\; t > 0 \end{cases}$$

Introducing the following Non – dimensional Variables

$$Z = \frac{C}{2h}X, \quad \tau = \frac{C}{2h}t, \quad \theta = \frac{(T - T_0)}{T_0}$$

From equations (1) and (2), we get

$$\frac{\partial^2 \theta}{\partial Z^2} - \frac{\partial^2 \theta}{\partial \tau^2} - 2\frac{\partial \theta}{\partial \tau} = 0 \qquad -------(3)$$

$$\theta\left(Z,0\right) = 0 = \frac{\partial \theta\left(Z,0\right)}{\partial \tau}$$

$$\lim_{Z \to \infty} \theta\left(Z,\tau\right) = 0 \qquad ------(4)$$

$$\frac{\partial \theta}{\partial Z}\left(0,\tau\right) = -Q_1, \ for\ \tau > 0$$

Where,

$$Q_1 = \frac{2h}{KCT_0} Q$$

Applying Laplace transform to equations (3) and (4), we get

$$\frac{d^2 \overline{\theta}}{dZ^2} - \left(P^2 + 2P\right)\overline{\theta} = 0 \qquad ------(5)$$

$$\lim_{Z \to \infty} \overline{\theta}\left(Z,P\right) = 0 \qquad -------(6)$$

$$\frac{d\overline{\theta}\left(0,P\right)}{dZ} = -\frac{Q_1}{P}$$

Where,

$$\overline{\theta}\left(Z,P\right) = \int_0^\infty e^{\left(-P\tau\right)} \theta\left(Z,\tau\right) d\tau$$

is the Laplace transform of temperature $\theta\left(Z,\tau\right)$ and p is the transform parameter. From the above equations, we get

$$\bar{\theta}\left(Z,P\right)=\frac{Q_1}{P\left(P^2+2P\right)^{\frac{1}{2}}}e^{\left[-Z(p^2+2p)^{\frac{1}{2}}\right]} \quad ----------(7)$$

Taking inverse Laplace Transform, we get

$$\theta\left(Z,\tau\right)=Q_1\int_0^\infty e^{\left(-\delta\right)}I_0\left[\left(\delta^2-Z^2\right)^{\frac{1}{2}}\right]\eta\left(\delta-Z\right)d\delta \quad -----(8)$$

Where η is the unit step function and is defined as

$$\eta\left(w\right)=\begin{cases}0, & for \quad w<0\\ 1, & for \quad w>0\end{cases} \quad -------------(9)$$

and I_0 is the modified Bessel function of order zero.

3. Determination of Thermal Stresses:

The field equations of thermoelasticity are:

1. Equations of motion.

$$\sigma_{ij,j}=\rho\ddot{u}_i \quad ---------(10)$$

2.
   a. Stress – Strain – Temperature relations.

$$\sigma_{ij}=\lambda e\delta_{ij}+2\mu e_{ij}-mT\delta_{ij} \quad -------(11)$$

3.
   a. Strain – displacement relations.

$$e_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \quad ------(12)$$

Where σ_{ij} are components of stress tensor, e_{ij} are those of strain tensor, u_i are displacement components. $e=e_{11}+e_{22}+e_{33}$ is dilatation. Here (,) a comma followed by a subscript denotes the differentiation with respect to that independent variable with that subscript and (.) dot represents the differentiation with respect to time and $m=(3\lambda+2\mu)\alpha$, λ and μ are Lame's constants and α is coefficient of linear thermal expansion.

We impose suitable constraints on the medium so that the displacement components u_1, u_2, u_3 in X, Y, Z directions may be taken as

$$u_1 = u(x, y), \quad u_2 = u_3 = 0$$

So, for the one dimensional case considered here, the field equations (10) to (12) can be written as

$$\frac{\partial \sigma_{XX}}{\partial X} = \rho \frac{\partial^2 u}{\partial t^2} \qquad \text{-----------------} (13)$$

$$\left.\begin{aligned} \sigma_{XX} &= (\lambda + 2\mu) e_{XX} - mT \\ \sigma_{YY} &= \sigma_{ZZ} = \lambda e_{XX} - mT \end{aligned}\right\} \text{---------} (14)$$

$$e_{XX} = \frac{\partial u}{\partial X} \qquad \text{-----------------} (15)$$

From equations (13), (14) and (15) we get

$$C_1^2 \frac{\partial^2 \sigma_{XX}}{\partial X^2} - \frac{\partial^2 \sigma_{XX}}{\partial t^2} = m \frac{\partial^2 T}{\partial t^2} \qquad \text{------} (16)$$

Where, $C_1^2 = \frac{(\lambda+2\mu)}{\rho}$ is the square of the velocity of an elastic longitudinal wave.
The initial and boundary conditions are

$$\left.\begin{aligned} \sigma_{XX}(X, 0) &= \frac{\partial \sigma_{XX}(X, 0)}{\partial t} = 0 \\ \sigma_{XX}(0, t) &= 0 \end{aligned}\right\} \text{------} (17)$$

and the regularity boundary condition is

$$\lim_{X \to \infty} \sigma_{XX}\left(X, t\right) = 0 \quad ----------(18)$$

Using the non dimensional variables and solving by using Laplace transform, we get,

$$\bar{\sigma}\left(Z, p\right) = \frac{Q_1 a^2}{1-a^2}\left[\frac{1}{\left(p+b^2\right)\left(p^2+2p\right)^{\frac{1}{2}}}\right]. \left[e^{\left(-Z\left(P^2+2P\right)^{\frac{1}{2}}\right)} - e^{\left(-aPZ\right)}\right] ----(19)$$

Where, $b^2 = \frac{2}{1-a^2}$

Above equation can be written as,

$$\bar{\sigma}\left(Z, p\right) = \frac{Q_1 a^2}{1-a^2}\left[\frac{1}{\left(p^2+b^2\right)\left(p^2+2p\right)^{\frac{1}{2}}} e^{\left(-Z\left(P^2+2P\right)^{\frac{1}{2}}\right)}\right] - \frac{Q_1 a^2}{1-a^2}\left[\frac{1}{\left(p^2+2p\right)^{\frac{1}{2}}\left(p+b^2\right)} e^{\left(-aPZ\right)}\right] ------------(20)$$

Taking inverse Laplace transform, we get the expression for the stress component as,

$$\sigma\left(Z, \tau\right) = \frac{Q_1 a^2}{1-a^2}\int_0^{\tau} e^{\left(-b^2\tau-\delta\left(1-b^2\right)\right)} .I_0\left(\left(\delta^2 - Z^2\right)^{\frac{1}{2}}\right)\eta\left(\delta - Z\right) d\delta - \frac{Q_1 a^2}{1-a^2}\int_0^{\tau} e^{\left(-\delta\left(b^2-1\right)-\left(\tau-aZ\right)\right)} .I_0\left(\tau-\delta\right)\eta\left(\delta - aZ\right) d\delta ------(21)$$

4 Discussion of the Results:

The non-dimensional temperature distribution (θ) given by equation (8) and the non-dimensional stress distribution (σ) given by equation (21) are evaluated numerically for various values of the parameters τ *and* Z.

The variation of the temperature distribution $\left(\frac{\theta}{Q_1}\right)$ behind and after the thermal wave fronts $\tau = 1$ *and* $\tau = 2$ is represented in Figure (1), while in Figure (2), it is given for Z=1, Z=2 and for Z=3. Thermal stress distribution $\left(\frac{1-a^2}{Q_1^2 a^2}\sigma\right)$ at the head and thermal wave fronts $\tau = 1,\ \tau = 2$ *and* $\tau = 3$ is represented in Figure (3).

Solutions obtained here reveal the inherent wave nature of the heat transfer process. The wave aspect is produced by the unit step function. It can be noticed that the temperatures as well as thermal stresses are having discontinuities at the wave fronts. The temperature and stress discontinuities at the wave fronts decay exponentially.

The results obtained here are more general, which include the effect of finite speed of heat propagation. By taking infinite velocity of propagation in equation (1), the results of Murthy, M.G.K [4] can be obtained as a particular case.

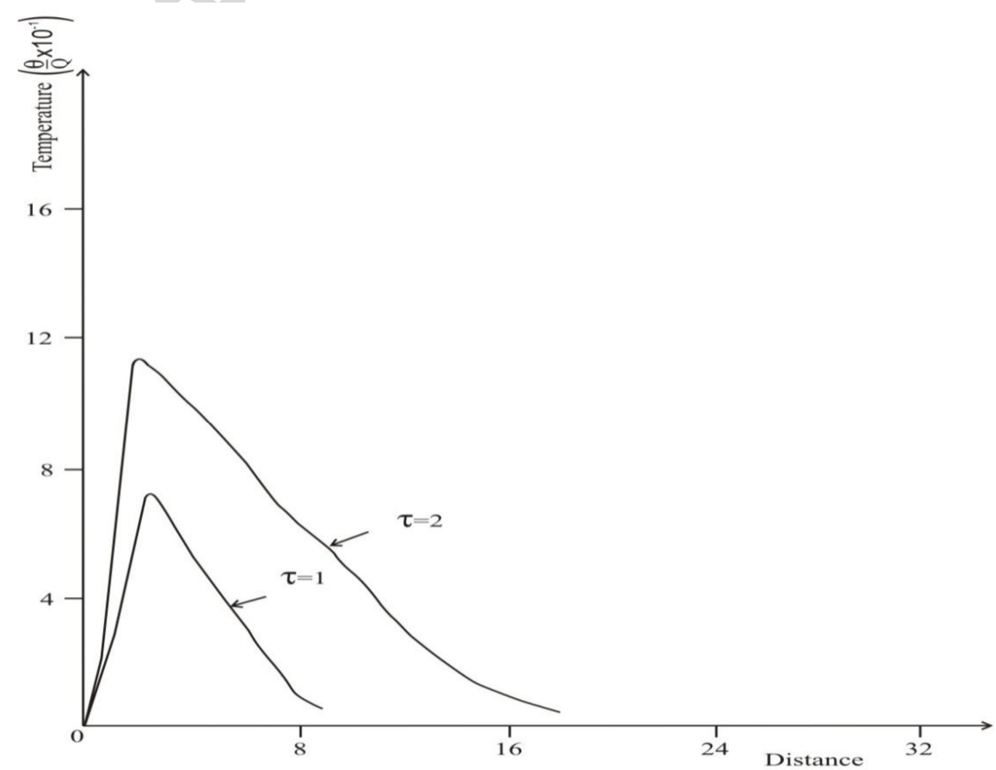

Figure 1 : DISTRIBUTION OF TEMPERATURE WITH DISTANCE

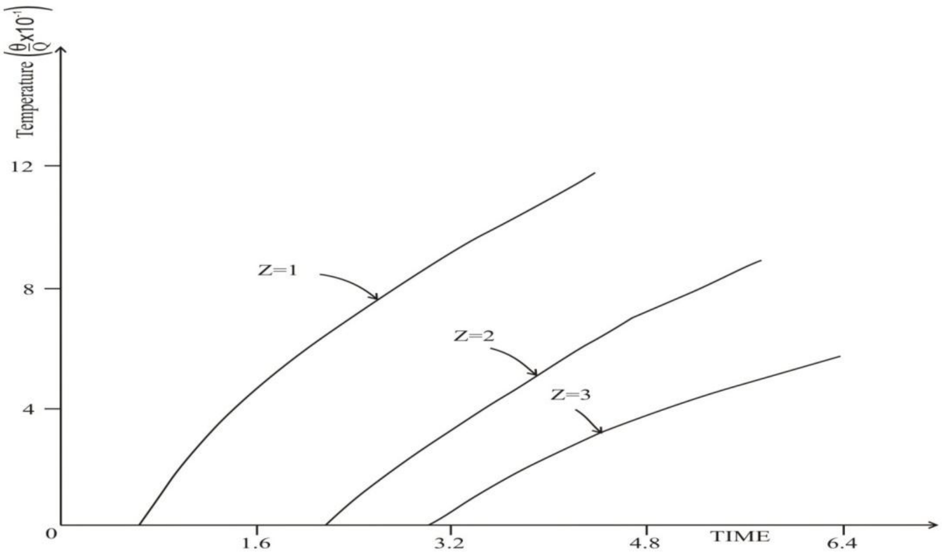

Figure 2 : VARIATION OF TEMPERATURE WITH TIME

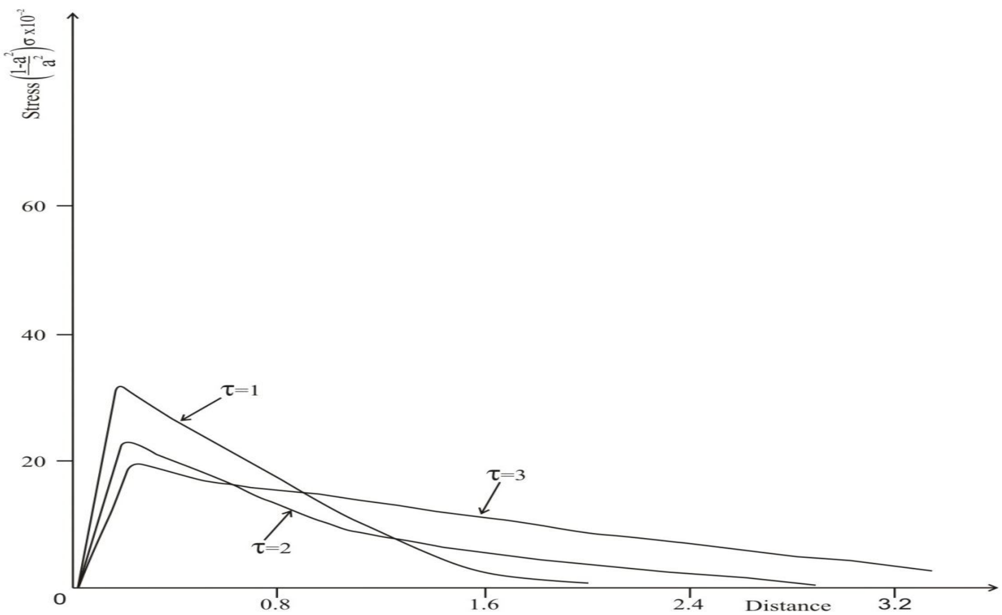

Figure 3 : STRESS DISTRIBUTION

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