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Pre-Homeomorphisms in Topological Ordered Spaces

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Abstract:

In this paper we introduce I-pre-homeomorphisms, D-pre-homeomorphisms and B-pre-homeomorphisms for topological ordered spaces after introducing I-pre-continuous, D-pre-continuous and B-pre-continuous maps, I-pre-open, D-pre-open, B-pre-open maps, I-pre-closed, D-pre-closed and B-pre-closed maps for topological ordered spaces together with their characterizations.

Keywords And Phrases: Topological ordered spaces, pre-open sets, increasing sets, decreasing sets, pre-continuous map, pre-open map, pre-closed map.

1. **INTRODUCTION**. Leopoldo Nachbin [4] inititated the study of topological ordered spaces. A topological ordered space is a triple $(X, \tau, \leq)$, where τ is a topology on X and $\leq$ is a partial order on X. Let $(X, \tau, \leq)$ be a topological ordered space. For any $x \in X$, $[x, \rightarrow] = \{y \in X / x \leq y\}$ and $[\leftarrow, x] = \{y \in X / y \leq x\}$. A subset A of a topological ordered space $(X, \tau, \leq)$ is said to be increasing if $A = i(A)$ decreasing if $A = d(A)$, where $i(A) = \underset{a \in A}{\cup} [a, \rightarrow]$ and $d(A) = \underset{a \in A}{\cup} [\leftarrow, a]$.

Observe that the complement of an increasing set is a decreasing set and the complement of a decreasing set is an increasing set. A subset of a topological ordered space $(X, \tau, \leq)$ is said to be balanced if it is both increasing and decreasing. M.K.R.S. Veera Kumar [5] studied different types of maps between topological ordered-spaces. A.S. Mashour [1] introduced pre-open sets and pre-closed sets. A subset A of a topological space (X, τ) is called a pre-open set [1] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.

Note that the complement of pre-open set is pre-closed and vice versa. We denote the complement of A by C(A).

For a subset A of a topological ordered space $(X, \tau, \leq)$ define $ipcl(A) = \cap\{F/F$ is an increasing pre-closed subset of X containing A $\}$, $dpcl(A) = \cap\{F/F$ is decreasing pre-closed subset of X containing A$\}$,

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bpcl(A) = ∩{F/F is a balanced pre-closed subset of X containing A},
A^{ipo} = ∪{G/G is an increasing pre-open subset of X contained in A},
A^{dpo} = ∪{ G/G is an decreasing pre-open subset of X contained in A} and
A^{bpo} = ∪{G/G is a balanced pre-open subset of X contained in A}.

Clearly ipcl(A) (resp. dpcl (A), bpcl (A)) is the smallest increasing (resp. decreasing, balanced) pre-closed set containing A.

IPO(X) (resp. DPO(X), BPO(X)) denotes the collection of all increasing (resp. decreasing, balanced) pre-open subset of a topological ordered space (X, τ, ≤). IPC(X) (resp. DPC(X), BPC(X)) denotes the collection of all increasing (resp. decreasing, balanced) pre-closed subsets of a topological ordered space (X, τ, ≤).

A function f:(X, τ)→ (Y, σ) is called pre-continuous [2] if $f^{-1}(V)$ is a pre-closed set of (X, τ) for every closed set V of (Y, σ).

A function f:(X, τ) → (Y, σ) is called pre-open [2] if f(G) is pre-open set in Y, for every open set G of X,.

A function f:(X, τ) → (Y, σ) is called pre-closed [2] if f(F) is pre-closed set in Y, for each closed set F of X,

2. I-PRE-CONTINUOUS, D-PRE-CONTINUOUS, B-PRE-CONTINUOUS MAPS

We introduce the following definition.

DEFINITION 2.01. A function f: (X, τ, ≤) → (X*, τ*, ≤*) is called a I-pre-continuous (resp. D-pre-continuous, B-pre-continuous) maps if $f^{-1}(V)$ ϵ IPC(X), (resp. $f^{-1}(V)$ ϵ DPC(X), $f^{-1}(V)$ ϵ BPC(X)) whenever V is closed in X.

It is evident that every x-pre-continuous map is pre-continuous for x = I, D, B and that every B-pre-continuous map is both I-pre-continuous and D-pre-continuous.

The following example shows that a pre-continuous map need not be x-pre-continuous for x = I,D,B.

EXAMPLE 2.01. LetX ={a,b,c} τ = {ϕ,X,{a},{b},{a,b}} and ≤ = {(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)}. Clearly (X, τ, ≤) is a topological ordered space. Let f be the identity map from (X, τ, ≤) onto itself.{a,c} is closed set in X. But $f^{-1}(\{a,c\})$ is neither an increasing nor a decreasing pre-closed set. Thus f is not x-pre-continuous for x = I,D,B.

The following example shows that a D-pre-continuous map need not be a B-pre-continuous.

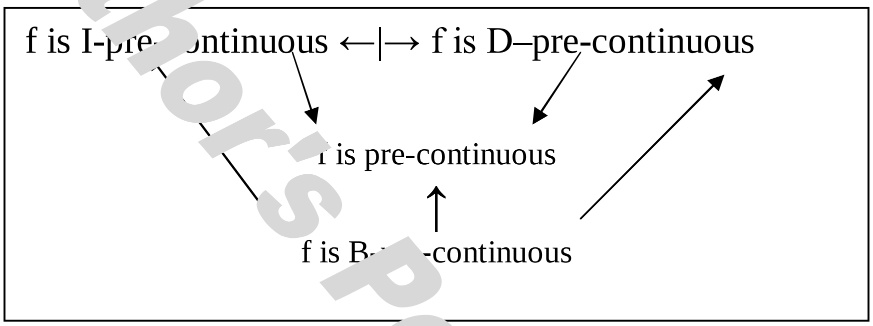
EXAMPLE 2.02. Let X ={a,b,c} = X*, τ = {ϕ,X,{a},{a,b}} and ≤ = {(a,a),(b,b),(c,c),(b,a)}, ≤* ={(a,b),(b,b),(c,c)}. Let g be the identity map from (X, τ, ≤) onto (X*,τ*,≤*). Then g is not B-pre-continuous, however g is D-pre-continuous.

The following example supports that an I-pre-continuous map need not be B-pre continuous map.

EXAMPLE 2.03. Let X ={a,b,c} = X*, τ = {ϕ,X,{a},{b},{a,b}}= τ* and let ≤ = {(a,a) ,(b,b), (c,c), (a,c), (b,c)} = ≤*. Define f: (X, τ, ≤) → (X*, τ*, ≤*) is an identity map then f is I-pre-continuous but not a B-pre-continuous map.

2.01 Thus we have the following diagram.

For a function f:(X, τ, ≤) →(X*, τ*, ≤*)



,where p → q (resp. p ←|→ q) represents p implies q but q need not imply p (resp. p and q are independent of each other)

The following theorem characterizes I-pre-continuous maps.

THEOREM 2.01. For a function f:(X, τ, ≤) → (X*, τ*, ≤*) the following statements are equivalent.

1) f is I-pre-continuous.
2) f (ipcl(A)) ⊆ cl(f (A)) for any A ⊆ X.
3) ipcl(f^{-1} (B)) ⊆ f^{-1}(cl(B)) for any B ⊆ X*.
4) For any closed subset K of (X*, τ*, ≤*), f^{-1}(K) is an increasing pre-closed subset of (X, τ, ≤).

Proof (1) => (2) Since cl(f(A)) is closed in X* and f is I-pre-continuous we have that f^{-1}(cl(f(A))) is an increasing pre-closed set in X. f(A) ⊆ cl(f(A)) => A ⊆ f^{-1}(cl(f(A))), and ipcl(A) is the smallest increasing pre-closed set containing A. Therefore ipcl(A) ⊆ f^{-1}(cl(f(A))). Hence f(ipcl(A)) ⊆ cl(f(A))).

(2) =>(3) Put A = f^{-1}(B). Then f(A) ⊆ B and cl(f(A)) ⊆ cl(B). Therefore ipcl (f^{-1}(B)) ⊆ f^{-1}(cl(B))

(3) => (4) Let K be any closed set in X*. From (3) ipcl(f^{-1}(K)) ⊆ f^{-1}(cl(K)) = f^{-1}(K). Clearly f^{-1}(K) ⊆ ipcl(f^{-1}(K)). Thus f^{-1}(K) is increasing pre-closed set in (X, τ, ≤) whenever K is closed in (X*, τ*, ≤*).

(4) => (1) follows from definition.

THEOREM 2.02. For a function f:(X, τ, ≤) →(X*, τ*, ≤*), the following statements are equivalent.

1) f is D-pre-continuous.
2) f (dpcl(A)) ⊆ cl(f(A)) for any A ⊆ X.
3) dpcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for any B ⊆ X*.
4) for every closed subset K of (X*, τ* , ≤*), f^{-1}(K) is a decreasing pre-closed subset of (X, τ, ≤).

THEOREM 2.03. For a function f:(X, τ, ≤) →(X*, τ,* ≤*), the following statements are equivalent.

1) f is B-pre-continuous.
2) f(bpcl(A)) ⊆ cl (f(A)) for any A ⊆ X.
3) bpcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for any B ⊆ X*.
4) For every closed subset K of (X*, τ*, ≤*), f^{-1}(K) is both increasing and decreasing pre-closed subset of (X, τ, ≤).

3. I–PRE-OPEN, D-PRE-OPEN AND B-PRE-OPEN MAPS.

DEFINITION 3.01. A function f:(X, τ, ≤) →(X*, τ*, ≤*) is called an I-pre-open map (resp. D-pre-open, B-pre-open map if f(G) ϵ IPO(X*) (resp.f(G) ϵ DPO(X*), f(G) ϵ BPO(X*) whenever G is an open subset of (X, τ, ≤).

It is evident that every x-pre-open map is an pre-open map for x= I,D,B and that every B-pre-open map is both I-pre-open and D-pre-open.

The following example shows that a pre-open map need not be x-pre-open for x= I,D,B.

EXAMPLE 3.01. Let X ={a,b,c} = X*, τ = {ϕ,X,{a},{b}} ,{a,b} = τ^* and ≤ ={(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)} clearly (X, τ, ≤) is a topological ordered space. Let f be the identity map from (X, τ, ≤) onto itself.{b}is a open set in X but f({b}) = {b} is not I-pre-open set, not D-pre-open set, not B-pre-open set in X^*. Therefore f is not I-pre-open map, not D-pre-open map and not B-pre-open map

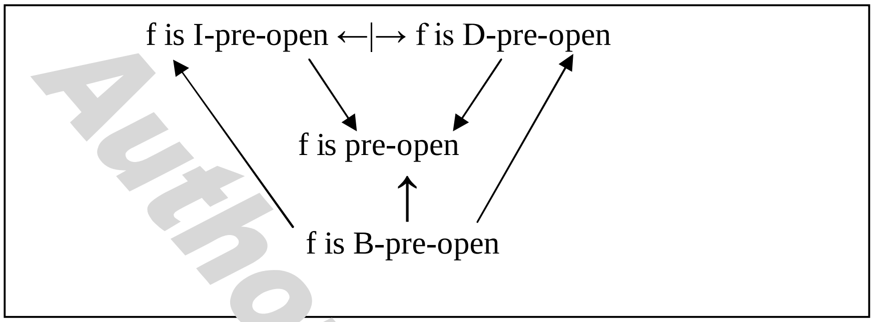
The following example shows that D-pre-open map need not be a B-pre-open map.

EXAMPLE 3.02. Let X ={a,b,c} = X*, τ = {ϕ,X,{a},{b},{a,b}}= τ*,≤ ={(a,a),(b,b),(c,c),(a,c)} and ≤* ={(a,a),(b,b),(c,c),(a,c),(b,c)}. Let θ be the identity map from (X, τ, ≤) onto **(X*, τ*, ≤*).** θ is D-pre-open map but not B-pre-open map.

EXAMPLE 3.03. Let X ={a,b,c} = X*, τ = {ϕ,X,{a},{a,c}} = τ *≤ ={(a,a),(b,b),(c,c),(c,a),(b,c),(b,a)}= ≤*. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be the identity map. Then f is an I-Pre-open map but not B-Pre-open map.

3.01 Thus we have the following diagram

For a function f.(X, τ, ≤) → (X*, τ*, ≤*)



,where p→ q (resp.p ←|→ q) represents p implies q but q need not imply p (resp. p and q are independent of each other.

LEMMA 3.01. Let A be any subset of a topological ordered space (X, τ, ≤). Then

1) C (dpcl(A)) = $(C(A))^{ipo}$.
2) C (ipcl(A)) = $(C(A))^{dpo}$.
3) C (bpcl(A)) = $(C(A))^{bpo}$.

Proof.1 C(dpcl(A)) = C{∩F/F is a decreasing pre-closed subset of X containing A}
= ∪{C(F)/F is a decreasing pre-closed subset of X containing A}
= ∪{G/G is an increasing pre-open subset of X contained in C(A)}
= $(C(A))^{ipo}$.

The proof of (2) and (3) are analogous to as that of (1) and hence omitted

The following theorem characterizes I-pre-open functions.

THEOREM 3.01. For any function f:(X, τ, ≤) → (X*, τ*, ≤*), the following statements are equivalent.

1) f is an I-pre-open map.
2) $f(A^0) \subseteq [f(A)]^{ipo}$ for any A ⊆ X.
3) $[f^{-1}(B)]^0 = f^{-1}(B^{ipo})$ for any B ⊆ X*.

Proof. (1) => (3). Since $[f^{-1}(B)]^0$ is open in X and f is an I-pre-open, $f((f^{-1}(B))^0) \subseteq f(f^{-1}(B)) \subseteq B$ and $f([f^{-1}(^1(B)]^0)$ is I-pre-open in X*.Then $f((f^{-1}(B))^0) \subseteq B^{ipo}$ since B^{ipo} is the largest increasing pre-open set contained in B. Therefore $[f^{-1}(B)]^0 \subseteq f^{-1}(B^{ipo})$

(3) => (2). Replacing B by f(A) in (3), we have $[f^{-1}(f(A))]^0 \subseteq f^{-1}([f(A)]^{ipo})$.

Since $A^0 \subseteq [f^{-1}(f(A)]^0$, we have $A^0 \subseteq f^{-1}([f(A)]^{ipo})$. $f(A^0) \subseteq f(f^{-1}([f(A)]^{ipo})) \subseteq [f(A)]^{ipo}$.

Hence $f(A^0) \subseteq [f(A)]^{ipo}$.

(2) => (1). Let G be any open set in X. Then $f(G) = f(G^0) \subseteq [f(G)]^{ipo} \subseteq f(G)$. Therefore f(G) is an increasing pre-open set in X* f is an I-Pre-open map.

The following two theorems give characterizations for D-pre-open map and B-pre-open maps, whose proofs are similar to as that of the above theorem.

THEOREM 3.02. For any function $f:(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$, the following statements are equivalent.

1) f is D-pre-open map.
2) $f(A^0) \subseteq [f(A)]^{dpo}$ for any $A \subseteq X$.
3) $[f^{-1}(B)]^0 \subseteq f^{-1}(B^{dpo})$ for any $B \subseteq X^*$.

THEOREM 3.03. For any function $f:(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$, the following statements are equivalent.

1) f is B-pre-open map.
2) $[f(A^0)] \subseteq [f(A)]^{bpo}$ for any $A \subseteq X$.
3) $[f^{-1}(B)]^0 \subseteq f^{-1}(B^{bpo})$ for any $B \subseteq X^*$.

THEOREM 3.04. Let $f:(X, \tau, \leq_1) \rightarrow (Y, \sigma, \leq_2)$ and $g: (Y, \sigma, \leq_2) \rightarrow (Z, \eta, \leq_3)$ be any two mappings. Then $gof:(X, \tau, \leq_1) \rightarrow (Z, \eta, \leq_3)$ is x-pre-open if f is open and g is x-pre-open for x = I,D,B.

Proof. Omitted.

4. I-PRE-CLOSED, D-PRE-CLOSED AND B-PRE-CLOSED MAPS

DEFINITION 4.01. A function $f:(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ is called an I-pre-closed (resp. D-Pre-closed, B-Pre-closed) map if f(G) ϵ IPC(X*) (resp. f(G) ϵ DPC(X*), f(G) ϵ BPC(X*)) whenever G is a closed subset of X. Clearly x-pre-closed map is a pre-closed map for x = I,D,B and every B-pre-closed map is both I-pre-closed and D-pre-closed map.

The following example shows that a pre-closed map need not be a x-pre-closed for x = I,D,B.

EXAMPLE 4.01. Let X ={a,b,c}, τ ={ϕ,X,{a},{b},{a,b}} and ≤ ={(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)}clearly (X, τ, ≤) is a topological ordered space. Let f be the identity map from (X, τ, ≤) onto itself. {a,c} is closed set but f({a,b}) is neither an increasing nor a decreasing pre-closed set. Thus f is not x-pre-closed for x = I, D, B.

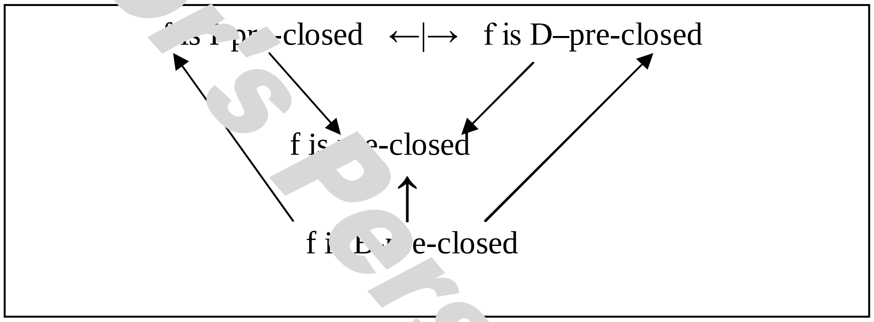
The following example shows that an I-pre-closed map need not be a B-pre-closed map.

EXAMPLE 4.02. Let X ={a,b,c}= X*, τ ={ϕ,X,{a},{b},{a,b}} = τ *,≤ ={(a,a),(b,b),(c,c), (a,c)} and ≤* ={(a,a),(b,b),(c,c),(a,c),(b,c)}. Let θ be the identity map from (X, τ, ≤) onto (X*, τ*, ≤*). θ is I-pre-closed but not a B-pre-closed map.

EXAMPLE 4.03. Let X ={a,b,c}= X*, τ ={ϕ,X,{c},{b,c}} = τ,*≤ ={(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)} = ≤ * and f:(X, τ, ≤) → (X*, τ*, ≤*) be an identity map. Then f is D-pre-closed map but not a B-pre-closed map.

4.01 Thus we have the following diagram

For a function f:(X, τ, ≤) → (X*, τ*, ≤*)



,where p → q (resp. p ←|→ q) represents p → q but q need not imply p
(p and q are independent of each other)

The following theorem characterizes I-pre-closed maps.

THEOREM 4.01. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be any map. Then f is I-pre-closed iff ipcl(f(A)) ⊆ f(cl(A)) for any A ⊆ X.

Proof. Necessity: Since f is I-pre-closed, f(cl(A)) is an increasing pre-closed subset of X. Clearly f(A) ⊆ f(cl(A)). Therefore ipcl(f(A)) ⊆ f (cl(A)) since ipcl(f(A)) is the smallest increasing pre-closed set in X* containing f(A). Sufficiency:Let F be any pre-closed subset of X. Then f(F) ⊆ ipcl(f(F)) ⊆ f(cl(F)) = f(F). Thus f(F) = ipcl(F). So f(F) is an increasing pre-closed subset of X*.Therefore f is an I-pre-closed map.

The following two theorems characterizes D-pre-closed maps and B-pre-closed maps.

THEOREM 4.02. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be any map. Then f is D-pre-closed iff dpcl(A) ⊆ f(cl(A)) for every A ⊆ X.

Proof. Omitted.

THEOREM 4.03. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be any map. Then f is B-pre-closed iff bpcl(A) ⊆ f(cl(A)) for every A ⊆ X.

Proof. Omitted

THEOREM 4.04. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a bijection. Then

1) f is I-pre-open iff f is D-pre-closed
2) f is I-pre-closed iff f is D-pre-open

3) f is B-pre-open iff f is B-pre-closed.

Proof. Necessity. Let F be any closed subset of X. Then f(C(F)) is an increasing pre-open subset of X*. Since f(C(F)) = C(f(F)) and C(f(F)) is an increasing pre-open subset of X*, f(F) is a decreasing pre-closed subset of X*. Therefore f is D–pre-closed.
Sufficiency. Let G be any open subset of X. Then f (C(G)) is a decreasing pre-closed subset of X*. Since f is a bijection, we have f(C(G)) = C(f(G)). So f(G) is an increasing pre-open subset of X*. Therefore f is an I-pre-open map.
Proof of (2) and (3) are similar to that of (1)

THEOREM 4.05. Let f:(X, τ, $\leq_1$) →(Y, σ, $\leq_2$) and g:(Y,σ, $\leq_2$) → (Z, η, $\leq_3$) be any two mappings then gof:(X, τ, $\leq_1$) →(Z, η, $\leq_3$) is x-pre-closed if f is closed and g is x-pre-closed for x=I,D,B.

Proof. Omitted

THEOREM 4.06. Let f:(X, τ, ≤) → (X*, τ*, ≤*), be a bijection. Then the following statements are equivalent.

1) f is an I-pre-open map.
2) f is a D-pre-closed map.
3) f^{-1} is a D-pre-continuous map.

Proof. (1) =>(2). Let f be I-pre-open map. Let F be closed set of X, then C(F) is open. f(C(F)) is an increasing pre-open set of X*. =>C(f(F)) is an increasing pre-open set of X*. =>f(F) is a decreasing pre-closed set of X*. =>f is D-pre-closed map.

(2)=>(3). Let f be a D-pre-closed map. Let F be closed in X, then f(F) is a decreasing pre-closed set of X*. =>[f^{-1}]$^{-1}$ (F) is a decreasing pre-closed set of X*. => f^{-1}: X*→ X is D-pre-continuous.

(3)=>(1) Let F be open in X. Then C(F) is closed in X. => [f^{-1}]$^{-1}$(C(F)) is a decreasing closed subset of X* => C(f(F)) is a decreasing closed set in X*. =>f(F) is an increasing open set in X*. => f is I-pre-open.

THEOREM 4.07. Let f:(X, τ, ≤) → (X*, τ*, ≤*), be a bijection. Then the following are equivalent.

1) f is a D-pre-open map.
2) f is an I-pre-closed map.
3) f^{-1} is D-pre-continuous map.

Proof. Omitted

THEOREM 4.08. Let f:(X, τ, ≤) → (X*, τ*, ≤*), be a bijection. Then the following statements are equivalent.

1) f is B-pre-open map.
2) f is an B-pre-closed map.
3) f^{-1} is a B-pre-continuous map.

Proof. Omitted

THEOREM 4.09. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be an I-pre-closed map and B,C ⊆ X*. Then.

1) If U is an open neighborhood of f^{-1}(B), then there exists a decreasing pre-open neighborhood V of B such that f^{-1}(B) ⊆ f^{-1}(V) ⊆ U.
2) If f^{-1}(B) and f^{-1}(C) have disjoint neighborhoods then B and C have disjoint pre-open neighborhoods.

Proof. Let U be an open neighborhood of f^{-1}(B). Take C(V) = f(C(U)). Since f is an I-Pre-closed map and C(V) is closed then C(V) = f(C(U)) is an increasing pre-closed subset of X. Since f^{-1} (B) ⊆ U , then C(V) = f(C(U)) ⊆ f(f^{-1}(C(U))) ⊆ C(B). Therefore B ⊆ V. Thus V is a decreasing pre-open neighborhood B. => f^{-1}(B) ⊆ f^{-1}(V). Further C(U) ⊆ f^{-1}(f(C(U))) = f^{-1}(C(V)) = C(f^{-1}(V)) => f^{-1}(V) ⊆ U. Thus f^{-1}(B) ⊆ f^{-1}(V) ⊆ U. Let U_B, U_C be disjoint open neighborhoods of f^{-1}(B), f^{-1}(C), where B,C ⊆ X* From (1) there exists V_B, V_C such that B ⊆ V_B, C ⊆ V_C. Also f^{-1}(B) ⊆ f^{-1}(V_B) ⊆ U_B, f^{-1}(C) ⊆ f^{-1}(V_C) ⊆U_C where V_B, V_C are decreasing closed neighborhoods of B and C respectively. Since U_B∩U_C = ϕ; f^{-1}(V_B) ∩ f^{-1}(V_C) = ϕ. => V_B ∩ V_C = ϕ

Similarly we have the following two theorems (proofs are omitted) regarding D-pre-closed maps and B-pre-closed maps.

THEOREM 4.10. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a D-pre-closed map and B,C, ⊆, X*. Then

1. If U is an open neighborhood of f^{-1} (B), then there exists an increasing pre-open neighborhood V of B such that f^{-1} (B) ⊆ f^{-1}(V) ⊆ U
2. If f^{-1}(B) and f^{-1}(C) have disjoint neighborhoods, then f^{-1}(B) and f^{-1}(C) have disjoint increasing pre-open neighborhoods.

THEOREM 4.11. Let f :(X, τ, ≤) → (X*, τ*, ≤*) be a B-pre-closed map and B,C, ⊆ X*, then

1. If U is an open neighborhood of f^{-1}(B), then there exists an pre-open neighborhood V of B, which is balanced such that f^{-1}(B) ⊆ f^{-1}(V) ⊆ U
2. If f^{-1}(B) and f^{-1}(C) have disjoint neighborhoods, then f^{-1}(B) and f^{-1}(C) have disjoint pre-open neighborhoods which are balanced.

5. I-PRE-HOMEOMORPHISMS, D-PRE-HOMEOMORPHISMS AND B-PRE-HOMEOMORPHISMS

We introduce the following definition

DEFINITION 5.01. A bijection f:(X, τ, ≤) → (X*, τ*, ≤*) is called I-pre-homeomorphism (resp. a D-pre-homeomorphism, B-pre-homeomorphism) if both f and f^{-1} are I-pre-continuous (resp. D-pre-continuous, B-pre-continuous).

Clearly every x-pre-homeomorphism is a pre-homeomorphism for x = I,D,B and every B-pre-homeomorphism is both I-pre-homeomorphism and D-pre-homeomorphism.

The following example shows that a pre-homeomorphism need not be a x-pre-homeomorphism for x= I,D,B.

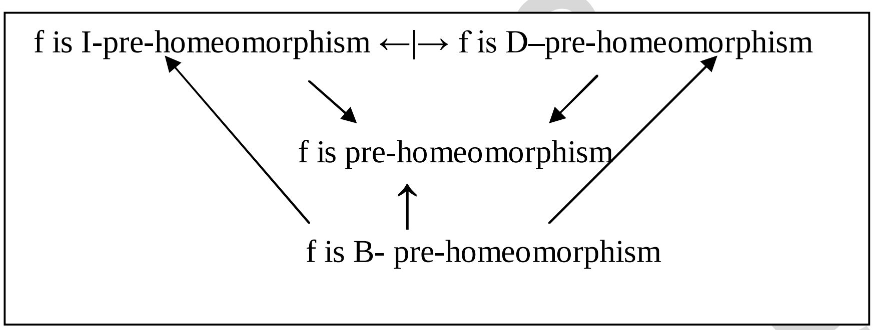
EXAMPLE 5.01. Let X ={a,b,c}, τ ={ϕ,X,{a},{b},{a,b}} and ≤ = {(a,a),(b,b),(c,c),(a,b), (b,c),(a,c)} clearly (X, τ, ≤) is a topological ordered space. Let f be the identity map from (X, τ, ≤) onto itself. {a,c} is closed set but f^{-1}({a,c}) = {a,c} is neither an increasing nor a decreasing pre-closed set.
Thus f is not x-pre-continuous for x= I,D,B. However f is pre-continuous. f is a pre-homeomorphism but not x-pre-homeomorphism for x = I,D,B.

The following example shows that D-pre-homeomorphism need not be a B-pre-homeomorphism

EXAMPLE 5.02. Let X ={a,b,c} = X*; τ = {ϕ,X,{a},{a,b}}= τ*, ≤ = {(a,a),(b,b),(c,c),(b,a)}. Let g be the identity map from (X, τ, ≤) onto (X*, τ*, ≤*). .g is a D-pre-homeomorphism but not a B-pre-homeomorphism.

EXAMPLE 5.03. Let X ={a,b,c} = X*; τ = {ϕ,X,{a},{b},{a,b}};≤ ={(a,a),(b,b),(c,c),(a,c), (b,c)} = ≤*. f: (X, τ, ≤) → (X*, τ*, ≤*) is a identity map then f is I-pre-homeomorphism but not B-pre-homeomorphism.

5.01 Thus we have the following diagram
For a function f:(X, τ, ≤) → (X*, τ*, ≤*)



,where p→ q (resp. p ←|→ q) represents p implies q but q does not imply p (resp. p and q are independent of each other)

THEOREM 5.01. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a bijective I-pre-continuous map. Then the following are equivalent.

1) f is I-pre-homeomorphism.

2) f is D-pre-open.
3) f is I-pre-closed.

Proof. follows from the theorem 4.07.

THEOREM 5.02. Let f:(X, τ, $\le$) → (X*, τ^*, $\le^*$) be bijective D-pre-continuous map. Then the following are equivalent.

1) f is D-pre-homeomorphism
2) f is I-pre-open
3) f is D-pre-closed

Proof. Follows from the theorem 4.06

THEOREM 5.03. Let f:(X, τ, $\le$) → (X*, τ^*, $\le^*$) be a bijection,B-pre-continuous map. Then the following are equivalent.

1) f is B-pre-homeomorphism.
2) f is a B-pre-open map.
3) f is a B-pre-closed map.

Standard separation axioms for topological ordered spaces have been studied systematically by S.D.McCartan. [2,3] Now we examine the separation properties of range spaces under some of these mappings.

DEFINITION 5.02. A topological ordered space (X, τ, $\le$) is said to be upper strongly T_1-ordered iff for each pair of elements a $\nleq$ b in X, there exists a decreasing τ-open neighborhood W of b such that a $\notin$W.

DEFINITION 5.03. A topological ordered space (X, τ, $\le$) is said to be lower strongly T_1-ordered iff for each pair of elements a $\nleq$ b in X, there exists an increasing open neighborhood W of a such that b $\notin$ W. (X, τ, $\le$) is said to be strongly T_1 ordered iff it is both lower and upper strongly T_1-ordered.

DEFINITION 5.04. A topological ordered space (X, τ, $\le$) is said to be upper strongly pre-T_1-ordered iff for each pair of elements a $\nleq$ b in X, there exists a decreasing τ pre-open neighborhood W of b such that a $\notin$ W.

DEFINITION 5.05. A topological order space (X, τ, $\le$) is said to be lower strongly pre-T_1-ordered iff for each pair of elements a $\nleq$ b in X there exists an increasing τ-pre-open neighborhood W of a such that b $\notin$ W. (X, τ, $\le$) is said to be strongly T_1-ordered iff it is both lower and upper strongly pre-T_1-ordered.

THEOREM 5.04. Let f.(X, τ, $\le$) → (X*, τ^*, $\le^*$) be a bijective I-pre-open map as well as a poset isomorphism (i.e. x $\le$ y iff f(x) $\le^*$ f(y) $\forall$x y € X). If (X, τ, $\le$) is a lower strongly T_1-ordered space then (X*, τ^*, $\le^*$) is a lower strongly pre-T_1-ordered space.

Proof. a, b € X* such that a $\nleq^*$ b. Then f^{-1}(a) $\nleq^*$ f^{-1}(b). Since (X, τ, $\le$) is a lower strongly T_1-ordered space, there exists an increasing open neighborhood U of f^{-1}(a) such that f^{-1}(b) $\notin$ U . Thus f(U) is an increasing pre-open neighborhood of f(f^{-1}(a)) = a such that b= f (f^{-1}(b)) € f (U). Therefore (X*, τ*, ≤*) is a lower strongly pre-T_1- ordered space.

THEOREM 5.05. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a bijective D-pre-open map as well as a po-set isomorphism if (X, τ, ≤) is an upper strongly pre-T_1-ordered space then (X*, τ*, ≤*) is upper strongly pre-T_1-ordered space.

THEOREM 5.06. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a bijective B-pre-open map. If f is a po-set isomorphism and (X, τ, ≤) is strongly T_1-ordered space then (X*,τ*, ≤*) is a strongly pre-T_1-ordered space.

DEFINITION 5.06. A topological ordered space (X,τ, ≤) is called strongly pre-T_2-ordered (or strongly pre-Hausdorff ordered or strongly pre-Hausdorff closed) if for each pair of elements a ≰ b in X, there exists pre-open neighborhoods U and V of a and b respectively such that U is an increasing set and V is a decreasing set.

THEOREM 5.07. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be bijective B-pre-open map. If (X,τ) is a Hausdorff space, then (X*,τ*, ≤*) is strongly pre-Hausdorff ordered space.

Proof. Let a, b € X* such that a ≰* b. Then f^{-1}(a) ≠ f^{-1}(b). Since H is Hausdorff, there exists disjoint τ-open neighborhoods U and V of f^{-1}(a) and f^{-1}(b) respectively. Since f is B-Pre-open then f(U) and f(V) are two disjoint τ*pre-open neighborhoods of a and b respectively such that f(U) is an increasing set and f(V) is a decreasing set. Therefore (X*,τ*, ≤*) is a strongly Hausdorff ordered space.

DEFINITION 5.07. A topological ordered space (X,τ, ≤) is said to be a lower (an upper) strongly regular ordered space iff for each element a ∉ F there exists τ open neighborhoods U of a and V of F such that U is an increasing (a decreasing) and V is a decreasing (an increasing) set in X and U ∩ V = ϕ .

DEFINITION 5.08. A topological ordered space (X,τ, ≤) is said to be a lower (upper) strongly pre-regular ordered space iff for each decreasing (an increasing) τ-closed set F and each element a € F, there exist τ pre-open neighborhoods U of a and V of F such that U is an increasing (a decreasing) and V is a decreasing (an increasing) set in X and U ∩V = ϕ. Strongly pre-regular if it is upper and lower strongly pre-regular ordered space.

THEOREM 5.08. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a bijective D-continuous map, B-pre-open map. If (X, τ) is regular space, then (X*, τ*, ≤*) is a lower strongly pre-regular ordered space.

Proof. Let F be decreasing closed subset of X* and a € X* such that a ∉ F. Since f is D-continuous f^{-1} (F) is a decreasing closed set in X. Since f^{-1}(a) ∉ f^{-1}(F) and X is regular there exists two disjoint open neighborhoods U of f^{-1}(a), V of f^{-1}(F) in X. Since f is B-pre-open clearly f^{-1}(U) is an increasing pre-open set and f(V) is a decreasing pre-open in X*. Also a € f(U), F ⊆ f(V). (X*, τ*, ≤*) is lower strongly pre-regular ordered space.

THEOREM 5.09. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a bijective D-continuous, B-open map. If (X, τ) is a regular space, then (X*, τ*, ≤*) is an upper strongly pre-regular ordered space.

Proof. Analogous to that of the theorem 5.08.

THEOREM 5.10. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a B-pre-homomorphism. If (X, τ) is a regular space, then (X*, τ*, ≤*) is a strongly pre-regular ordered space.

DEFINITION 5.09. A topological ordered space (X, τ, ≤) is said to be a strongly pre-normally ordered space iff for each pair of disjoint τ-closed sets F_1 and F_2 in X, where F_1 is increasing F_2 is decreasing there exists two disjoint τ-pre-open neighborhoods U_1 of F_1 and U_2 of F_2 such that U_1 is increasing and U_2 is a decreasing in X.

DEFINITION 5.10. A topological ordered space (X, τ, ≤) is said to be a strongly pre-T_3 -ordered iff it is both strongly pre-T_1-ordered and strongly pre-regular ordered.

DEFINITION 5.11. A topological ordered space (X, τ, ≤) is said to be a strongly pre-T_4-ordered space iff it is both strongly pre-T_1-ordered and strongly pre-normally ordered.

THEOREM 5.11. Let f:(X, τ, ≤) → (X*, τ*, ≤*) be a bijective B-continuous and B-pre-open map then.

1) If (X, τ,) is normal then (X*, τ*, ≤*) is a strongly pre-normally ordered space.
2) If f is a po-set isomorphism and (X, τ) is T_3, then (X*, τ*, ≤*) is strongly Pre-T_3-ordered space (Follows from the 5.06, 5.08)
3) If f is a po-set isomorphism and (X, τ) is T_4, then (X*, τ*, ≤*) is strongly pre-T_4-ordered space. (Follows from 5.06, 5.11)

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REFERENCES:

[1]. A. S. Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb. On pre-continuous and weak pre-continuous mappings, Proc. Math and Phys. Soc. Egypt **53**(1982), 47-53.

[2] McCartan S.D., J.London Math. Soc, Second Series, 1, Part 2(1969), 274-278.

[3] McCartan S.D., Proc. Camb.Phil Soc, **64**(1968) 965-973.

[4]. Nachbin. L., Topology and order, D. Van Nostrand Inc, Princeton, Newjersey (1965)

[5]. M. K. R. S. Veera Kumar, Homeomorphisms in topological ordered spaces, Acta Ciencia Indica, **XXVIII (M)**(1)(2002), 67-76.