INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES, TECHNOLOGY AND HUMANITIES

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International Journal of Mathematical Sciences, Technology and Humanities 45 (2012) 453 – 465

Flow of an Incompressible Viscous Fluid through Straight Circular Tube with Porous Lining

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Abstract:

In the present paper the flow of an incompressible viscous fluid through a straight circular tube is considered. The innerside of the circular cylindrical tube is coated with porous lining of thickness δ, radious of the tube is taken to be 'b' and the clear region (excluding the porous lining) will be of radious 'a'. The problem becomes the two layered flow one in porous region and other in clear region. This has analogy with blood flow which considered to be two layered. The effect of magnetic field, porous lining on the flow of the fluid is investigated.

Key words: Incompressible viscous fluid, Two layered, Permeability, Porous lining.

1. Introduction:

The study of flow through porous medium assumed importance because of its interesting applications in the diverse field of science, Engineering and Technology. The practical applications are in the percolation of water through soil., extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter-disciplinary fields such as bio-medical engineering etc. The lung alveolar is an example that finds application in an animal body. The classical Darcy's law Musakat [4] states that the pressure gradient pushes the fluid against the body forces excreted by the medium which can be expressed as $\vec{V} = -\left(\frac{K}{\mu}\right)\nabla P$.

The law gives good results in the situations when the flow is uni-directional or the flow is at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to it's viscous nature and produces distortions in the velocity field. In the case of highly porous media such as fiber glass, papus of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by the Beavers and Joseph [1], Saffman [9] and others. A generalized Darcy's law proposed by Brinkman [2] is given by

$$0 = -\nabla P - \left(\frac{\mu}{k}\right)\vec{V} + \mu\nabla^2\vec{V}$$

Where μ and K are coefficients of viscosity of the fluid and permeability of the porous medium.

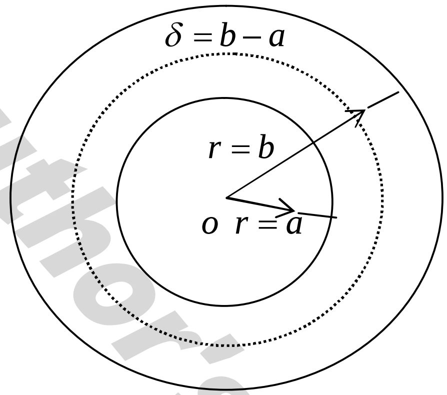
The generalized equation of momentum for the flow through the porous medium is

$$\rho\left[\frac{\partial\vec{V}}{\partial t} + \left(\vec{V}.\nabla\right)\vec{V}\right] = -\nabla P + \mu\nabla^2\vec{V} - \left(\frac{\mu}{K}\right)\vec{V}$$

The classical Darcy's law helps in studying flows through porous medium. In the case of highly porous medium such as papus of dandelion etc. The Darcy's law fails to explain the flow near the surface in the absence of pressure gradient. The non-Darcian approach is employed to study the problem of flow through highly porous medium by several investigators. Narasimhacharyulu and Pattabhi Ramacharyulu [5, 6] Narsimhacharyulu [7] and Singh [8] etc. Studied the flow employing Brinkman law [2] for the flow through highly porous medium.

Earlier the flow of Newtonian fluid between two parallel plates with porous lining is examined by V.Narsimhacharyulu & VenkatrRaman [9]. In the present study, the flow of an incompressible viscous fluid is considered through a straight circular tube of infinite length. The tube is coated with porous lining of thickness δ. The flow will be two layered one region is porous region other region is clear region. Fluid flows through the tube in two layers. The magnetic field is applied in the porous region, the clear region is made free from the magnetic field. Such type of flows find applications in the inter-disciplinary fields such as bio-medical engineering etc. the flow of the blood is one such application. The blood may be represented as a Newtonian fluid and the flow of the blood is two layered light foot [3] and Shukla et al [10]. The effect of the coefficient of the porous medium and the effect of the thickness of the porous lining on the physical quantities at the fluid flow are discussed.

2. Formulation of the problem:



Consider the flow of an incompressible viscous fluid through a infinite straight circular tube of radious 'b'. The inner region of the tube is coated with porous lining of thickness δ. There exists two regions, one clear region of radious $r = b - \delta = a$ and porous region of radious $\delta = b - a$. A magnetic field is applied in the porous region perpendicular to the flow of the fluid. The clear medium is free from magnetic field (Such situation occur in practical problems of blood flow where plasma region will be away from magnetic field).

A cylindrical polar coordinate system (r, θ, z) is taken such that z is the axis of the tube, r radius of the tube, θ is azimuthal angle. The velocity is independent of z and θ because of infinite length of the tube and symmetric of the flow. Therefore velocity depends on $'r'$ only.

The velocities of the fluid in the two regions is given by

$[0, 0, w_c(r)]$ in clear region, $0 < r < a$ (2.1)

and $[0, 0, w_p(r)]$ in porous region, $a < r < b$ (2.11)

Where w_c is velocity in clear medium, w_p is velocity in the porous medium.

The equation of continuity

$$\nabla . \vec{V} = 0 \tag{2.12}$$

is satisfied by the choice of velocity.

The constant pressure gradient acting in the clear region is assumed to induce flow in porous region.

$$\frac{\partial p}{\partial t} = G \text{ in the clear region} \tag{2.13}$$

$$\frac{\partial p}{\partial t} = 0 \text{ in the porous region} \tag{2.14}$$

The equation of motion in the two regions

$$0 = -\frac{\partial p}{\partial z} + \mu \nabla^2 w_c ;\ o < r < a \tag{2.15}$$

$$0 = \mu \nabla^2 w_p - \frac{\mu}{k} w_p ;\ a < r < b \tag{2.16}$$

Together with the Boundary conditions

$$w_c = \textit{finite at } r = 0 \tag{2.17}$$

$$w_c = w_p \ \textit{at } r = a \tag{2.18}$$

$$\frac{\partial w_c}{\partial r} = \frac{\partial w_p}{\partial r} \ \textit{at } r = a \tag{2.19}$$

$$w_p = 0 \ \textit{at } r = b \tag{2.20}$$

The equation of motion in the two regions will be

$$\frac{d^2 w_c}{dr^2} + \frac{1}{r}\frac{dw_c}{dr} = \frac{-G}{\mu} \tag{2.21}$$

$$\frac{d^2 w_p}{dr^2} + \frac{1}{r}\frac{dw_p}{dr} = -\alpha^2 w_p \tag{2.22}$$

Where $M^2 = \frac{\mu_e \sigma^2 \beta_0^2}{\mu}$, $\alpha^2 = M^2 + \frac{1}{k}$

The velocities in the two regions will be given by

$$w_c(r) = A_1 \log r + B_1 - \frac{r^2 G}{4\mu} \tag{2.23}$$

$$w_p(r) = A_2 I_0(\alpha r) + B_2 K_0(\alpha r) \tag{2.24}$$

Where A_1, B_1, A_2, B_2 are constants.

The Boundary conditions give the following equations

$$A_2 I_0(b\alpha) + B_2 K_0(b\alpha) = 0 \tag{2.25}$$

$$A_2 I_1(a\alpha) - B_2 K_1(a\alpha) = -\frac{Ga}{2\mu\alpha} \tag{2.26}$$

$$B_1 = \frac{Ga^2}{4\mu} + A_2 I_0(a\alpha) + B_2 K_0(a\alpha) \tag{2.27}$$

$A_1 = 0$ as the velocity is finite at $r = a$ (2.28)

By solving the above equations

$$B_1 = \frac{Ga^2}{4\mu} + \frac{Ga}{2\alpha\mu}\left[\frac{K_0(a\alpha)I_0(b\alpha) - I_0(a\alpha)K_0(b\alpha)}{K_0(b\alpha)I_1(a\alpha) + K_1(a\alpha)I_0(b\alpha)}\right] \tag{2.29}$$

$$A_2 = \frac{-K_0(b\alpha)\left(\frac{aG}{\alpha}\right)}{2\mu\left[K_1(a\alpha)I_0(b\alpha) + K_0(b\alpha)I_1(a\alpha)\right]} \tag{2.30}$$

$$B_2 = \frac{aG}{2\mu\alpha} I_0(b\alpha)\left[\frac{1}{K_0(b\alpha)I_1(a\alpha) + K_1(a\alpha)I_0(b\alpha)}\right] \tag{2.31}$$

Thus we obtain velocity field, in the clear zone w_c

$$w_c(r) = \frac{G}{4\mu}\left(a^2 - r^2\right) + \frac{aG}{2\alpha\mu}\left[\frac{K_0(a\alpha)I_0(b\alpha) - I_0(a\alpha)K_0(b\alpha)}{K_0(b\alpha)I_1(a\alpha) + K_1(a\alpha)I_0(b\alpha)}\right] \tag{2.32}$$

Velocity in the porous region w_p

$$w_p(r) = A_2 I_0(r\alpha) + B_2 K_0(r\alpha)$$

$$w_p(r) = \frac{Ga}{2\alpha\mu}\left[\frac{K_0(r\alpha)I_0(b\alpha) - I_0(r\alpha)K_0(b\alpha)}{K_1(a\alpha)I_0(b\alpha) + K_0(b\alpha)I_1(a\alpha)}\right] \tag{2.33}$$

Flow rate in clear zone

$$Q_c = 2\pi\int_0^a r w_c \, dr$$

$$Q_c = \frac{G\pi a^4}{8\mu} + \frac{G\pi a^3}{2\mu\alpha}\left[\frac{K_0(a\alpha)I_0(b\alpha) - I_0(a\alpha)K_0(b\alpha)}{K_0(b\alpha)I_1(a\alpha) + K_1(a\alpha)I_0(b\alpha)}\right] \qquad (2.34)$$

Flow rate in porous lining

$$Q_p = 2\pi\int_a^b r w_p \, dr$$

$$Q_p = \frac{G\pi a^2}{\mu\alpha^2}\left[1 - \frac{b}{a}\left\{\frac{K_0(b\alpha)I_1(b\alpha) + I_0(b\alpha)K_1(b\alpha)}{K_0(b\alpha)I_1(a\alpha) + K_1(a\alpha)I_0(b\alpha)}\right\}\right] \qquad (2.35)$$

$$Q_p = \frac{G\pi a^2}{\mu\alpha^2}\left[1 - \frac{1}{a\alpha\left\{K_0(b\alpha)I_1(a\alpha) + K_1(a\alpha)I_0(b\alpha)\right\}}\right] \qquad (2.36)$$

(from Watson [11] page 80, we have $I_0(z)k_1(z) + I_1(z)k_0(z) = \frac{1}{Z}$)

Deductions :

CASE – I :

Flow of the fluid in the absence of magnetic field is given by :

$$w_c(r) = \frac{G(a^2 - r^2)}{4\mu} + \frac{Ga\sqrt{K}}{2\mu}\left[\frac{K_0\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right) - I_0\left(\frac{a}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right) + K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)}\right] \qquad (2.37)$$

$$w_p(r) = \frac{Ga\sqrt{K}}{2\mu}\left[\frac{K_0\left(\frac{r}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right) - I_0\left(\frac{r}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right) + K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right)}\right] \qquad (2.38)$$

$$Q_c = \frac{G\pi a^4}{8\mu} + \frac{G\pi a^3\sqrt{K}}{2\mu}\left[\frac{K_0\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right) - I_0\left(\frac{a}{\sqrt{k}}\right)K_0\left(\frac{b}{\sqrt{k}}\right)}{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right) + K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)}\right] \qquad (2.39)$$

$$Q_p = \frac{G\pi a^2 k}{\mu}\left[1-\frac{\sqrt{k}}{a}\frac{1}{\left\{K_0\left(\frac{b}{\sqrt{k}}\right)I_1\left(\frac{a}{\sqrt{k}}\right)+K_1\left(\frac{a}{\sqrt{k}}\right)I_0\left(\frac{b}{\sqrt{k}}\right)\right\}}\right] \tag{2.40}$$

CASE – A :

If the permeability is large.

$$w_c(r) = \frac{G\left(a^2-r^2\right)}{4\mu}+\frac{Ga^2}{2\mu}\left(1+\frac{a^2}{2k}\right)\log\left(\frac{b}{a}\right)$$

$$= \frac{a^2 G}{4\mu}\left[1-\left(\frac{r}{a}\right)^2+2\left(1+\frac{a^2}{2k}\right)\log\left(1+\frac{\delta}{a}\right)\right] \tag{2.41}$$

$$w_p(r) = \frac{Gr}{2\mu}\left(1+\frac{r^2}{2k}\right)\log\left(\frac{b}{r}\right)$$

$$= \frac{Gr}{2\mu}\left(1+\frac{r^2}{2k}\right)\log\left(1+\frac{\delta}{a}\right) \tag{2.42}$$

$$Q_c = \frac{G\pi a^4}{8\mu}+\frac{G\pi a^4}{4\mu}\left[2+\frac{a^2}{k}\right]\log\left(\frac{b}{a}\right)$$

$$= \frac{G\pi a^4}{8\mu}\left[1+2\left(2+\frac{a^2}{k}\right)\log\left(1+\frac{\delta}{a}\right)\right] \tag{2.43}$$

$$Q_p = \frac{\pi G}{9\mu}\left[\left(b^3-a^3\right)-3a^3\log\left(\frac{b}{a}\right)\right]+\frac{\pi G}{50\mu k}\left[\left(b^5-a^5\right)-5a^5\log\left(\frac{b}{a}\right)\right]$$

$$= \frac{\pi b a^3}{\mu}\left\{\frac{a^2}{50k}\left(\left(1+\frac{\delta}{a}\right)^5-\log\left(1+\frac{\delta}{a}\right)^5-1\right)+\frac{1}{9}\left(\left(1+\frac{\delta}{a}\right)^3-\log\left(1+\frac{\delta}{a}\right)^3-1\right)\right\} \tag{2.44}$$

CASE – B :

If the permeability is very small.

$$w_c(r) = \frac{G(a^2 - r^2)}{4\mu} + \frac{32}{67}\frac{Gak}{\mu}\frac{\sinh\left(\frac{b-a}{\sqrt{k}}\right)}{\cosh\left(\frac{a-b}{\sqrt{k}}\right)} \tag{2.45}$$

$$w_p(r) = \frac{32Ga^{3/2}\sqrt{k}}{67\mu}\frac{\sinh\left(\frac{b-r}{\sqrt{k}}\right)}{\cosh\left(\frac{a-b}{\sqrt{k}}\right)} \tag{2.46}$$

$$Q_c = \frac{G\pi a^4}{8\mu} + \frac{32}{67}G\pi a^3 k \tag{2.47}$$

$$Q_p = \frac{64G\pi a^{3/2}k}{67\mu}\left[a + k\tanh\left(\frac{b-a}{\sqrt{k}}\right) - b\sec h\left(\frac{b-a}{\sqrt{k}}\right)\right] \tag{2.48}$$

CASE – II :

In the absence of porous lining. The flow of fluid becomes flow in clear medium under magnetic field :

$$w_c(r) = \frac{G(a^2 - r^2)}{4\mu} + \frac{aG}{2\mu M}\left[\frac{K_0(aM)I_0(bM) - I_0(aM)K_0(bM)}{K_0(bM)I_1(aM) + K_1(aM)I_0(bM)}\right] \tag{2.49}$$

$$w_p(r) = \frac{Ga}{2\mu M}\left[\frac{K_0(rM)I_0(bM) - I_0(rM)K_0(bM)}{K_1(aM)I_0(bM) + K_0(bM)I_1(aM)}\right] \tag{2.50}$$

$$Q_c = \frac{G\pi a^4}{8\mu} + \frac{G\pi a^3}{2M\mu}\left[\frac{K_0(aM)I_0(bM) - I_0(aM)K_0(bM)}{K_0(bM)I_1(aM) + K_1(aM)I_0(bM)}\right] \tag{2.51}$$

$$Q_p = \frac{G\pi a^2}{\mu M^2}\left[1 - \frac{1}{aM}\frac{1}{\left[K_0(bM)I_1(aM) + K_1(aM)I_0(bM)\right]}\right] \tag{2.52}$$

The magnetic parameter decreases the flow of the fluid as M increases.

Conclusions:

The flow of Newtonianfluid through a straight circular tube, the inner side of which is having porous lining of the thickness δ is examined. The flow rate in the clear medium is increasing with increasing the thickness, at the same the flow rate is decreasing with increasing values of k, where as the flow rate in the porous region is decreasing with increasing the thickness and also as k increases the flow rate decrease. Which can be observed from fig.3, 4.

From fig. 1, 2 it is observed that as thickness increases velocity in the clear region is increasing. Therefore the effect of porous lining is to increase the velocity of the fluid in the clear region. The velocity profile of w_C is become more parabolic with increasing δ. The velocity profile w_p in the fig.2 shows an increasing velocity with increasing thickness.

Increase in magnetic parameter effects the velocity in two regions. The velocity decreases with increase in M.

Velocity Profile of w_c for different δ

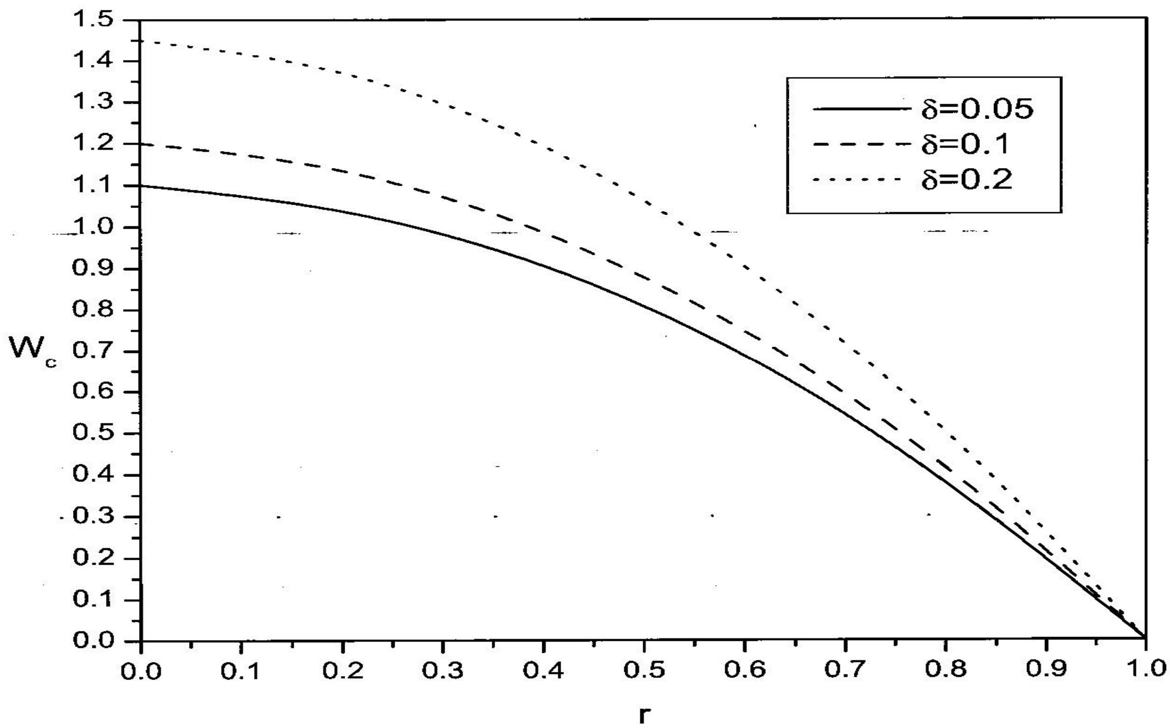


Fig. 1

Velocity Profile of w_p for different δ

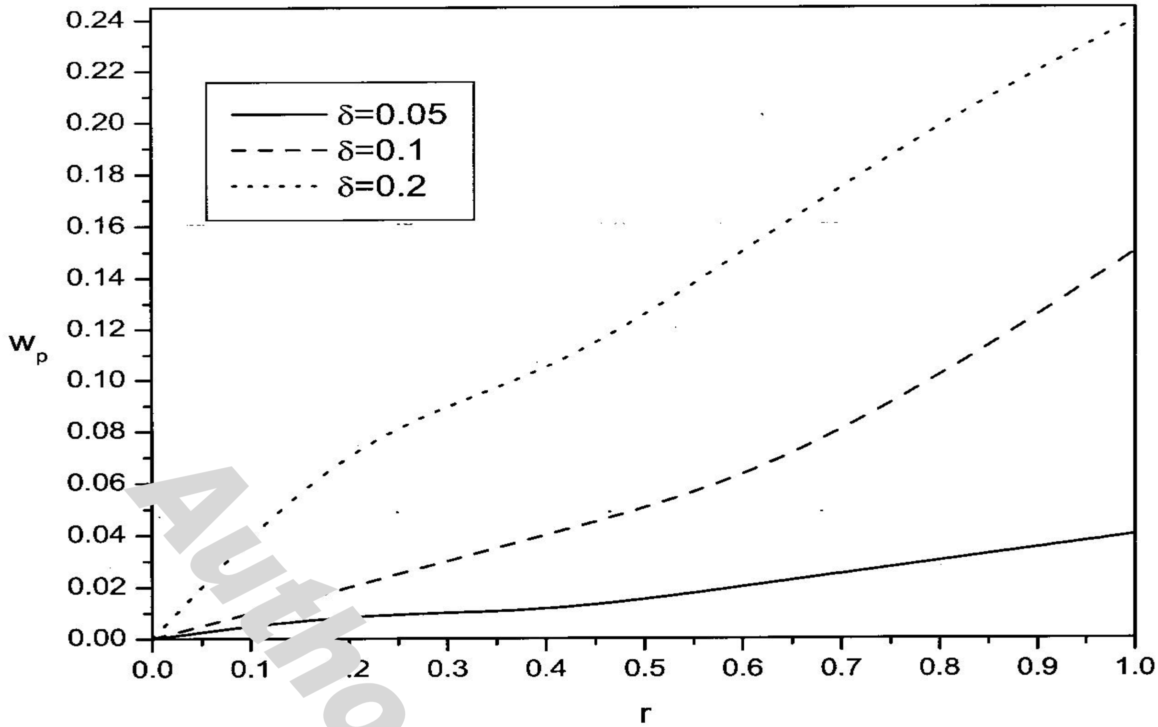


Fig. 2

Flow rate Q_c against thickness of porous lining δ

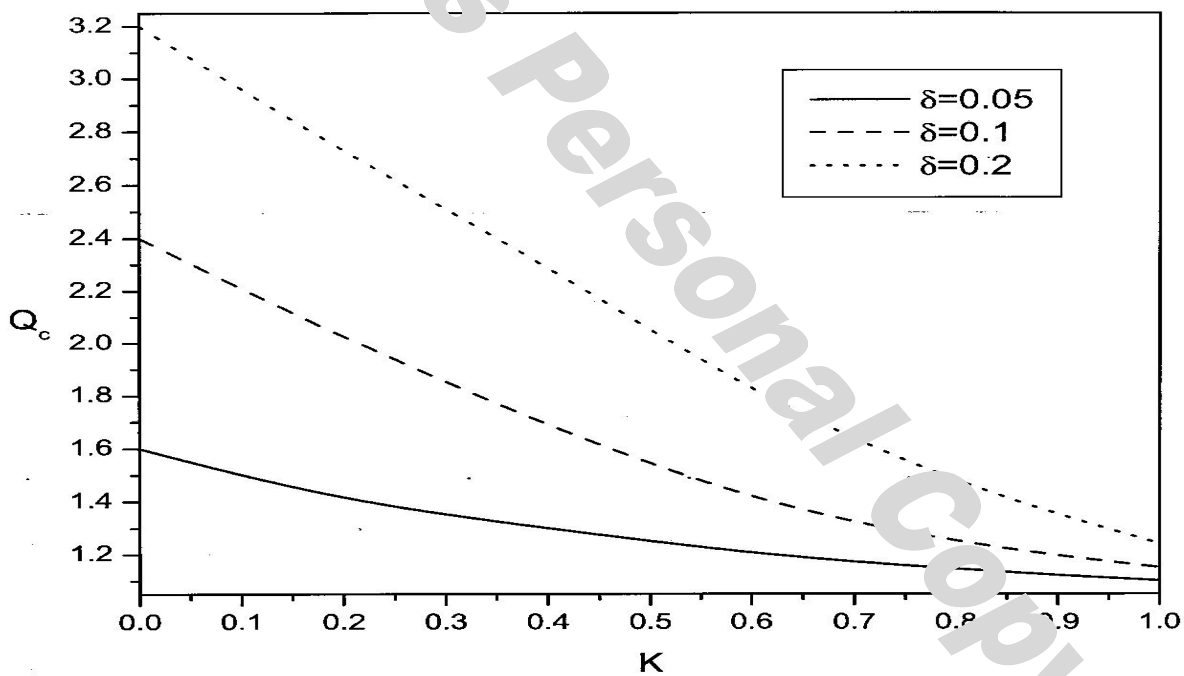


Fig. 3

Flow rate Q_p against thickness of porous lining δ

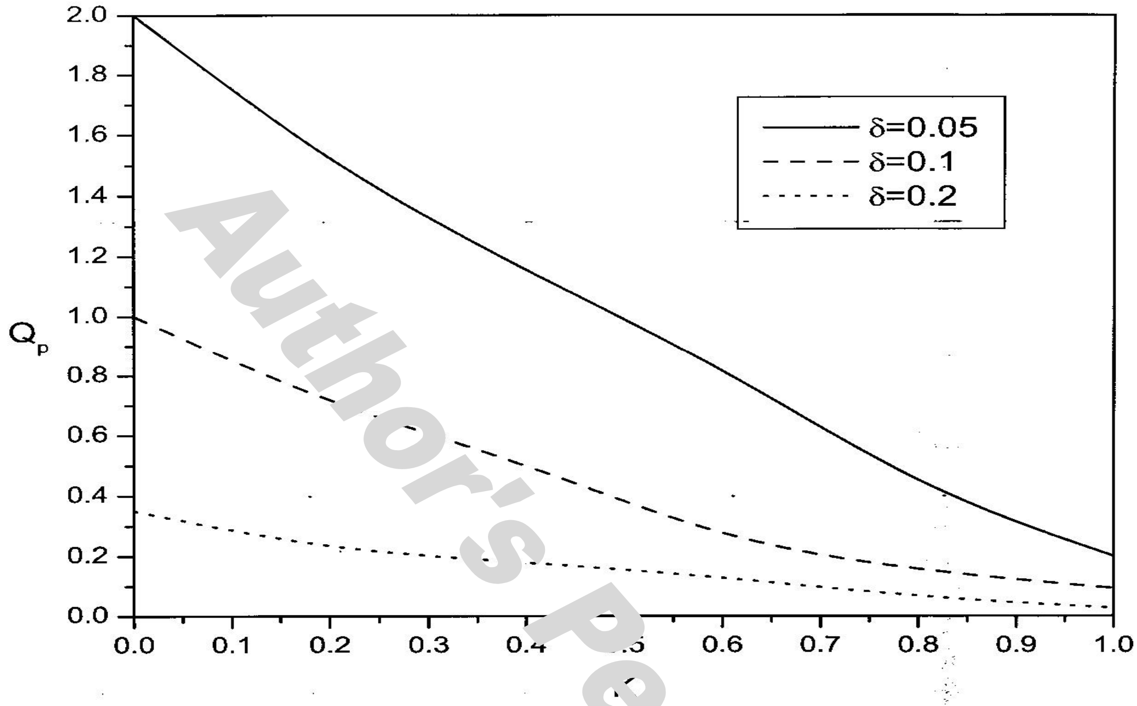

δ=0.05
δ=0.1
δ=0.2
Q_p
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.1
0.2
0.3
0.6
0.7
0.8
0.9
1.0

Fig. 4

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APPENDIX: FLOW FOR LARGE POROSITIES

Following Watson [11], we note that for small x

$$I_0(x) = 1 + \frac{x^2}{4} + \frac{x^4}{64} \ldots\ldots\ldots\ldots$$

$$I_1(x) = \frac{x}{2} + \frac{x^3}{16} + \frac{x^5}{384} \ldots\ldots\ldots\ldots$$

$$K_0(x) = -\left(r + \log\frac{x}{2}\right)\left(1 + \frac{x^2}{4} + \frac{x^4}{64} + \ldots\ldots\right) + \frac{x^2}{4} + \frac{3x^4}{128}$$

$$K_1(x) = \left\{r + \log\left(\frac{x}{2}\right)\right\} I_1(x) - \frac{1}{2}\left[\left\{\phi(0) + \phi(1)\right\}\frac{x}{2} + \left\{\phi(1) + \phi(2)\right\}\frac{x^3}{8} \ldots\ldots\right]$$

FLOW FOR SMALL POROSITES :

Following Watson for large Z , we have

$$I_0(Z) = \frac{e^z}{\sqrt{2\pi z}}\left\{1 + \frac{1}{8z} + \frac{9}{1024z^3} \ldots\ldots\right\}$$

$$I_1(Z) = \frac{e^z}{\sqrt{2\pi z}}\left\{1 - \frac{3}{8z} - \frac{15}{1024z^3} \ldots\ldots\right\}$$

$$K_0(Z) = \sqrt{\frac{\pi}{2z}}e^{-z}\left\{1 - \frac{1}{8z} + \frac{9}{1024z^3} \ldots\ldots\right\}$$

$$K_1(Z) = \sqrt{\frac{\pi}{2z}}e^{-z}\left\{1 + \frac{3}{8z} - \frac{15}{1024z^3} \ldots\ldots\right\}$$