ISSN 0976 – 1411

Available online at www.internationaleJournals.com

International eJournals

International eJournal of Mathematics and Engineering 90 (2010) 857 - 865

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

www.InternationaleJournals.com

UNSTEADY NONSIMILAR TWO-DIMENSIONAL ETHANOL BOUNDARY LAYERS WITH TEMPERATURE-DEPENDENT VISCOSITY AND PRANDTL NUMBER

A.H. SRINIVASA

Department of Mathematics, Maharaja Institute of Technology Mysore, Belawadi – 571438, India

K.R. JAYAKUMAR

Department of Mathematics, K.S. Group of Institutions, Bangalore – 560062, India

A.T. ESWARA

Department of Mathematics, PES College of Engineering, Mandya – 571401, India

ABSTRACT

In this paper, we study the effects of temperature dependent viscosity and Prandtl number on the unsteady, laminar boundary layer flow of ethanol over a cylinder. The viscosity and Prandtl number are assumed to vary as inverse linear functions of temperature. The coupled non linear partial differential equations governing the flow have been solved numerically by using an implicit finite-difference scheme, along with quasilinearization technique. The skin friction increases while heat transfer decreases in the case of variable viscosity and Prandtl number as compared to constant fluid properties. It is also noticed that the unsteadiness and suction causes the point of zero skin friction is moved down stream, delaying the boundary layer separation. The heat transfer depends strongly on viscous dissipation, but skin friction is little affected by it. The velocity and temperature fields are to found to be influenced by accelerating free stream.

1. INTRODUCTION

A wide range of non-similar boundary layer flow and heat transfer problems have attracted several investigators [1-3]. These studies are of practical importance as they involve non-similarities in the flow arising due to the free stream velocity, the curvature of the body, the surface mass transfer, or a combined effect of all these factors. Further, several studies also have been made on laminar boundary layer flows over heated bodies with temperature-dependent fluid properties [4-6]. Indeed, fluid viscosity and thermal conductivity are the main governing fluid properties in laminar boundary layer forced flow, and hence their variations also can be expected to affect separation of boundary layer from the solid surface. As these properties are temperature dependent, variations are most easily accomplished in the boundary layer by maintaining a temperature difference between the solid wall and the fluid. In practice, wall heating has been shown to be an efficient way to stabilize boundary layer flow and to delay the flow transition when fluid viscosity decreases by heating. In fact, for technological applications, surface heating is an effective means of controlling boundary layer separation since heating promotes stability through the interplay among the thermal boundary layer, the temperature dependent viscosity, and momentum balance in the crucial region near the wall.

Corresponding author: A.T. Eswara

E-mail address: eswaram@ymail.com

In this paper we investigate the effect of variation viscosity and Prandtl number on the unsteady, laminar incompressible boundary layer flow of ethanol over a two-dimensional body (cylinder). It is remarked here that ethanol, produced from molasses in the sugarcane industry, is one of environmental friendly blended fuel. Further, ethanol is one of the most commonly used fluid found in all engineering applications, particularly in automobile/ pharmaceutical industries.

NOMENCLATURE

F	dimensionless velocity along surface	β	pressure gradient parameter
G	dimensionless temperature	η	transformed variable
L	characteristic length	μ	dynamic viscosity
N	viscosity ratio	ψ	dimensional stream function
T	dimensional temperature	ξ	scaled stream wise co-ordinate
Pr	Prandtl numbers	**Subscripts**	
Re_L	Reynolds number	∞	conditions in the free stream
R	radius of the cylinder	e, w	denote conditions at the edge of the boundary layer and on the surface, respectively
u, v	velocity components along x – and y – directions, respectively	ξ	partial derivatives with respect ξ
x, y	cartesion co-ordinates along and normal to surface respectively	t, t*	partial derivatives with respect to t and t*
u_e	potential flow velocity	f_w	surface mass transfer
α_1, P	dimensionless parameters	u_∞	free stream velocity
Ec	viscous dissipation parameter (Eckert number)	x, y	partial derivatives with respect to x and y
A	surface mass transfer parameter	**Superscripts**	
U	steady state velocity at the edge of the boundary layer	$(')$	partial derivatives with respect to η
t, t*	dimensional and dimensionless times		

2. MATHEMATICAL FORMULATION

Consider the unsteady, laminar nonsimilar boundary – layer forced convection flow (of ethanol) over a two – dimensional body (cylinder) when the free stream velocity vary with the axial distance (x) along the surface. The fluid is assumed to flow with moderate velocities, and the temperature difference between the wall and the free stream is small (< 40^0C). In the range of temperature considered (i.e. < 40^0C), the variation of both density (ρ) and specific heat (c_p) of ethanol with temperature, is less than 1% (See Table 1) and hence they are taken as constants. However, since the thermal conductivity (k) and viscosity (μ) [and hence Prandtl number (Pr)] variation with temperature is quite significant, the viscosity and Prandtl number are assumed to vary as an inverse liner function of temperature [7,8]:

$$\mu = 1/(b_1 + b_2 T) \tag{1}$$

$$\Pr = 1/(c_1 + c_2 T) \tag{2}$$

where $b_1 = 53.804$, $b_2 = 1.584$, $c_1 = 0.0428$ and $c_2 = 0.0006$ (3)

Table. 1. Values of thermo physical properties of ethanol at different temperature [9]

Temperature(T) (^{0}C)	Density(ρ) (g/cm^3)	Specific heat(c_p) (J × 10^7/kg K)	Thermal conductivity (k) (erg × 10^5/cms K)	Viscosity(μ) (g × 10^{-2}/cms)	Prandtl number (Pr)
0	0.8037	2.2416	0.1721	1.7730	23.0906
10	0.7981	2.2416	0.1703	1.4662	20.1064
20	0.7901	2.4283	0.1670	1.2003	17.4532
30	0.7821	2.5296	0.1611	1.0035	13.7531
40	0.7741	2.6242	0.1585	0.8340	12.6316

The numerical data, used for these correlations, are taken from Ref. [9]. The relation (1) and (2) are reasonably good approximations for liquids such as ethanol, particularly for small wall and ambient temperature differences. As the fluid is incompressible, the contribution of heating due to compression is very small and it has been neglected. The effect of viscous dissipation is included in the analysis. It is assumed that the injected fluid possess the same physical properties as the boundary -layer fluid. Under the foregoing assumptions, the equations governing the above two dimensional flow over cylinder are [10]:

$$\left(r^j u\right)_x + \left(r^j v\right)_y = 0 \tag{4}$$

$$u_t + uu_x + vu_y = \left(u_e\right)_t + u_e\left(u_e\right)_x + \rho^{-1}\left(\mu u_y\right)_y \tag{5}$$

$$T_t + uT_x + vT_y = \rho^{-1}\left(\frac{\mu}{\Pr}T_y\right)_y + \left(\frac{\mu}{\rho c_p}\right)\left(u_y\right)^2 \tag{6}$$

The initial and boundary conditions are:

$$u(x,y,0) = u_i(x,y),\quad v(x,y,0) = v_i(x,y),\quad T(x,y,0) = T_i(x,y) \tag{7}$$

$$\left.\begin{array}{l} u(x,0,t) = 0,\quad v(x,0,t) = v_w(x,t),\quad T(x,0,t) = T_w(x,t) \\ u(x,\infty,t) = u_e(x,t),\quad T(x,\infty,t) = T_\infty \end{array}\right\} \tag{8}$$

Applying the following transformation

$$\xi = \int_0^x \left(\frac{U}{u_\infty}\right)\left(\frac{r}{L}\right)^{2j} d\left(\frac{x}{L}\right),\quad t^* = \left(\frac{3}{2}\right)\left(\mathrm{Re}_L\right)\left(\frac{\mu_e}{\rho L^2}\right)t,\quad \eta = \left(\frac{U}{u_\infty}\right)\left(\frac{\mathrm{Re}_L}{2\xi}\right)^{1/2}\left(\frac{r}{L}\right)^j\left(\frac{y}{L}\right),$$

$$\psi(x,y,t) = u_\infty L\phi(t^*)\, f(\xi,\eta,t^*)\left(\frac{2\xi}{\mathrm{Re}_L}\right)^{1/2},\quad \mathrm{Re}_L = \frac{\rho u_\infty L}{\mu},$$

$$u = \left(\frac{L}{r}\right)^j \frac{\partial\psi}{\partial y},\quad v = -\left(\frac{L}{r}\right)^j \frac{\partial\psi}{\partial x},\qquad G = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

to Eqs. (4)-(6), we see that the continuity Eq. (4) is identically satisfied, and Eqs. (5) and (6) reduce to a non - dimensional form, respectively, as:

$$\left(NF'\right)' + \phi\left[fF' + \beta\left(1 - F^2\right)\right] - P\left[F_{t^*} - \phi^{-1}\phi_{t^*}\left(1 - F\right)\right] = 2\xi\phi\left(FF_\xi - f_\xi F'\right) \tag{10}$$

$$\left(NPr^{-1}G'\right)' + \phi fG' + NEc\left(\frac{u_e}{u_\infty}\right)^2 (F')^2 - PG_{t^*} = 2\xi\phi\left(FG_\xi - f_\xi G'\right) \tag{11}$$

where $N = \left(\dfrac{\mu}{\mu_\infty}\right) = \dfrac{b_1 + b_2 T_\infty}{b_1 + b_2 T} = \dfrac{1}{1 + a_1 G}$, $\Pr = \dfrac{1}{c_1 + c_2 T} = \dfrac{1}{a_2 + a_3 G}$, $a_1 = \left(\dfrac{b_2}{b_1 + b_2 T_\infty}\right)\Delta T_w$

$$a_2 = c_1 + c_2 T_\infty,\quad a_3 = c_2 \Delta T_w,\quad \Delta T_w = (T_w - T_\infty),\quad \beta(\xi) = \frac{2\xi}{U}\left(\frac{dU}{d\xi}\right),\quad \frac{u}{u_e} = f' = F$$

$$u_e = U\phi(t^*),\ v = -\left(\frac{r}{L}\right)^j 2\xi (\mathrm{Re}_L)^{-1/2} U\phi[f + 2\xi f_\xi + (\beta + \alpha_1 - 1)\eta F],$$

$$Ec = \frac{u_\infty^2}{[c_p (T_w - T_\infty)]},\ P = 3\xi\left(\frac{L}{r}\right)^{2j}\left(\frac{u_\infty}{U}\right)^2,\quad \alpha_1 = \left(\frac{2j\xi}{r}\right)\left(\frac{dr}{d\xi}\right),\quad f = \int_0^\eta F d\eta + f_w,$$

$$f_w = -(\xi)^{-1/2}\left(\frac{\mathrm{Re}_L}{2}\right)^{1/2}\phi^{-1}\int_0^x \left(\frac{v_w}{u_\infty}\right)\left(\frac{r}{L}\right)^j d\left(\frac{x}{L}\right) \tag{12}$$

The transformed boundary conditions are

$$F(\xi, 0) = 0 \quad G(\xi, 0) = 1$$
$$F(\xi, \infty) = 1 \quad G(\xi\ \infty) = 0, \quad \text{for } \xi \ge 0 \text{ and } t^* \ge 0.$$

(13)

Here it is assumed that the flow is, initially, steady and changes to unsteady state for t*>0. Therefore, the initial conditions for F and G at t* = 0 are given by steady-flow equations obtained by putting

$$\phi = 1,\quad \phi_{t^*} = F_{t^*} = G_{t^*} = 0 \tag{14}$$

in Eqs. (10) and (11). Consequently, the initial conditions at t* = 0 can be written as

$$\left.\begin{aligned} (NF')' + fF' + \beta(1 - F^2) &= 2\xi(FF_\xi - f_\xi F') \\ (NPr^{-1}G')' + fG' + NEc\left(\frac{u_e}{u_\infty}\right)^2 (F')^2 &= 2\xi(FG_\xi - f_\xi G') \end{aligned}\right\} \tag{15}$$

It may be remarked here that Eqs. (10 and (11) are exactly same as those of Eswara and Nath [11] who have studied the effect of variable viscosity and Prandtl number on unsteady water boundary layers.

For a circular cylinder, unsteadiness as well as, nonsimilarity are both due to the external velocity at the edge of the boundary-layer, $u_e(\bar{x}, t)$ (where $\bar{x}$ is the dimensionless distance along the surface) and the normal component of the velocity at the surface, $f_w(\bar{x}, t)$. The free stream velocity distribution for the case of a circular cylinder, the pressure gradient parameter and the distance from the axis of body are given by Schlichting [8] as

$$\frac{u_e}{u_\infty} = 2\sin\bar{x}\varphi(t^*),\quad \frac{U}{u_\infty} = 2\sin\bar{x},\quad \bar{x} = \frac{x}{R},\ J = 0,\quad L = R\ ,$$

$$\beta = \frac{2\cos\bar{x}}{(1 + \cos\bar{x})},\ \xi = 2(1 - \cos\bar{x}),\ \alpha_1 = 0,\ P = \frac{3}{2(1 + \cos\bar{x})},$$

$$f_w = A\phi^{-1}(t^*)\left[\frac{(\bar{x}/2)}{\sin(\bar{x}/2)}\right],\quad A = -\left(\frac{v_w}{u_\infty}\right)\left(\frac{\mathrm{Re}_L}{2}\right)^{1/2} \tag{16}$$

The skin friction coefficient and heat transfer coefficient in terms of Nusselt number can be expressed, respectively, as

$$C_f\left(\mathrm{Re}_L\right)^{1/2}=\frac{2\tau_w}{\rho u_\infty{}^2}=\frac{2\mu\dfrac{\partial u}{\partial y}}{\rho u_\infty{}^2}=2\left(\frac{u_e}{u_\infty}\right)^2\phi^{-1}(t^*)\left(\frac{r}{L}\right)^j\left(2\xi\,\mathrm{Re}_L\right)^{-1/2}F'_w \qquad (17)$$
$$=4\left(1+\cos\bar{x}\right)\left(\sqrt{1-\cos(\bar{x})}\right)\phi(t^*)\,F'_w$$

$$Nu\left(\mathrm{Re}_L\right)^{-1/2}=-\frac{L\left(\dfrac{\partial T}{\partial y}\right)_w}{\Delta T_w}=-\left(\frac{\mathrm{Re}_L}{2\xi}\right)^{1/2}\left(\frac{r}{L}\right)^j\left(\frac{U}{u_\infty}\right)G'_w=-\sqrt{2}\,\cos\left(\frac{\bar{x}}{2}\right)G'_w \qquad (18)$$

If the normal velocity at the wall v_w is taken as constant, then A is a constant since u_∞ and Re_L are constants. The mass transfer parameter A>0 or A<0, according as there is suction or injection.

3. RESULTS AND DISCUSSION

The set of partial differential equations (10) and (11) along with the boundary conditions (13) using the relations (14)-(16) has been solved numerically using an implicit finite difference scheme along with quasilinearization technique. Since the method is described in great detail in Ref [12], for the sake of brevity, its description is omitted here. In order to assess the accuracy of our method, we have compared the skin friction $[C_f(\mathrm{Re}_L)^{1/2}]$ and heat transfer $[Nu(\mathrm{Re}_L)^{-1/2}]$ coefficients [See Fig. 1(a) & Fig 1(b)] with Eswara and Nath [11] who have studied the effect of variable viscosity and Prandtl number on unsteady water boundary–layers. Our results are found to be in good agreement with these of [11] for both the cases of constant viscosity (Pr = 7.0) and variable viscosity (ΔT_w = 10.0).

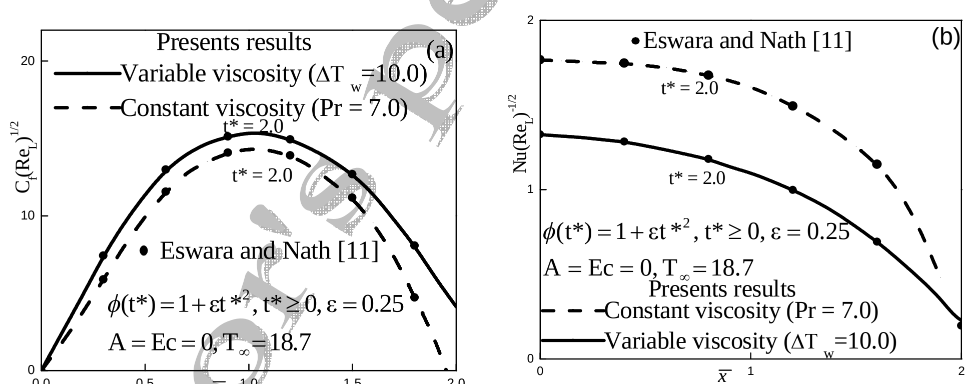

Fig.1. Comparison of unsteady state results of water (Pr = 7.0) boundary layers for a cylinder

The effect of unsteadiness on skin friction $C_f(\mathrm{Re}_L)^{1/2}$ and heat transfer $Nu(\mathrm{Re}_L)^{-1/2}$coefficients is displayed in the Fig. 2. It is observed from Fig. 2(a) that skin friction coefficient $C_f\,(\mathrm{Re}_L)^{1/2}$ increases from zero to a maximum value in a certain range of $\bar{x}$ ($\bar{x}$ = 1.05) and then decreases as $\bar{x}$ further increases. It is also observed that the effect of variable fluid properties is to increases the skin friction coefficient and decrease the heat transfer coefficient [See Fig. 2(b)]. When t* = 2.0, skin friction coefficient $C_f(\mathrm{Re}_L)^{1/2}$ for variable fluid properties differs from those of constant fluid properties by about 16.33% at $\bar{x}$ = 1.05. On the other hand the percentage between constant and variable fluid properties, in the case of heat transfer $Nu(\mathrm{Re}_L)^{-1/2}$coefficient is 52.76% at $\bar{x}$ =1.05. It is also observed from this figure in the case of variable fluid properties, the point of zero skin friction for unsteady flow is moved down stream as compared to constant fluid properties. For circular cylinder it may be noted that the point of skin friction predicted by our method is $\bar{x}$ = 1.98 for constant fluid properties 2.178 for variable fluid properties.

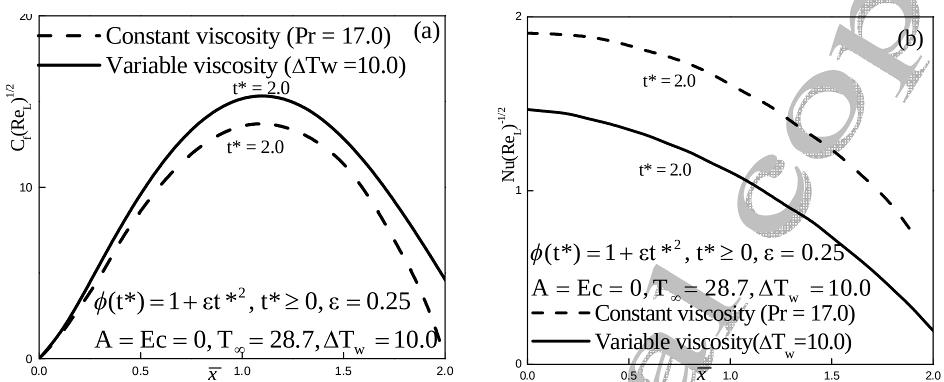

Fig.2. Comparison of variable fluid property results with constant fluid properties results. (a) Skin friction coefficient. (b) Heat transfer coefficient.

To see the effect of difference in the temperature (ΔT_w) between the wall and fluid, which is actually causes the variation of viscosity and Prandtl number across the boundary-layer, the skin friction and heat transfer coefficients have been plotted against ΔT_w [Fig. 3(a) and 3(b)]. Since T_∞ =28.7^{o}C, the maximum value of ΔT_w taken is 30 ^{o}C so as to keep the temperature within the allowed value (40 ^{o}C). It is observed from this figure that $C_f(Re_L)^{1/2}$ increases with increase of ΔT_w whereas $Nu(Re_L)^{-1/2}$ decreases.

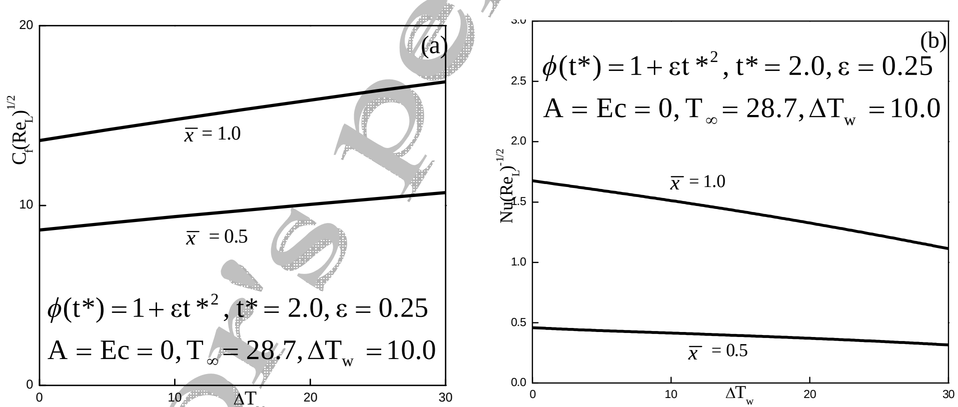

Fig. 3. Variation of skin friction and heat transfer with ΔT_w

The variation of skin friction and heat transfer coefficients [$C_f(Re_L)^{1/2}$, $Nu(Re_L)^{-1/2}$] with time at different stream wise locations, in the presence of variable fluid properties [T_∞ =28.7^{o}C, ΔT_w=10.0], is shown in Fig. 4(a) . It is found that $C_f(Re_L)^{1/2}$ and $Nu(Re_L)^{-1/2}$ increase with the increase of time. This behavior is same at all stream wise locations. In fact, the percentages of increase in $C_f(Re_L)^{1/2}$, at $\bar{x}$ =1.0 is 110.23 % , for an increase of t* from 0 to 2.0 whereas in the case of $Nu(Re_L)^{-1/2}$, at $\bar{x}$ = 1.5, it is about 60%. The corresponding velocity (F) and temperature (G) profiles are displayed in Fig. 4(b). It is clear from these figures that both velocity and temperature profiles become steep with the increase of time (t*). Thus, the effect of unsteadiness, when the free stream, is accelerating, is to decrease the thickness of both momentum and thermal boundary – layer.

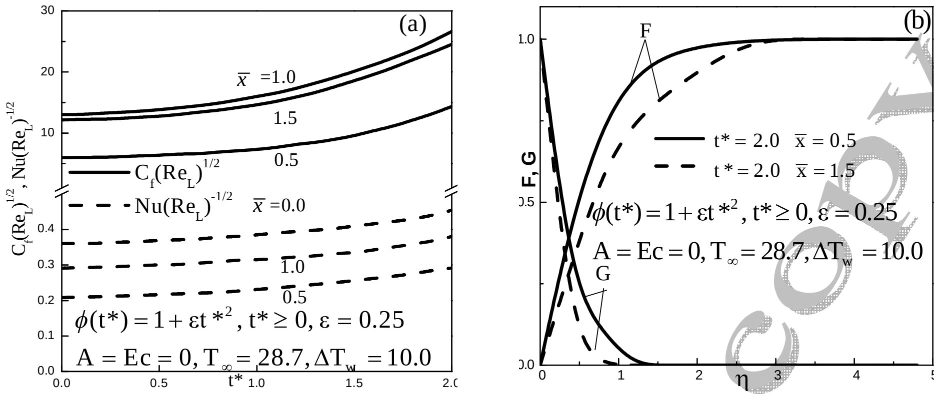

Fig. 4.The effect of time on (a) skin friction and heat transfer coefficients
(b) velocity and temperature profiles

The effect of viscous dissipation parameter (Ec) on skin friction and heat transfer $[C_f(Re_L)^{1/2}, Nu(Re_L)^{-1/2}]$ in the presence of variable fluid properties $[T_\infty = 28.7^oC, \Delta T_w = 10.0]$, is depicted in Fig. 5(a). It is observed that both skin friction $[C_f(Re_L)^{1/2}]$ and heat transfer $[Nu(Re_L)^{-1/2}]$ decreases with the increases of Ec and the effect of viscous dissipation is more pronounced on the heat transfer than on skin friction. Though the viscous dissipation parameter Ec appears only in energy equation, its effect becomes prominent on the skin friction since, the momentum equation is coupled with energy equation due to the temperature–dependent viscosity and Prandtl number. It is also found that $C_f(Re_L)^{1/2}$ increases with the increase of time, irrespective of the value of Ec. On the other hand, $Nu(Re_L)^{-1/2}$ reduces with the increase of Ec ($Ec \neq 0$) and time t*. In fact, it is found that the percentage decrease of $Nu(Re_L)^{-1/2}$ for an increase in Ec from 0 to 2.0, for t* = 1.0 and $\bar{x}$ = 1.5, is 68.7% as compared to 22.4% of $C_f(Re_L)^{1/2}$ for the same data. This means that heat transfer is more affected by the viscous dissipation as compared to skin friction. Due to viscous dissipation, the fluid near the wall heats up and its temperature becomes more than the wall, although originally the wall was at higher temperature $[T_\infty = 28.7^oC, T_w = 38.7^0C, \Delta T_w = 10.0]$. Thus, the cooler free stream is unable to cool the hot wall due to the heat cushion provided by frictional heating. This results in the reduction of the heat transfer (from the wall to the fluid). Further, it is observed that, for $Ec \neq 0$, $Nu(Re_L)^{-1/2}$ becomes negative indicating the reversal of the direction of heat transfer from the initial, wall to fluid, to fluid to wall. Similar trend has been observed in the case of constant fluid properties parallel flow past a flat plate at zero incidence [7]. However, in the absence of viscous dissipation (Ec = 0) heat transfer takes place in the usual way (from wall to the fluid).

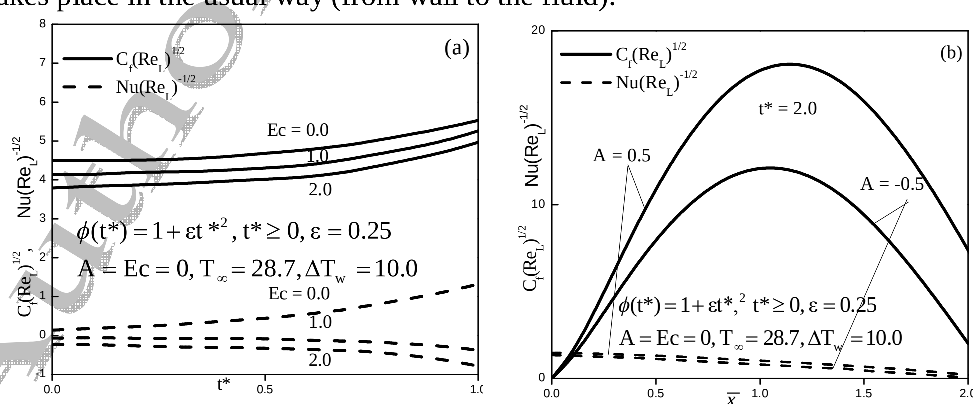

Fig. 5. Effect of (a) viscous dissipation on skin friction and heat transfer coefficients
(b) mass transfer on skin friction and heat transfer coefficients

Fig. 5(b) shows the effect of mass transfer parameter (A) and time (t*) on $C_f(Re_L)^{1/2}$ and $Nu(Re_L)^{-1/2}$. Whatever may be the value of A, both skin friction and heat transfer are found to increases with time t*. Further, it is also found that suction (A > 0) increases both $C_f(Re_L)^{1/2}$ and $Nu(Re_L)^{-1/2}$ while injection (A< 0) does the reverse. From the Fig. 5(b) it is observed that $C_f(Re_L)^{1/2}$ increases from zero to maximum up to a value of $\bar{x} = 1.1$ in the case of injection (A< 0) and up to $\bar{x} = 1.2$ in the case of suction (A>0), and then decreases as $\bar{x}$ further increases. On the other hand $Nu(Re_L)^{-1/2}$ continues to decrease with the increase of $\bar{x}$. Furthermore, it is found that injection (A< 0) causes the location of zero skin friction to move upstream, as compared to suction (A > 0).

4. CONCLUSIONS

The skin friction coefficient increases in the case of variable viscosity as compared to constant viscosity, where as the effect of variable viscosity on the heat transfer coefficient is just opposite. The unsteadiness and suction cause the point of zero skin friction to move downstream. However, the effect of variable fluid properties is to move it downstream. In general injection reduces both skin friction and heat transfer while suction does the reverse. The heat transfer is found to depend strongly on viscous dissipation, but the skin friction is little affected by it. Both momentum and thermal boundary layer thicknesses decrease boundary-layer in the stream wise location. From our study, it can be concluded that the effect of variation of viscosity and Prandtl number with temperature has to be taken into consideration to avoid significant errors in the prediction of skin friction coefficient and heat transfer rate.

ACKNOWLEDGEMENT

One of the authors (AHS) thanks the Principal and the Management of MIT Mysore for their kind support. The authors are also thankful to PETRF and the Department of Mathematics, PES College of Engineering, Mandya-571 401 for providing excellent facilities for research.

REFERENCES

[1] C.F. Dewey, J. F. Gross, Exact Solutions of the Laminar Boundary Layer Equations: Advances in Heat Transfer, vol. IV. Academic Press, New York, **pp**. 313-367. (1967).

[2] T. Davis, G. Walker, On solution of the compressible laminar boundary layer equations their behavior near separation, J. Fluid Mech. **80** 279 – 292. (1977).

[3] B.J. Venkatachala G. Nath, Nonsimilar laminar incompressible boundary layers with vectored mass transfer, proc. Indian Acad Sci.(Eng. Sci) **3** (1980)129-142.

[4] K.R.Jayakumar, A.H. Srinivasa, A.T. Eswara, Nonsimilar MHD boundary layers in two-dimensional forced flow with temperature-dependent viscosity *J. Curr.Sci*. **14(1)**: 407- 417 (2009)

[5] A. Pantokratoras, Forced and Mixed convection boundary layer flow along a flat Plate with variable viscosity and variable Prandtl number: new results, *Heat Mass Transfer* **41** 1085-1094 (2005).

[6] J.J. Eisenhuth, H. Hoffman, wall temperature estimation for heated under water bodies, *J. Hydronaut*. **15** 90-96 (1981)

[7] L.S.Yao and I. Catton, *Int. J. Heat and Mass Transfer* **21,** 407 (1978).

[8] I. Pop, R. S. R. Gorla and M. Rashidi, *Int. J. Engng Sci*. **30,** 1 (1992).

[9] N.B. Vargaftik, Thermo physical properties of liquids and gases, John Wiley and Sons, Inc. London (1975).

[10] H.Schlichting, *Boundary Layer Theory*, Springer-Verlag, New York (2000).

[11] A.T.Eswara, G. Nath, Unsteady nonsimilar two dimensional and axi-symmetric water boundary layers with variable viscosity and Prandtl number, *Int. J. Eng.Sci.***32** No.2, (1994) 267-279.

[12] K. Inouye and A. Tate. Finite difference version of quasi-linearization applied boundary layer equations, ***AIAA J***. **12**, 558-560 (1974).