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Flow of Incompressible Viscous Fluid Through Porous Medium Between Two Semi-Infinite Parallel Plates

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ABSTRACT

The flow of incompressible viscous liquid is examined between two semi-infinite parallel plates. The space between the parallel plates is filled with porous medium. Brinkman equation is applied to study the fluid flow. Results are graphically represented.

Keywords: Porous medium, permeability parameter.

1. INTRODUCTION

The study of flow through porous medium assumed importance because of the interesting applications in the diverse fields of science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter disciplinary fields such as biomedical engineering etc. The lung alveolar is an example that finds applications in an animal body. The classical

Darcy's law Musakat [1] states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as

$$\vec{V} = -\left(\frac{k}{\mu}\right)\nabla P \ .$$

The law gives good results in the situations when the flow is uni-directional or the flow is at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to its viscous nature and produces distortions in the velocity field. In the case of highly porous medium such as fiber glass, papus of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by the Beavers and Joseph [2], Saffman [3] and others. A generalized Darcy's law proposed by Brinkmann [4] is given by

$$O = -\nabla P - \left(\frac{\mu}{K}\right)\vec{v} + \mu\nabla^2 \vec{v}$$

where μ and K are coefficients of viscosity of the fluid and permeability of the porous medium.

The applications of flows through porous medium bears wide spread interest in Geophysics, biology and medicine. In many of these areas the flow consists of more than one phase, such type of flows find applications in the inter disciplinary fields such as bio-medical engineering etc., the flow of blood is one such application. The blood may be represented as Newtonian fluid and the flow of the blood is in two layered. Lightfoot [5], Shukla *et al.* [6] and Chaturani [7]. Bird *et al*. [8] found an exact solution for the laminar flow of two immiscible fluids between two parallel plates. Bhattacharya [9] discussed the flow of immiscible fluids between rigid plates with a time dependent pressure gradient. Vajravelu *et al*. [10] have discussed the effect of magnetic field on unsteady flow of two immiscible conducting fluids between two permeable beds. Transciet couette flow in a rotating non-Darcian porous medium parallel plate configuration is studied by Anwarbeg *et al*. [11] Kandryzakaria *et al*. [12] discussed magneto hydrodynamics instability of interfacial waves between two immiscible cylindrical fluids.

Earlier Narasimhacharyulu *et al*. [13] studied the problem of two phase fluid flow between parallel plates with porous lining and Narasimhacharyulu *et al*. [14] examined the flow of micro polar fluid between parallel plates coated with porous lining.

In this present paper we are considering the fluid flow between two parallel plates, the space between the plates is filled with porous medium. Special cases are discussed and graphical representation of the results is given.

2. Mathematical Formulation of the Problem

The flow of an incompressible viscous liquid is considered between two semi infinite parallel plates given by $y = \pm h$. The space between the plates is filled with porous region. The coordinate system is taken such that x-axis lies parallel to the length of the plates and y-axis perpendicular to the length of the plates. The fluid flows under a constant pressure gradient.

$$G = -\frac{\partial p}{\partial x}$$

The velocity of the fluid $\vec{V} = (u, 0, 0)$ satisfies the equation of continuity, the physical quantity depends only on y.

The equation of motion is given by

$$\frac{d^2u}{dy^2} - \frac{u}{k} = -\frac{G}{\nu} \qquad \ldots \qquad (2.1)$$

$$-h < y < h$$

Where $G = -\frac{\partial p}{\partial x}$ is a constant pressure gradient, in the x direction, ν is coefficient of viscosity of the fluid, k is permeability of the porous medium.

Using the following Non dimensional quantities.

$$u^* = \frac{uh}{\nu},\; y^* = \frac{y}{h},\; G^* = \frac{Gh^3}{\nu},\; \beta^2 = \frac{h^2}{K} \qquad \ldots \qquad (2.2)$$

After removing *, the non-dimensional form of equation of motion is

$$\frac{d^2u}{dy^2} - \alpha^2 u = -\frac{G}{\nu}; \qquad -1 < y < 1 \qquad \ldots \qquad (2.3)$$

The boundary conditions are given by

$$u = 0 \quad \text{at} \quad y = \pm 1 \qquad \ldots \qquad (2.4)$$

3. Solution of the problem :

Solving the equation (2.3) employing boundary conditions (2.4) we get

$$u = \frac{G}{\nu\alpha^2}\left(1 - \frac{\cosh \alpha y}{\cosh \alpha}\right) \qquad \ldots \qquad (2.5)$$

Flow rate $Q = \int_{-1}^{1} u\,dy$

$$Q = \frac{2G}{\nu\alpha^2}\left(1 - \frac{Tanh\alpha}{\alpha}\right) \qquad \ldots \qquad (2.6)$$

Deductions

Case 1 : The permeability of the porous region is very large i.e. $\alpha \to 0$.

If α is very small, permeability coefficient K is very large. The velocity is given by

$$u = \frac{G}{2\nu}\left(1 - y^2\right) \qquad \ldots \qquad (2.7)$$

Flow rate $$Q = \frac{2G}{3\nu} \qquad \ldots \qquad (2.8)$$

Case 2 : The permeability of the porous region is very small i.e. $\alpha \to \infty$.

$$u = \frac{GK}{h^2 \nu} \qquad \ldots \qquad (2.9)$$

Flow rate $$Q = \frac{2GK}{3h\nu} \qquad \ldots \qquad (2.10)$$

CONCLUSION:

The flow of an incompressible viscous liquid is examined between two semi-infinite parallel plates. The space between the parallel plates is filled with porous medium.

In Fig.1, the effect of the permeability of the porous medium on the fluid velocity is observed. As permeability of the porous medium is increasing the velocity of the fluid is decreasing.

From Fig.2, it is observed that as permeability of the porous medium is increasing, the flow rate is decreasing. Further it is also observed that as thickness of the porous medium is increasing the flow rate is decreasing.

The results of the problem have great importance to the petroleum engineer concerned with the movement of oil, gas and water through the reservoir of an oil or gas field. Beyond this, the results of present problem are widely applicable in soil mechanics, water purification, ceramic engineering and power metallurgy.

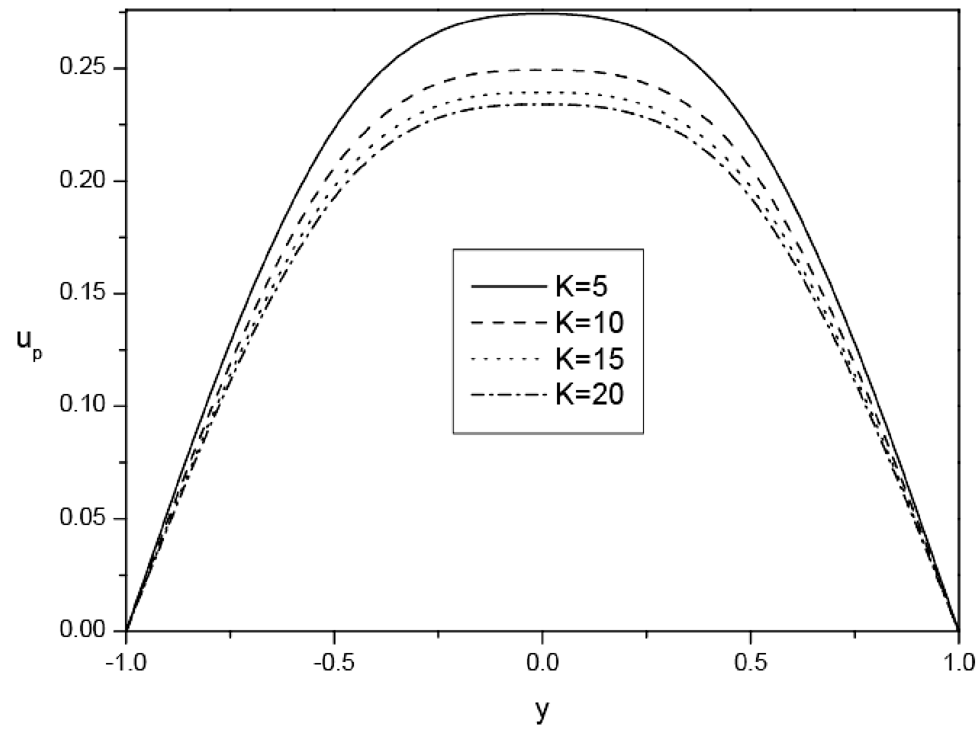

Fig. 1 : Variation of u with permeability parameter K

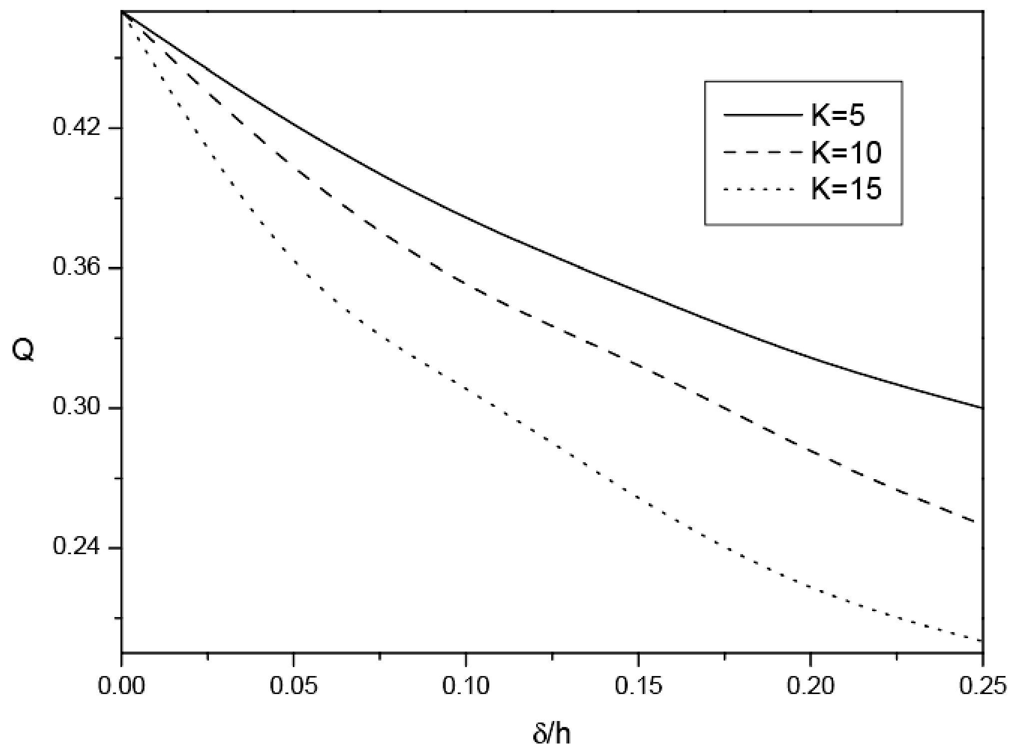

Fig. 2 : Flow rate for different values of permeability parameter K

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