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Fuzzy Translations of Fuzzy p-Ideals in BCI-Algebras

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ABSTRACT

In this paper, the concepts of fuzzy translation to fuzzy p-ideals in BCK/BCIalgebras are introduced. The notion of fuzzy extensions and fuzzy multiplications of fuzzy p-ideals with several related properties are investigated. Also, the relationships between fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy ideals are investigated.

AMS Mathematics Subject Classifications: 06F35, 03G25, 08A72.

Keywords: Fuzzy ideal, fuzzy p-ideal, fuzzy translation, fuzzy extension, fuzzy multiplication.

1. Introduction

BCK/BCI-algebras are two important classes of logical algebras introduced by Iski in 1966 (see [9, 10, 20]). Since then, several works have been dedicated to the theory of BCI/BCK/MV/BL-algebras with a focus on ideals and filters of these classes of algebras. From the logical point of view, various ideals correspond to various sets of provable formulas [2, 11, 12]. In 1965, Zadeh [18] introduced the concept of fuzzy sets which has been successfully applied to many mathematical disciplines. In 1991, O. Xi [17] applied the concept of fuzzy sets to BCI-algebras and introduced the notion of fuzzy ideals in BCI-algebras. In 1994, Jun et al. [15] introduced fuzzy p-ideals in BCI-algebras and in 2010, Kordi et al. [16], extend it to notation of (m, n) $-$fold p-ideals, see also [17]. Lee et al. [18] and Jun [14] discussed fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy sub algebras and ideals in BCK/BCI algebras. They investigated relations among fuzzy translations, fuzzy extensions and fuzzy multiplications. In this paper, fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy p-ideals in BAK/BCI-algebras are discussed. Relations among fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy p-ideals in BAK/BCI-algebras are also investigated.

2. Preliminaries

By a BCI-algebra we mean an algebra $(X; *, 0)$ of type $(2\ , 0)$ satisfying following axioms:

(1) $\big((x * y) * (x * z)\big) * (z * y) = 0,$

(2) $\big(x * (x * y)\big) * y\big) = 0,$

(3) $x * x = 0,$

(4) $x * y = 0$ *and* $y * x = 0$ *imply* $x = y.$

for all $x, y, z \in X$. We can define a partial ordering " $\leq$ " on X by $x \leq y$ if and only if $x * y = 0.$

The following statements are true in any BCI-algebra X:

(1.1) $(x * y) * z = (x * z) * y,$

(1.2) $x * 0 = x,$

(1.3) $(x * z) * (y * z) \leq x * y,$

(1.4) $x \leq y$ *implies* $x * z \leq y * z$ *and* $z * y \leq z * x,$

(1.5) $0 * (x * y) = (0 * x) * (0 * y),$

(1.6) $x * \big(x * (x * y)\big) = x * y$.

Definition 2.1. A nonempty subset I of X is called an ideal of X if it satisfies:

(I_1) $0 \in I,$

(I_2) $x * y \in I$ *and* $y \in I$ *imply* $x \in I$.

Definition 2.2. A nonempty subset I of X is called an ideal of X if it satisfies condition (I_1) and

(I_3) $(x * z) * (y * z) \in I$ *and* $y \in I$ *imply* $x \in I$.

Putting $z = 0$ in (I_3), we can see that every p-ideal is an ideal.

Definition 2.3. A fuzzy set μ of BCI-algebra X is called fuzzy ideal of X if it satisfies

(FI_1) $\mu(0) \geq \mu(x)$

(FI_2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$.

Definition 2.4. A fuzzy set μ of BCI-algebra X is called fuzzy p-ideal of X if it satisfies (FI_1) and

(FI_3) $\mu(x) \geq \min\{((x * z) * (y * z)), \mu(y)\}$

Proposition 2.5. ([17]) Let μ be a fuzzy set in a BCI-algebra X . Then μ is a fuzzy p-ideal of X if and only if for all $t \in [0,1]$,

$$\mu_t \neq \emptyset \Longrightarrow \mu_t \text{ is a } p\text{-ideal of } X,$$

Where $\mu_t = \{x \in X | \mu(x) t\}$.

3. Main Results

Throughout this paper, we take $\dagger = 1 - sup\{\mu(x) | x \in X\}$ for any fuzzy set μ of X.

Definition 3.1. ([18]) Let μ be a fuzzy subset of X and let $\alpha \in [0, \dagger]$. A mapping $\mu_\alpha^\dagger \longrightarrow [0,1]$ is called a fuzzy α- translation of μ if it satisfies $\mu_\alpha^\dagger(x) = \mu(x) + \alpha$ for all $x \in X$.

Theorem 3.2. if μ is a fuzzy p-ideal of X , then the fuzzy α- translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal of X , for all $\alpha \in [0, \dagger]$.

Proof. Assume that μ is a fuzzy p-ideal X and let $\alpha \in [0, \dagger]$. Then we have

$$\mu_\alpha^\dagger = \mu(0) + \alpha \geq \mu(x) + \alpha = \mu_\alpha^\dagger(x),$$

and for all $x, y, z \in X$ we have

$$\mu_\alpha^\dagger(x) = \mu(x) + \alpha$$

$$\geq \min\{\mu((x * z) * (y * z)), \mu(y)\} + \alpha$$

$$= \min\{\mu((x * z) * (y * z)) + \alpha, \mu(y)\} + \alpha$$

$$\text{Min}\{\mu_\alpha^\dagger((x * z) * (y * z)), \mu_\alpha^\dagger(y)\}.$$

Hence, the fuzzy α- translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal of X.

Theorem 3.3. *let μ be a fuzzy subset of X such that the fuzzy α-translation $\mu_\alpha^\dagger$ of μ is a fuzzy p -ideal of X , for some $\alpha \in [0, \dagger]$. Then, μ is a fuzzy of X.*

Proof. Assume that $\mu_\alpha^\dagger$ is a fuzzy p-ideal of X , for some $\alpha \in [0, \dagger]$. Let $x \in X$, then

$$\mu(0) + \alpha = \mu_\alpha^\dagger(0) \geq \mu_\alpha^\dagger(x) = \mu(x) + \alpha\ ,$$

So $\mu(0) \geq \mu(x)$. Also, for all $x, y, z \in X$ we have

$$\begin{aligned}\mu(x) + \alpha &= \mu_\alpha^\dagger(x) \\ &\geq \min\{\mu_\alpha^\dagger((x * z) * (y * z)), \mu_\alpha^\dagger(y)\} \\ &= \min\{\mu((x * z) * (y * z)) + \alpha, \mu(y)\} + \alpha \\ &= \min\{\ \mu((x * z) * (y * z)),\ \mu(y)\} + \alpha.\end{aligned}$$

So, $\mu(x) \geq \min\{\mu((x * z) * (y * z)), \mu(y)\}$. Therefore μ is a fuzzy p-ideal of X.

Theorem 3.4. *If the fuzzy α-translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal of X , for some $\alpha \in [0, \dagger]$. Then, μ is a fuzzy of X.*

Proof. Let the fuzzy α- translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal of X. then, we have $\mu_\alpha^\dagger(x) \geq \min\{\mu_\alpha^\dagger((x * z) * (y * z)), \mu_\alpha^\dagger(y)\}$. Since by ([15]), μ is a subalgebra, we have

$$\begin{aligned}\mu_\alpha^\dagger(x) &= \ \mu(x) + \alpha \\ &\geq \min\{\mu(x), \mu(y)\} + \alpha \\ &= \min\{\mu(x) + \alpha, \mu(y) + \alpha\} \\ &= mim\ \{\mu_\alpha^\dagger(x), \mu_\alpha^\dagger(y)\}\ .\end{aligned}$$

Therefore , $\mu_\alpha^\dagger$ is a fuzzy sub algebra of X.

Theorem 3.5 *let μ be a fuzzy subset of X such that the fuzzy α- translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal of X , for some $\alpha \in [0, \dagger]$. Then, μ is a fuzzy of X.*

Proof. Clearly, $0 \in I_\mu$. Assume that $x, y, z \in X$ such that $(x * z) * (y * z), y \in I_\mu$, then

$$\mu_\alpha^\dagger((x * z) * (y * z)) = \mu_\alpha^\dagger(x) = \mu_\alpha^\dagger(y)$$

Thus, we have

$$\mu_\alpha^\dagger(x) \geq \min\{\mu_\alpha^\dagger((x * z) * (y * z)), \mu_\alpha^\dagger(y)\ \} = \mu_\alpha^\dagger(0)$$

Since, $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal of X, we conclude that $\mu_\alpha^\dagger(x) = \mu_\alpha^\dagger(0)$. Therefore $\mu(x) + \alpha =, \mu(0) + \alpha$, i.e. , $\mu(x) = \mu(0)$, so that $x \in I_\mu$. Therefore, I_μ is an p-ideal of X.

Proposition 3.6. ([25]), *If the fuzzy α- translation $\mu_\alpha^\dagger$ of μ is an p-ideal of X, then it is order reversing.*

Theorem 3.7. *let μ be a fuzzy subset of X such that the fuzzy α-translation $\mu_\alpha^\dagger$ of μ is a fuzzy ideal of X , then the following statements are equivalent:*

(i) $\mu_\alpha^\dagger$ *is a fuzzy p-ideal of X,*

(ii) $\mu_\alpha^\dagger\big(0 * (0 * x)\big) \le \mu_\alpha^\dagger\,(x)$

Proof. $(i) \Longrightarrow (ii)$: It is enough to put *x=z=0* and *y=x* in definition of fuzzy p-ideal.

$(ii) \Longrightarrow (i)$: for all $x, y, z \in X$ we have

$$\Big(\big(0 * (0 * x)\big) * y\Big) * \big((x * z) * (y * z)\big) = \Big(0 * (x * z) * (y * z)\big)\Big) * (0 * x)\Big) * y =$$

$$\Big(\big[\big((0 * x)(0 * z)\big) * \big((0 * y) * (0 * z)\big)\big] * (0 * x)\Big) * y$$

$$\le \Big(\big((0 * x) * (0 * y)\big) * (0 * x)\Big) * y$$

$$= \big(0 * (0 * y)\big) * y = 0$$

Now, by Corollary 3.6 of ([25]), we have:

$$\mu_\alpha^\dagger\big(0 * (0 * x)\big) \ge \min\{\mu_\alpha^\dagger\big((x * z) * (y * z)\big), \mu_\alpha^\dagger(y)\},$$

So $\mu_\alpha^\dagger(x) \ge \min\{\mu_\alpha^\dagger\big((x * z) * (y * z)\big), \mu_\alpha^\dagger(y)\}$. Hence $\mu_\alpha^\dagger$ is a fuzzy p-ideal of X.

Definition 3.8. ([18]) Let $\mu_1$and μ_2 be fuzzy subsets of *X*. If $\mu_1 \le \mu_2$, for all $x \in X$, then we say that μ_2 is a fuzzy extension of μ_1.

Definition 3.9. Let μ_1 and μ_2 be fuzzy subsets of *X*. Then μ_2 is called a fuzzy p-ideal extension of μ_1 if following statements are valid:

(i) μ_2 is a fuzzy extension of *X*, then μ_1,

(ii) If μ_1 is a fuzzy p-ideal of *X*, then μ_2 us a fuzzy p-ideal of *X*.

Theorem 3.10. *let μ be a fuzzy p-ideal of X subset of X and $\alpha \in [0, \dagger]$. then the fuzzy α-translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal extension of μ.*

Proof. It's clear from the definition of fuzzy α- translation.

The following example show that a fuzzy p-ideal extension of a fuzzy p-ideal μ may not be represented as a fuzzy α- translation of μ:

Example 3.11. Consider a BIC-algebra $X = \{0, a, b, c\}$ with the following Cayley table:

*	0	a	b	c
0	0	a	b	c
A	a	0	c	b
B	b	c	0	a
C	c	b	a	0

Let μ be a fuzzy subset of X defined by:

X	0	a	b	c
μ	0.6	0.6	0.4	0.4

Then, μ is a fuzzy p-ideal of X. Let ϑ be a fuzzy subset of X given by:

X	0	a	b	c
ϑ	0.63	0.63	0.41	0.41

Then, ϑ is a fuzzy p-ideal extension of μ . But it is not the fuzzy α- translation $\mu_\alpha^\dagger$ of μ , for all $\alpha \in [0, \dagger]$.

Theorem 3.12. *let μ be a fuzzy subset of X and $\alpha \in [0, \dagger]$. Then, the fuzzy α- translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal of X if and only if $U_\alpha(\mu; t)$.is a p-ideal of X, for all $t \in Im(\mu)$ with $t > \alpha$.*

Proof. Suppose that $\mu_\alpha^\dagger$ is a fuzzy p-ideal of X and $t \in Im(\mu)$ with $t > \alpha$. Since $\mu_\alpha^\dagger(0) \geq \mu_\alpha^\dagger(x)$, for all $x \in X$, we have $\mu(0) + \alpha = \mu_\alpha^\dagger(0) \geq \mu_\alpha^\dagger(x) = \mu(x) + \alpha \geq t$, for $x \in U_\alpha(\mu; t)$, so $0 \in U_\alpha(\mu; t)$. Let, $x, y, z \in X$ such that $(x * z) * (y * z), y \in U_\alpha(\mu; t)$, then

$$\mu\big((x * z) * (y * z)\big) \geq t - \alpha \quad , \quad \mu(y) \geq t - \alpha$$

i .e .,

$$\mu_\alpha^\dagger\big((x*z)*(y*z)\big) \geq t \quad , \quad \mu_\alpha^\dagger(y) \geq t$$

Since $\mu_\alpha^\dagger$ is a fuzzy p-ideal. So, we have

$$\mu(x) + \alpha = \mu_\alpha^\dagger(x) \geq \min\{\mu_\alpha^\dagger\big((x*z)*(y*z)\big), \mu_\alpha^\dagger(y)\} \geq t\ ,$$

that is $\mu(x) \geq t - \alpha$ so that $x \in U_\alpha(\mu; t)$. Therefore, $U_\alpha(\mu; t)$ is a p-ideal of X.

Conversely, suppose that for all $t \in Im(\mu)$ with $t > \alpha$, $U_\alpha(\mu; t)$ is a p-ideal of X. If there exists $a \in X$ such that $\mu_\alpha^\dagger(0) < \beta \leq \mu_\alpha^\dagger(a)$, then $\mu(a) \geq \beta - \alpha$ but $\mu(0) \geq \beta - \alpha$.

Therefore $a \in U_\alpha(\mu; t)$ and $0 \notin U_\alpha(\mu; t)$. Hence it's contradiction and so for all $x \in X, \mu_\alpha^\dagger(0) \geq \mu_\alpha^\dagger(x)$, Now assume that there exist $a, b, c \in X$ such that,

$\mu_\alpha^\dagger(a) < \gamma \leq \min\{\mu_\alpha^\dagger\big((a*c)*(b*c)\big), \mu_\alpha^\dagger(b)\}$. Then $\mu(a*c)*(b*c) \geq \gamma - \alpha$ and $\mu(b) \geq \gamma - \alpha$. Therefore $(a*c)*(b*c), b \in U_\alpha(\mu; t)$ but $b \notin U_\alpha(\mu; t)$, which is a contradiction. Hence, $\mu_\alpha^\dagger$ is a fuzzy p-ideal of X.

Theorem 3.13. *let μ be a fuzzy p-ideal of X and let $\alpha, \beta \in [0, \dagger]$. If $\alpha \geq \beta$ Then, the fuzzy α- translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal extension of the fuzzy β- translation $\mu_\beta^\dagger$ of μ.*

Proof. It's straightforward.

Theorem 3.14. *let μ be a fuzzy p-ideal of X and $\beta \in [0, \dagger]$. For every fuzzy p-ideal extension v of the fuzzy β- translation $\mu_\beta^\dagger$ of μ, there exists $\alpha \in [0, \dagger]$. Such that $\alpha \geq \beta$* and *v* is a fuzzy *p-ideal extension of the fuzzy α-translation $\mu_\beta^\dagger$ of μ.*

Proof. For every fuzzy p-ideal μ of X and $\beta \in [0, \dagger]$, the *fuzzy β-translation $\mu_\beta^\dagger$ of μ* is a fuzzy p-ideal extension of $\mu_\beta^\dagger$ then there exists $\alpha \in [0, \dagger]$ such that $\alpha \geq \beta$ and for all $x \in X, v(x) \geq \mu_\alpha^\dagger$.

Definition 3.15. Let μ be a fuzzy subset of X and $\gamma \in [0,1]$. A fuzzy γ- multiplication of μ denoted by μ_γ^m, is defined to be a mapping $\mu_\gamma^m : X \to [0,1]$ by $\mu_\gamma^m(x) = \mu(x).\gamma$.

Theorem 3.16. *If μ is a fuzzy p-ideal of X, then the γ- multiplication of μ is a a fuzzy p-ideal of X for all $\gamma \in [0,1]$.*

Proof. It's clear.

Theorem 3.17. *Let μ be a fuzzy subset of X. Then μ is a fuzzy p-ideal of X if and only if the fuzzy γ-multiplication μ_γ^m of μ is a fuzzy p-ideal of X, for all* $\gamma \in [0,1]$.

Proof. (⟹) By Theorem 3.15, is clear.

(⟸) Assume that μ_γ^m of μ is a fuzzy p-ideal of *X*, for all $\gamma \in [0,1]$.Thus ,

$$\mu(0).\gamma = \mu_\gamma^m(0) \geq \mu_\gamma^m(x) = \mu(x).\gamma$$

i.e., for all $x \in X, \mu(0) \geq \mu(x)$ Also, for $x, y, z \in X$, we have

$$\begin{aligned}\mu(x).\gamma &= \mu_\gamma^m(x)\\ &\geq \min\{\mu_\gamma^m((x*z)*(y*z)), \mu_\gamma^m(y)\}\\ &= \min\{\mu((x*z)*(y*z)).\gamma, \mu(y).\gamma\}\\ &= \min\{\mu((x*z)*(y*z)), \mu(y)\}.\gamma\end{aligned}$$

Which implies that $\mu(x) \geq \min\{\mu((x*z)*(y*z)), \mu(y)\}$. Therefore μ is a fuzzy p-ideal of X.

Theorem 3.18. *Let μ be a fuzzy subset of X. $\alpha \in [0,\dagger]$ and* $\gamma \in [0,1]$. *Then, every fuzzy α-translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal extensio*n *of the fuzzy γ-multiplication μ_γ^m of μ .*

Proof. For all $x \in X$, we have

$$\mu_\alpha^\dagger(x) = \mu(x) + \alpha \geq \mu(x) > \mu(x).\gamma = \mu_\gamma^m(x)$$

and so $\mu_\alpha^\dagger$ is s fuzzy extension of μ_γ^m. Assume that μ_γ^m is a fuzzy *p*-ideal of *X*. Then by Theorem 3.16, μ is a fuzzy p-ideal of X. It follows from Theorem 3.2 that the fuzzy α-translation $\mu_\alpha^\dagger$ of μ is a fuzzy *p*-ideal of *X*, for all $\alpha \in [0,\dagger]$.

Therefore, every fuzzy α-translation $\mu_\alpha^\dagger$ of μ is a fuzzy p-ideal extension of the fuzzy γ-multiplication μ_γ^m of μ .

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