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Unsteady Dusty Viscous Flow between Parallel Plates

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ABSTRACT

The present investigation is confined the numerical solution to unsteady dusty viscous flow between two parallel plates, under three pressure gradients. Periodic pressure gradient, Constant pressure gradient and Exponentially decreasing pressure gradient. The governing differential equations with initial and boundary conditions in dimensionless form are solved by making use of successive over relaxation and forward difference formula. An interesting observation in this study is that dust particles take more time to arrive at steady state and velocity of clean gas is always greater than that of dusty gas throughout the region of the flow for all Reynolds numbers. Also this investigation brings to light and that at large Reynold numbers, in case of exponentially decreasing pressure gradient there exists a zone of silence in which the dust particle velocity is negligibly small.

Keywords: Dusty viscous flow, parallel plates, successive over relaxation method

INTRODUCTION

In recent years extensive research work has been carried out on Saffman [1] model for dusty gas as it has wider applications in the studies of movement of dust laden air, in the problems of fluidisation and in the use of dusty gas in cooling systems to enhance heat transfer process. Nawal Kishore and Pandy [2], Rao.P.S[3-4] and Gupta R.K [5] studied unsteady flow of dusty gas in circular and elliptic cylinders. Sastry.D.V.S and Sitaramaswamy.R [6] studied magnetohydrodynamic dusty viscous flow through a circular pipe under transverse magnetic field. These authors obtained closed form solutions in infinite series for velocity components. The series solutions so obtained are not very convenient for numerical computation owing to very slow convergence characterstics. In this problem we have investigated numerical solution to unsteady dusty viscous flow between two parallel plates, under three pressure gradients:

1. Periodic pressure gradient
2. Constant pressure gradient
3. Exponentially decreasing pressure gradient

MATHEMATICAL FORMULATION

Consider the motion of an incompressible dusty viscous fluid between two parallel plates. The fluid is initially at rest, and put into motion by applying a given pressure gradient in X-direction. The equations governing unsteady motion of dusty viscous liquid in Saffman [35] model are given by

$$\overline{\nabla}.\overline{\mathrm{U}} = 0 \qquad (1)$$

$$\boldsymbol{\nabla}.\overline{\boldsymbol{V}} = \boldsymbol{0} \qquad (2)$$

$$\boldsymbol{\rho}\vec{\boldsymbol{U}}.\boldsymbol{\nabla}\vec{\boldsymbol{U}} + \boldsymbol{\rho}\frac{\partial\overline{\boldsymbol{U}}}{\partial \boldsymbol{t}} = -\boldsymbol{\nabla P} \qquad (3)$$

$$\frac{\partial \mathrm{N}}{\partial \mathrm{t}} + \left(\nabla.\mathrm{N}\vec{\mathrm{V}}\right) = 0 \qquad (4)$$

$$\rho\vec{V}.\nabla\vec{V} + m\frac{\partial V}{\partial t} = K(\vec{U}\text{-}\vec{V}) \qquad (5)$$

Where $\vec{U}$, $\vec{V}$are the velocities of clean and dust particles, p is density of clean gas, P is the pressure, μ is the coefficient of viscosity.

K	stokes coefficient of resistance
N	number density
m	mass of dust particles

The fluid is confined between the planes Z=0 and Z=d, flow is induced in X-direction by the pressure gradient. For the present problem J, K components of velocities of dust and clean gas are zero hence

$$\vec{U} = U(z,t)\hat{\imath} \qquad (6)$$

$$\vec{V}=\mathrm{V(z,t)}\hat{\imath} \qquad (7)$$

Introducing dimensionless quantities

$$Z^{+} = \frac{Z}{d}\,, t^{+} = \frac{tU}{d}\,, U^{+} = \frac{U}{U} \quad V^{+} = \frac{V}{U} \qquad (8)$$

The above equations reduces to dimensionless formas (suppression +)

$$\mathrm{R}\frac{\partial \mathrm{U}}{\partial \mathrm{t}} = -\mathrm{Lf(t)} + \frac{\partial^2 \mathrm{U}}{\partial \mathrm{Z}^2} + \lambda_1\,(\mathrm{V}-\mathrm{U}) \qquad (9)$$

$$\frac{\partial \mathbf{V}}{\partial \boldsymbol{t}} = \boldsymbol{\lambda}_{\mathbf{2}}\,_{(\mathbf{U}-\mathbf{V})} \qquad (10)$$

Where

$$\frac{\partial p}{\partial z} = Lf(t) \tag{11}$$

F(t) = 0; t ≤ 0

= e , t > 0 (Exponential pressure gradient)
= cost (periodic pressure gradient) (12)
= 1 (constant pressure gradient)

Where

R = Reynold number
λ_1 = KNd^2 / μ = Dimensionless mass concentration
λ_2 = Kd / mU = Relaxation time of dust particles
L = $1d^2/ \mu_e U$
λ = λ d / U

The initial and boundary conditions are

U (0, t) = V (0, t) = 0 (13)
U (1, t) = V (1, t) = 0 (14)
U (Z,0) = V (Z,0) = 0 (15)

Applying successive over relaxation method to the equation (9) we get the following difference equation.

$$-\frac{r}{R} U_{i-1,j+1} + \left(2+\frac{2r}{R}\right) U_{i,j+1} = \frac{r}{R} U_{i-1,j} + \left(2-\frac{2r}{R}\right) U_{i,j} \tag{16}$$

$$+\frac{r}{R} U_{i+1,j} - f(k,j) + \lambda_1 V_{ij} - \lambda_1 U_{ij}$$

$$\begin{bmatrix} 2+2r/R & -r/R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -r/R & 2+2r/R & -r/R & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -r/R & 2+2r/R & -r/R & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r/R & 2+2r/R & -r/R & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r/R & 2+2r/R & -r/R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -r/R & 2+2r/R & -r/R & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -r/R & 2+2r/R & -r/R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -r/R & 2+2r/R & -r/R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r/R & 2+2r/R \end{bmatrix} \begin{bmatrix} U(1,J) \\ U(2,J) \\ U(3,J) \\ U(4,J) \\ U(5,J) \\ U(6,J) \\ U(7,J) \\ U(8,J) \\ U(9,J) \end{bmatrix} = B\,[I,J]$$

B[1] = (2-2r1 / r) * U [1]+r1 / r *U[2]- (f*k) / r + (λ_1 *k) / r * V[1] – ($\lambda_1 * k$) / r * U [1]
For i = 2 to 8 do

B[i] = r1/r * U[i-1] + (2-2r1/ r) * U[i] + r1/r * U[i+1] – (f*k) / r + (λ_1*k) /r * V[i] – $(\lambda_1 * k)/r * U[i]$

B[9] = r1 / r *U[8] + (2-2r1/r) * U[9] – (f*k) / r + $(\lambda_1 * k)\ r * V[9] -$ (λ_1 * k) / r * U[9]

Where A is the square matrix of order 9, U and B are column vectors of order 9 each. The successive over relaxation method is employed to solve the above equations and is found to be more efficient than algorithms due to Thomas and Evan's for the solution of tridiagonal system of equations. The values U (i,j) obtain are substituted in the equation (17) and V is determined at the nodal points. Applying forward difference formula to the equation (10) giving

$$V_{i,j+1} = (1-\lambda_2 k)\ V_{i,j} + \lambda_2 k U_{i,j} \qquad (17)$$

First the value of U and j time step is determine and V is evaluated at j+1 time step, using equation (17)

The system of equation (16) are tridiagonal, these equations are solved for j = 1 to 100. At each j V (j+1) is determined for the three pressure gradients. For constant pressure gradient

$$F\ (t) = 1 \qquad (18)$$

Periodic pressure gradient

$$F\ (t) = \cos\ (j \text{ x } k) \qquad (19)$$

Exponentially decreasing pressure gradient

$$F\ (t) = \exp(-j \text{ x } k) \qquad (20)$$

Where k is dimensionless time step choosen as k = .01

DISCUSSION:

The dimensionless length 1, between the plates is discretised with h=1/10, U(z,t) and V(z,t) is evaluated for 100 times steps for λ_1 = 0.1, λ_2 = 0.2, k = 0.1 and R = 5,10,100 graphs are drawn showing the distribution of velocity of clean and dust particles at different Reynold numbers and pressure gradients. Fig .1 shows velocity distribution of dust gas in case of periodic pressure gradient at the mid point of the planes. It is observed that at large Reynold numbers the dust particle velocity almost assumes the shape of cos curve, while for small Reynold numbers the dust particles velocity curve assumes the shape of a damping cos curve, and assuming maximum at 50th time step. In case of large Reynold numbers the period of dust particules is found to be 55th time steps. Fig .2 shows velocity distribution of clean gas in case of periodic pressure gradient at mid point of the planes. The effect of increasing Reynold number is to decrease the velocity of the clean gas. The period is found to be nonuniform with maximum velocity decreasing with time. At small Reynold numbers the maximum velocity obtained is same. Fig .3 shows velocity distribution of clean gas at different nodal points between the parallel plates for Reynold number 10. Maximum velocity for all times is found to obtain in a small neighbourhood of z = 4. It is evident from Fig .4 that maximum clean gas velocity is confined to the region z = 3 to 7 at Reynold number 100. The behavior of dust particles is observed to be different form that of clean gas as can be observed from Figures 5 and 6. The dusty gas particles obtained maximum velocity at middle of the plane irrespective of Reynold numbers. Fig .7 show velocity distribution of clean gas at constant pressure gradient at R = 5, it can be seen that steady state is obtained in time step 10 and maximum velocity is obtained at z = 4. Fig .8 shows velocity distribution of dust particles at constant pressure gradient when R = 5. The dust particles found to take more time to arrive at steady state, which is round about 40 time steps. Maximum dust particle velocity is obtained at z = 4 for all times.

Fig 7 and 8 show that velocity of clean gas is more than that of dusty gas at any point between the planes for all times. Fig .9 shows velocity distribution at constant pressure gradient at R = 100 steady state is reached round about 60 time step, obtaining maximum at z = 4. Fig .10 shows velocity distribution of dusty gas at constant pressure gradient R=100. Steady state is reached round about 120 time steps. An interesting observation in this investigation is that dust particles take more time to arrive at steady state and velocity of clean gas is always greater than that of dusty gas throughout the region of the flow for all Reynold numbers. Fig .11 shows velocity distribution of dust particles at exponentially decreasing pressure gradient at Reynold number 5 maximum velocity throughout the region is obtained in time step 10, and decreases there after words throughout the region. Maximum velocity is obtained at midway between the planes for both clean and dust gas for all times. Fig .12 shows velocity distribution of clean gas for exponentially decreasing pressure gradient at Reynold number 5. Maximum velocity is obtained at time step 5 and then onwards velocity decreases. Maximum velocity is obtained at time 5 and then onwards velocity decreases. Maximum velocity is obtained at z = 4 for all times.

Fig .11 and 12 also shows that the velocity of clean gas is always more than that of dust gas throughout the region. Fig .13 and 14 shows that when Reynold number is increased to 10, the dust particles (m) takes more time i.e. 20 time step to arrive at maximum velocity, while clean gas takes the same time steps as in the previous case to arrive at maximum velocity. Fig .15 shows velocity distribution of dust particles for a exponentially decreasing pressure gradient. An interesting observation is that at Reynold number 100 there are variations in the dust particle velocity in the regions very close to the two plates i.e. z = 0 to z = 2 and z = 7 to z = 10. In the remaining region the velocity distribution of dust particles is negligible small. In case of clean gas the velocity is obtained maximum in 10 time step and later it decreases obtaining maximum at z = 4. This investigation brings to light and that at large Reynold numbers, in case of exponentially decreasing pressure gradient there exists a zone of silence in which the dust particle velocity is negligibly small.

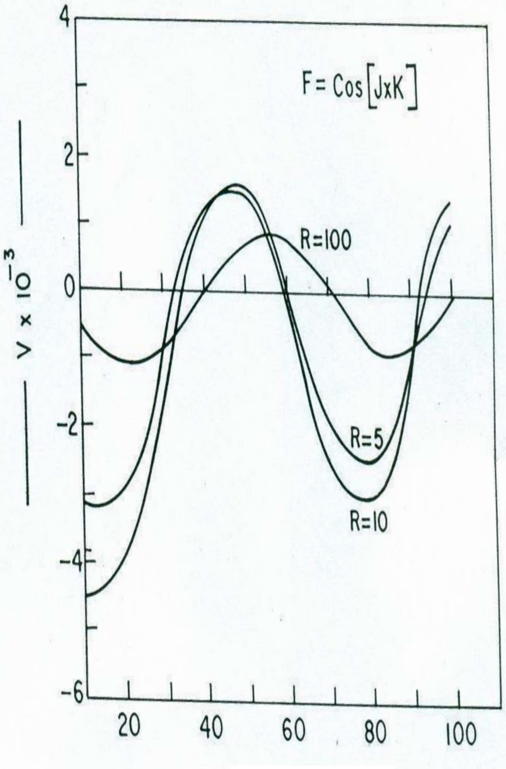

Fig. 1

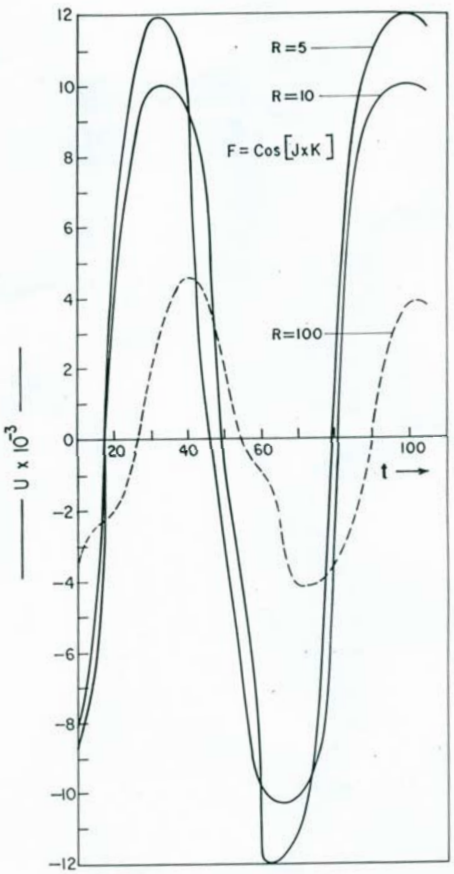

Fig. 2

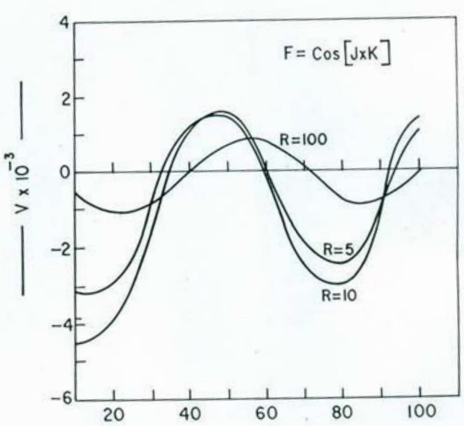

Fig. 3

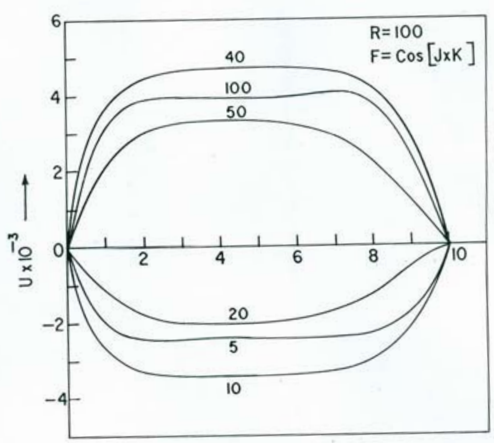

Fig. 4

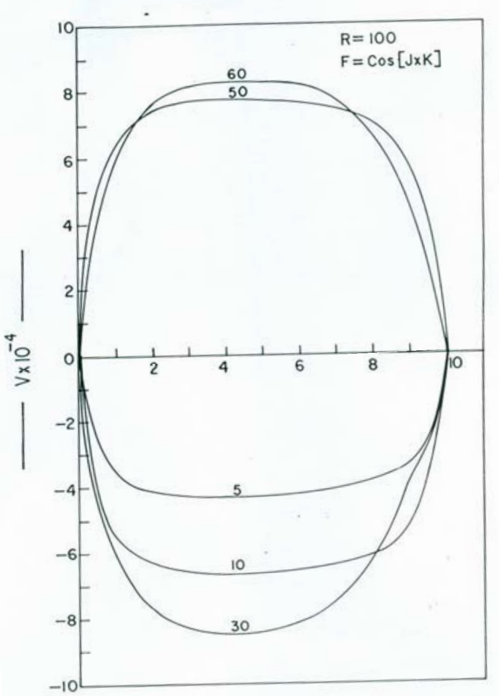

Fig. 5

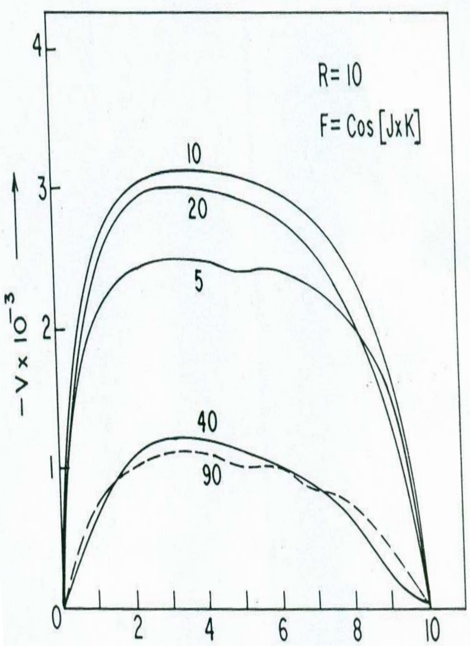

Fig. 6

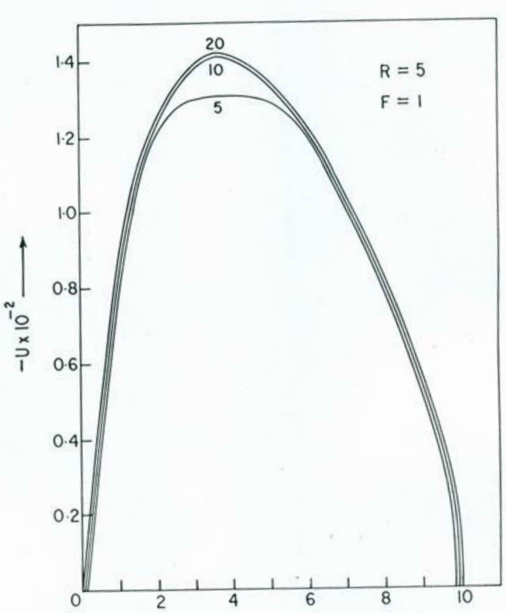

Fig. 7

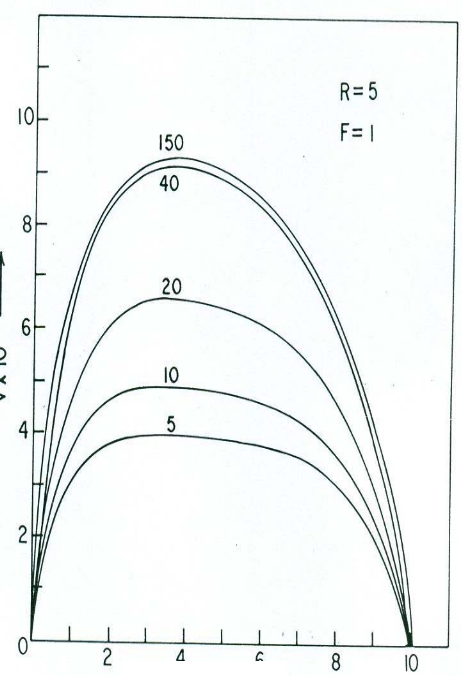

Fig. 8

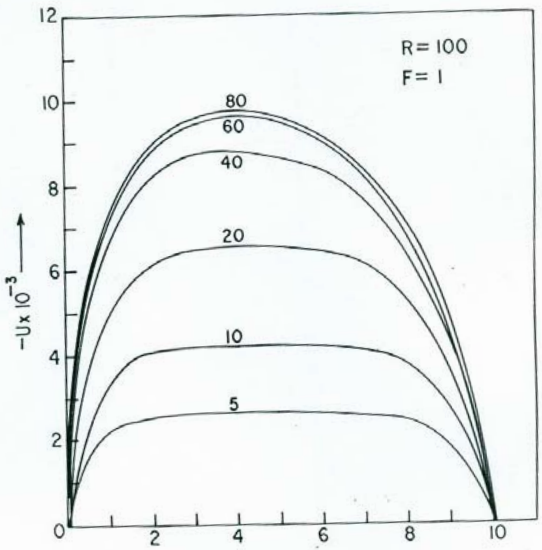

Fig. 9

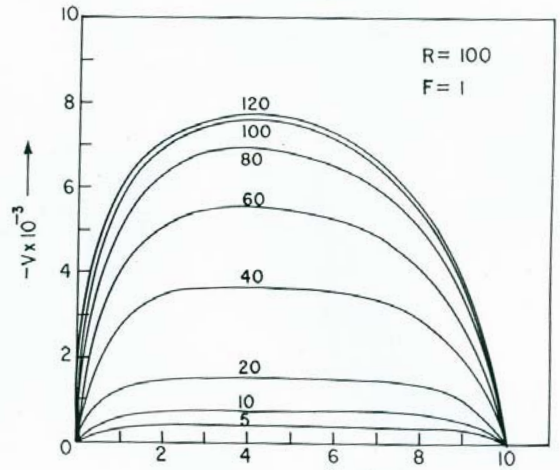

Fig. 10

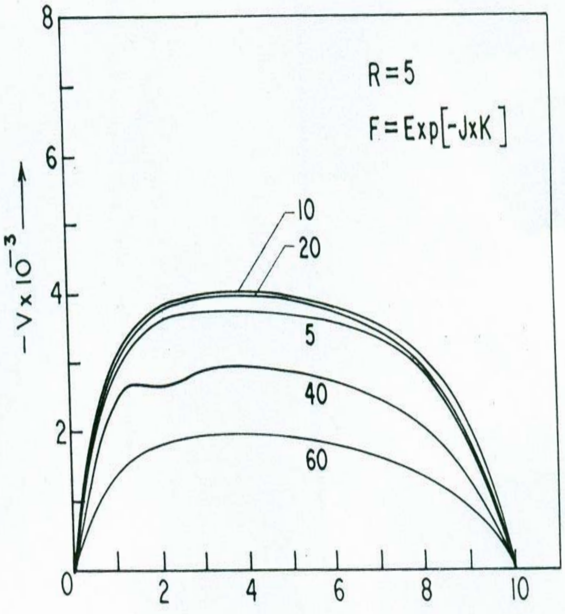

Fig. 11

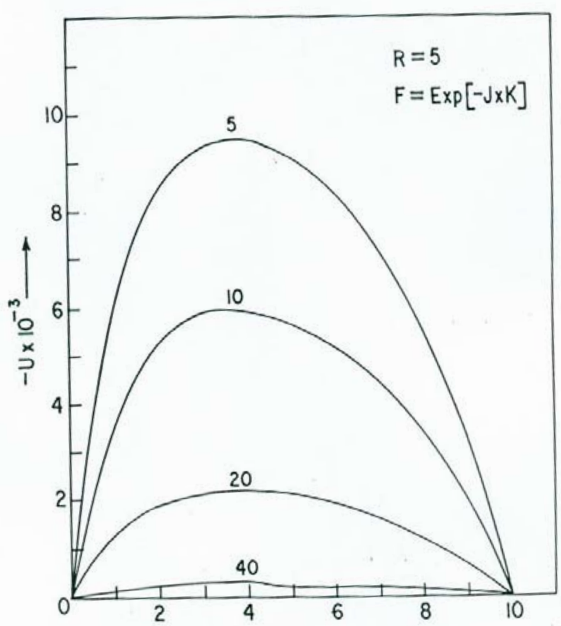

Fig. 12

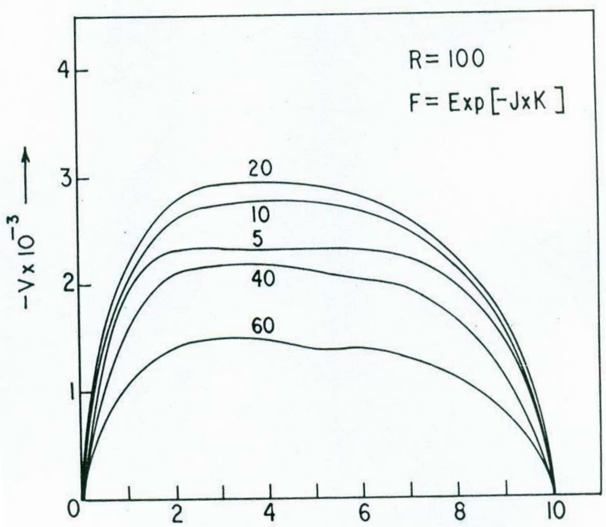

Fig. 13

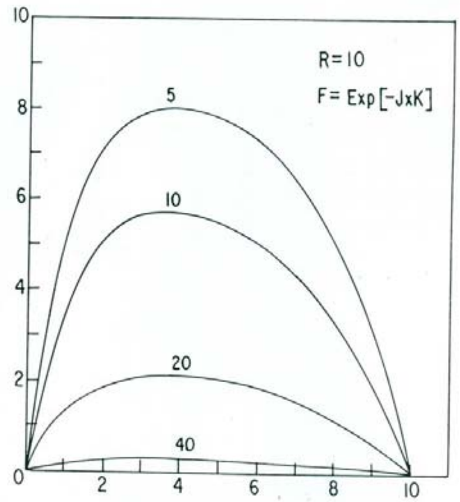

Fig. 14

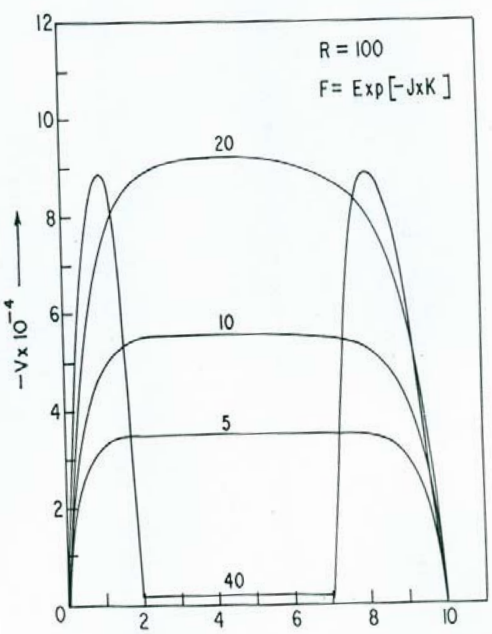

Fig. 15

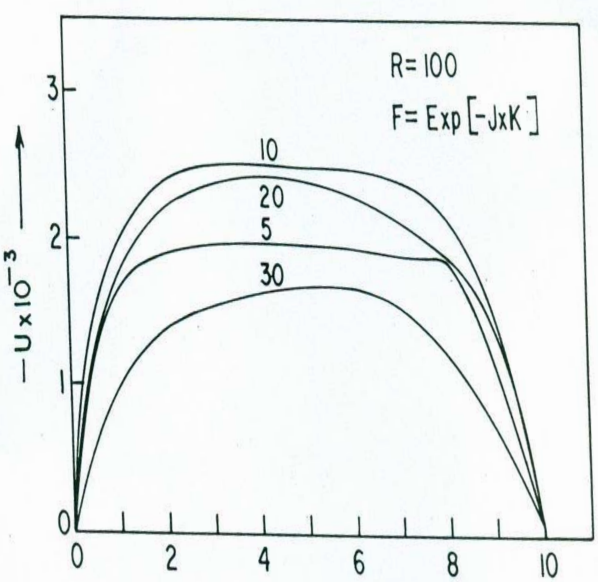

Fig. 16

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