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Solution of Coupled Diffusion of Temperature and Moisture Problem

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ABSTRACT

The process of heat and moisture transfer can be coupled depeding on the specified environmental conditions.The governing equations for the interdependence of heat and moisture are based on conservation laws in mechanics and linear symmetric flow force relations in irreversible thermodynamics. In this paper we have presented general solution of one-dimensional coupled diffusion of temperature and moisture problem.
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1. INTRODUCTION

The interdependence of heat and moisture in solids has been discussed by J. Crank [2], R.J. Hartrantt and G.C. Sih [3,4,5] and by few other researchers. According to G. King and A.B.D. Cossie [1], if an initially uniform dry solid is suddenly immersed in a body of water of the same temperature which is kept constant, the solid will absorb moisture at a rate which is initially proportional to the square root of time. The slope of the moisture content versus squate root of time curve can be used to compute the effective diffusion coefficient. It depends on the diffusion coefficients for both heat and moisture and on the coupling parameters associated with the heat and moisture flow rate. For conditions in which moisture diffuses into the solid, heat also diffuse into the solid at a rate proportional to the square root of time initially resulting in the increase in temperature of the solid. Ultimately, as the diffusion of moisture slows down, the flow of heat is reversed and the temperature of the solid decreases to its initial value. These are similar reciprocal effects for the case in which a slab is suddenly subjected to a change in surface temperature with no change in the surface moisture concentration.

The equations governing the interdependence of heat and moisture can be derived from the conservation laws in mechanics and linear symmetric flow force relations in irreversible thermodynamics. Depending on the definition of physical parameters, several models can be proposed and all lead to the same system of equations differing only in the interpretation of the moisture concentration.

In this paper we have presented general solution of one-dimensional coupled diffusion of temperature and moisture problem.

2. Governing Equations

The Coupling of heat and moisture is interpretted by considering Dufour and Soret effects. The governing coupled equations in temperature T and moisture concentration C are given by,

$$\mathrm{D}\nabla^2 C = \frac{\partial C}{\partial t} - \lambda \frac{\partial T}{\partial t} \qquad (1)$$

$$\mathsf{D}\nabla^2 T = \frac{\partial T}{\partial t} - \gamma \frac{\partial C}{\partial t} \qquad (2)$$

Where, D and D are given by,

$$\mathrm{D} = \left(1 - \lambda\gamma\right)\mathrm{D}_m \qquad (3)$$

$$\mathsf{D} = \left(1 - \lambda\gamma\right)\mathrm{D}_h \qquad (4)$$

Here λ and γ are coupling coefficients and are given by

$$\lambda = \frac{\mathrm{D}_m}{\mathrm{D}_h} \cdot \frac{C}{R_g T^2} Q_h \qquad (5)$$

$$\gamma = \frac{\mathrm{D}_h}{K} Q_h \qquad (6)$$

Here R_g is gas constant, Q_h is heat transport defined as the ratio of heat to moisture flow rate in an isothermal solid. D_m is moisture diffusion coefficient and D_h is temperature diffusion coefficient.

3. Solution

The governing coupled equations are [Equations (1) & (2)]

$$\left.\begin{aligned} D\nabla^2 C &= \frac{\partial C}{\partial t} - \lambda \frac{\partial T}{\partial t} \\ \mathsf{D}\nabla^2 C &= \frac{\partial T}{\partial t} - \gamma \frac{\partial C}{\partial t} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A)$$

Above equations may be decoupled to two simple diffusion equations provided that λ and γ are constants. This can be done by introducing normal coordinates consisting of a linear combination of C and T in the form ($m^1C + n^1T$). Multiplying first equation of (A) by m^1/**D** and second equation of (A) by n^1/D and adding, we get

$$\nabla^2\left(m^1C+n^1T\right)-\frac{\partial}{\partial t}\left[\left(\frac{m^1}{\mathbf{D}}-\frac{n^1\gamma}{\mathrm{D}}\right)C+\left(\frac{n^1}{\mathrm{D}}-\frac{m^1\lambda}{\mathbf{D}}\right)T\right]=0 \qquad (7)$$

Let the physical parameters in the above equation can be arranged according to

$$\frac{1}{\mathrm{D}}-\frac{n^1}{m^1}\frac{\gamma}{D}=\frac{1}{D}-\frac{m^1}{n^1}\frac{\lambda}{\mathrm{D}}=\mu^2 \qquad (8)$$

Such that the quantity (m^1C+n^1T) obeys the simple diffusion equation.

$$\nabla^2\left(m^1C+n^1T\right)-\frac{\partial}{\partial t}\left[\mu^2\left(m^1C+n^1T\right)\right]=0 \qquad (9)$$

Equation (8) may be expressed by eliminating m^1/n^1 as

$$\left(\mu^2-\frac{1}{\mathrm{D}}\right)\left(\mu^2-\frac{1}{D}\right)=\frac{\lambda\gamma}{\pi D} \qquad (10)$$

Solving the above equation, we get

$$\mu^2=\frac{\left(\mathrm{D}+D\right)\pm\sqrt{\left(D-\mathrm{D}\right)^2+4\lambda\gamma\mathrm{D}D}}{2\mathrm{D}D} \qquad (11)$$

μ^2 has two values μ_1^2 and μ_2^2.

$$\therefore \mu_1^2+\mu_2^2=\frac{1}{\mathrm{D}}+\frac{1}{D} \qquad (12)$$

From the above equation, we get

$$D\left(1-\mu_1^2\mathrm{D}\right)=-\mathrm{D}\left(1-\mu_2^2D\right) \qquad (13)$$

If coupling vanishes, μ_1^2 tends to $\frac{1}{D}$ and μ_2^2 tends to $\frac{1}{D}$.

Using equations (8) and (13),

The corresonding values of m_j^1 and n_j^1 (j = 1,2) are

$$\frac{n_1^1}{m_1^1} = -\frac{1-\mu_2^2 D}{\gamma} \tag{14}$$

$$\frac{m_2^1}{n_2^1} = -\frac{1-\mu_1^2 D}{\lambda} \tag{15}$$

By taking arbitrarily the quantities m_1^1 and n_2^1 as unity, a solution for equations (9) can be obtained as

$$\Delta C - \left(\frac{1-\mu_2^2 D}{\gamma}\right)\Delta T = \Psi_1\left(x, y, z, t\right) \tag{16}$$

$$\Delta T - \left(\frac{1-\mu_1^2 D}{\lambda}\right)\Delta C = \Psi_2\left(x, y, z, t\right) \tag{17}$$

Where $\Delta C = C - C_0$ and $\Delta T = T - T_0$

Where C_0 and T_0 are initial values at time t = 0.

Solving the above equations for ΔC and ΔT , we get,

$$\Delta C = \Psi_1^1\left(x, y, z, t\right) + \frac{1-\mu_2^2 D}{\gamma}\Psi_2^1\left(x, y, z, t\right) \tag{18}$$

$$\Delta T = \Psi_2^1\left(x, y, z, t\right) + \frac{1-\mu_1^2 D}{\lambda}\Psi_1^1\left(x, y, z, t\right) \tag{19}$$

Where

$$\Psi_1^1(x, y, z, t) = \frac{1-\mu_2^2 D}{D(\mu_1^2 - \mu_2^2)}\Psi_1(x, y, z, t) \tag{20}$$

$$\Psi_2^1(x, y, z, t) = \frac{1-\mu_2^2 D}{D\ (\mu_1^2 - \mu_2^2)}\Psi_2(x, y, z, t) \tag{21}$$

The functions $\Psi_j = j(1,2)$ are governed by the ordinary diffusion equations with coefficients $\frac{1}{\mu_1^2}$ and $\frac{1}{\mu_2^2}$ as follows:

$$\frac{1}{\mu_j^2}\nabla^2\Psi_j - \frac{\partial \Psi_j}{\partial t} = 0,\ j = 1,2 \tag{22}$$

Substituting , $t_j = t/\mu_j^2$ above equation can be written as,

$$\nabla^2\Psi_j = \frac{\partial \Psi_j}{\partial t_j},\ j = 1,2 \tag{23}$$

and can be easily solved by applying Laplace transform.

4. Solution of one dimensional problem

Consider a slab of solid with thickness $2h_x$ in the x-direction. The dimensions in the y and z directions are assumed to extend to infinity. Since heat and moisture transfer occurs in one direction ,Ψ_j depends only on x and t. Let Ψ_j on the edges $x = \pm h_x$ be specifies as

$$\Psi_j = \left(\pm h_x, t\right) = \Psi_0, t \geq 0 \tag{24}$$

Introducing the non-dimensional space and time variables ξ and τ_j as

$$\xi = \frac{x}{h_x},\quad \tau_j = \frac{t}{\left(2h_x\mu_j\right)^2} \tag{25}$$

the solution of equation (22) can be obtained as,

$$\frac{\Psi_j}{\left(\Psi_0\right)_j} = 1 - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}.\cos\left[\pi\left(2n-1\right)\xi\right].\exp\left[-\pi^2\left(2n-1\right)^2\tau_j\right] \tag{26}$$

The series in the above equation converge rapidly for moderate and large values of τ_j.

5. Conclusion

In this paper we have presented series solution of one dimensional coupled diffusion of temperature and moisture problem.

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