

**DARCY – WEISH BACH FRICTION FACTOR THROUGH POROUS  
RECTANGULER OPEN – CHANNELS – AN ANALYTICAL  
APPROACH**

**A.Venkateshwarlu**  
Lecturer in Mathematics  
C.K.M Arts&Science college  
Warangal – 506 006  
India

**\*N.Ch.Pattabhiramacharyulu**  
professor (Retd) of Mathematics  
National Institute of technology  
Warangal-506004  
India

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**ABSTRACT**

Open channel flows widely occur in the design of irrigation canal net works. Most of the investigation on open channel flows are experimental studies only. This may be due to geometric complexity of channels and channel networks. The present paper deals with an analytical investigation on the flow through an open porous straight channel of rectangular cross-section under a constant pressure gradient. The momentum equation takes care of the convective acceleration and the viscous stresses generated in the flow region. Fourier series technique has been employed to solve the flow equations. Expression for the Darcy-Weish Bach Friction factor has been computed for a wide spectrum of the values of the channel depth and porosity coefficient of the flow media and its variation graphically illustrated.\*

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**Introduction:**

Fluid flows through pipes and channels have been a subject of intensive study by both experimental and analytical scientists- Hydraulic Engineers and Mathematicians for the last several centuries because of their wide applications. In a pipe flow, the pipe is completely filled with the fluids. As such it is subjected to hydraulic pressure only and is not influenced by the atmospheric pressure.

In contrast to this, the channel flow, quite often referred as open channel flow has a free surface which is in contact with the atmosphere and so the atmosphere pressure influences the flow in the channel. Open channel flows are widely employed in the design of irrigation canal net works. Most of the investigations on open channel flows are experimental studies only. This may be due to geometric complexity of channels and channel networks. Many experimental studies on friction losses, wall shears, seepage and evaporation in flows through channels of different non-circular cross-sections are presented in the treatises of Bakhmeteff.B.A<sup>[1]</sup>, Posey.C.S<sup>[2]</sup>, Ven Te Chow<sup>[3]</sup> on the subject of Open Channels flows. The drop in the flow rate due to friction in channels is invariably expressed in terms of a friction factor which is referred in the literature as Darcy-Weish Bach friction factor ( $f$ ).

An Empirical relation has been given to find the variation of the friction factor with equivalent hydraulic radius of the channels. The treatises cited above amply illustration such relation. Recently Prabhakar Reddy<sup>[4]</sup> studied analytically for the “Darcy’s Weish Bach’ friction factor for a wide spectrum of channel – cross sections Rectangular, Trapezoidal, Isosceles triangular, general triangle, semicircle, circular segments and parabola.

In this paper a problem is investigated analytically when fluid is viscous, incompressible homogenous and the channel is of rectangular cross section and flow medium is porous. The momentum equation adopted is the one given by YAMAMOTO.K and YOSHIDA.K<sup>[5]</sup> and YAMAMOTO.K and IUMAMURA<sup>[6]</sup> which takes into consideration of convective acceleration and the viscous stresses generated in the flow region. The channel walls are taken to be impermeable by allowing no cross flow into the channel a constant pressure gradient down the channel length.

### MATHEMATICAL FORMULATION

Consider a uniform free surface flow of a homogenous incompressible fluid of constant density  $\rho = rg$  and constant viscosity  $\mu$  through a straight open channel of depth  $H$ . Consider a rectangular Cartesian frame  $O (XYZ)$  with the origin  $O$  on the free surface, the axis of  $Z$  parallel to the flow direction,  $Y$  – axis vertically upwards (consequently the  $X$  and  $Y$  axis in the plane of the channel cross section – perpendicular to its length (i.e. the  $Z$  – direction).

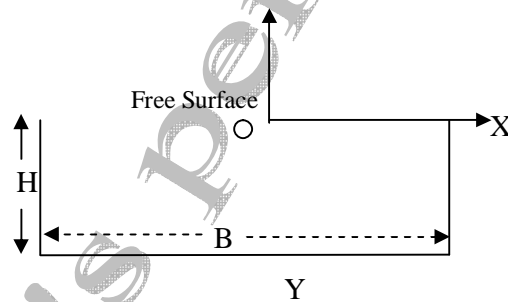


Fig.1:channel cross section

With reference to this frame of reference the velocity is given by

$\vec{V} = (0, 0, W(X, Y))$  which evidently satisfies the continuity equation

$$\text{div } \vec{V} = 0 \quad - \quad (1) \quad \text{where } \vec{V} \text{ is the}$$

fluid velocity

The momentum equation in the  $Z$  – direction reduces to

$$\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} - \frac{W}{K} = \frac{-C}{\mu} \quad - \quad (2)$$

Here  $\mu$  is the coefficient of viscosity of the fluid and K the porosity coefficient of the medium. Further C stands for the pressure gradient  $C = -\frac{\partial P}{\partial Z}$ , where P stands for the pizometric head in the channel.

Also the velocity  $W(x, y)$  satisfies the following boundary condition:

$$w/y=-.H = 0 \quad - (3.1)$$

$$w/y= \pm B/2 = 0 \quad - (3.2)$$

on the free surface which is stress free:  $\frac{\partial W}{\partial Y} \Big|_{y=0} = 0 \quad - (3.3)$

### DARCY – WEISHBACH FRICTION FACTOR

The average velocity  $\bar{W}$  in the channel defined by the equation

$$\bar{W} = \iint_A W dA \quad - (7)$$

the integration being carried over the channel cross – section area A.

The Darcy – Weish Bach friction factor in the channel

(p – 8 of [3]) is given by the definition equation  $f = \frac{8Rgs}{(\bar{W})^2} = \frac{K_1}{R^*} \quad - (8)$

where R represents the Hydraulic radius  
 channel cross-section area

$$R = \frac{\text{Channel wetted perimeter}}{\text{channel cross-section area}}, \quad s = \frac{\mu c}{\rho g} \text{ and } R^* = \frac{\gamma s^2}{Rg\mu^2} \quad - (9)$$

Further  $K_1$ , is the parameter characteristic of the Darcy – Weish Bach Friction Factor associated with the channel cross – section.

We introduce non-dimensional quantities as per the following scheme:

$$\left. \begin{aligned} X &= Bx & Y &= By, & H &= Bh \\ W &= \frac{B^2 C}{\mu} w, & K &= \frac{B^2}{\sigma} \end{aligned} \right\} \quad - (10)$$

Flow rate  $Q = \iint W dX dY = \frac{B^2 C}{\mu} . B^2 \iint w dx dy$

$$Q = \frac{B^4 C}{\mu} \iint w dx dy, \quad \text{where } q = \iint w dx dy$$

$$\bar{W} = \frac{Q}{BH} = \frac{Q}{B^2h} = \frac{1}{B^2h} \times \frac{B^4C}{\mu} \iint w dx dy = \frac{B^2C}{\mu h} \times \iint w dx dy = \frac{B^2C}{\mu h} . q$$

$$\bar{W} = \frac{B^2C}{\mu} . \bar{w} \quad - \quad (11)$$

where  $\bar{w} = \frac{1}{h} \iint w dx dy = \frac{q}{h}$

The basic equation and the boundary conditions in terms of the non – dimensional quantities introduced above is rewritten by

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - \sigma^2 w = -1 \quad \text{with the rectangle} \quad - \quad (12)$$

on the bottom  $w /_{y=-h} = 0 \quad - \quad (13)$

on the sides  $w /_{x=\pm \frac{1}{2}} = 0 \quad - \quad (14)$

and on the free surface

$$\frac{\partial w}{\partial y} /_{y=0} = 0 \quad - \quad (15)$$

Let the velocity field  $w(x, y)$  be split as

$$w = w_1(y) + w_2(x, y) \quad - \quad (16)$$

Substituting (16) in the (12) and separating the terms, we get the equation for  $w_1(y)$

$$\frac{d^2 w_1}{dy^2} - \sigma^2 w_1(x, y) = -1$$

with the conditions:

$$w_1(y) /_{y=-h} = 0 \quad - \quad (17.1)$$

$$\frac{dw_1}{dy} /_{y=-b} = 0 \quad - \quad (17.2)$$

and the equation for  $w_2(x, y)$

$$\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} - \sigma^2 w_2(x, y) = 0 \quad - \quad (18)$$

with the conditions

$$w_2(x, y) /_{y=-h} = 0 \quad - \quad (18.1)$$

$$\frac{\partial w_2}{\partial y} /_{y=0} = 0 \quad - \quad (18.2)$$

$$w_2 /_{x=\pm \frac{1}{2}} (\frac{1}{2}, y) = -w_1(y) \quad - \quad (18.3)$$

The solution of the equation (17) satisfies the conditions (17.1) and (17.2) is

$$w_1 = \frac{1}{\sigma^2} \left[ 1 - \frac{\text{Cosh}(\sigma y)}{\text{Cosh}(\sigma h)} \right] \quad - \quad (19)$$

Let the solution of the equation (18) be assumed as

$$w_2 = \sum_{n=0}^{\infty} f_n(x) \cdot \cos\left(\frac{(2n+1)\pi}{2h}\right) \Pi y \quad (20)$$

It can be noted that  $f_n(x)$  satisfies the equation

$$f_n^{11}(x) - r_n^2 f_n(x) = 0 \quad (21)$$

where  $r_n^2 = \left(\frac{(2n+1)\pi}{2h}\right)^2 + \sigma^2$  - (22)

The general solution of eq(21) is

$$f_n(x) = A_n \cosh(r_n x) + B_n \sinh(r_n x) \quad (23)$$

where  $A_n$  and  $B_n$  are constants to be determined

$w$  is **symmetric** about the Y – axis,  $w_2$  is also symmetric about Y – axis this leads to  $B_n = 0$

Hence

$$w_2(x, y) = \sum_{n=0}^{\infty} A_n \cosh(r_n x) \cos\left(\frac{(2n+1)\pi}{2h}\right) \Pi y \quad (24)$$

The constant  $\{A_n\}$  can be determined by employing the boundary condition (14)

$$w_1(y) + w_2(\pm 1/2, y) = 0$$

$$w_2\left(\frac{1}{2}, y\right) = -w_1(y)$$

$$\sum_{n=0}^{\infty} A_n \cosh(r_n x) \cos\left(\frac{(2n+1)\pi}{2h}\right) \Pi y = \frac{1}{\sigma^2} \left[ \frac{\cosh(\sigma y)}{\cosh(\sigma h)} - 1 \right] \quad (25)$$

Expanding the R.H.S. as a Fourier series in the interval  $(-1/2, 1/2)$  and comparing the like terms in the equation (25) we get

$$A_n = \frac{2(-1)^{n+1}}{h} \times \left[ \frac{1}{\lambda_n (r_n)^2 \cosh(r_n / 2)} \right] \quad (26)$$

The non dimensional flow rate

$$q = \iint w dx dy = \frac{1}{\sigma^3} \{ \sigma h - \tanh(\sigma h) \} + \left\{ \frac{4}{h} \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{\lambda_n^2 \gamma_n^3} \tanh(\lambda_n / 2) \right\} \quad (27)$$

and the average velocity

$$\bar{w} = \frac{1}{h} q = \frac{1}{h} \left\{ \frac{1}{\sigma^3} \{ \sigma h - \tanh(\sigma h) \} + \frac{4}{h} \sum_{n=0}^{\infty} (-1)^{2n+1} \frac{\tanh(\gamma_n / 2)}{\lambda_n^2 \gamma_n^3} \right\} \quad (28)$$

The average velocity in the dimensional form

$$\bar{W} = \frac{B^2 C \bar{w}}{\mu}$$

$$= \frac{B^2 C}{\mu h} \left\{ \frac{1}{\sigma^3} \{ \sigma h - \tanh(\sigma h) \} + \frac{4}{h} \sum_{n=0}^{\infty} (-1)^{2n+1} \frac{\tanh(\gamma_n / 2)}{\lambda_n^2 \gamma_n^3} \right\} \quad - (29)$$

Using this, the Darcy-Weish Bach friction factor can be expressed as

$$f = \frac{8Rgs}{W^2}, \quad \text{Here } s = \frac{\mu c}{\rho g}, \quad R = \frac{Bh}{2h+1}$$

$$f = \frac{8\mu c}{\rho} \times \frac{Bh}{(2h+1)} \times \frac{\mu^2 h^2}{B^4 C^2} \left\{ \frac{1}{\sigma^3} \{ \sigma h - \tanh(\sigma h) \} + \frac{4}{h} \sum_{n=0}^{\infty} (-1)^{2n+1} \frac{\tanh(\gamma_n / 2)}{\lambda_n^2 \gamma_n^3} \right\}^{-2} \quad - (30)$$

$$\text{Where, } \gamma_n^2 = \sigma^2 + \lambda_n^2 = \sigma^2 + \{(2n+1)\pi / 2h\}^2 \quad - (31)$$

For simplicity of the expression for the friction factor

$$F^* = \frac{f}{\rho C B^3} = \frac{h^3}{(2h+1)} \left\{ \frac{1}{\sigma^3} \{ \sigma h - \tanh(\sigma h) \} + \frac{4}{h} \sum_{n=0}^{\infty} (-1)^{2n+1} \frac{\tanh(\gamma_n / 2)}{\lambda_n^2 \gamma_n^3} \right\}^{-2} \quad - (32)$$

F\* is computed for wide spectra of h and  $\sigma$  :

$$h = 0.1, 0.2, 0.3, 0.5, 1, 5, 10 \dots\dots\dots$$

$$\sigma = 0.1, 0.2, 0.3, 0.5, 1, 5, 10 \dots\dots\dots$$

and the results are illustrated in Figs 2 and 3.

**Conclusion:**

From the fig. 2 and fig.3 it is observed that the friction factor increases with porosity coefficient  $\sigma$  increases. The fig.2 shows that for  $h < 1$  not much variation of friction factor for different  $\sigma$ , the friction factor linearly falls. As channel depth h is increased. Beyond that it is asymptotic to the h axis raising as the porosity parameter increases. From fig.3 the channel depth has no significant influence as friction factor upto the porosity coefficient  $\sigma = 20.5$  approximately. Beyond that the friction factor increases as the porosity  $\sigma$  increases and increases with the channel depth.

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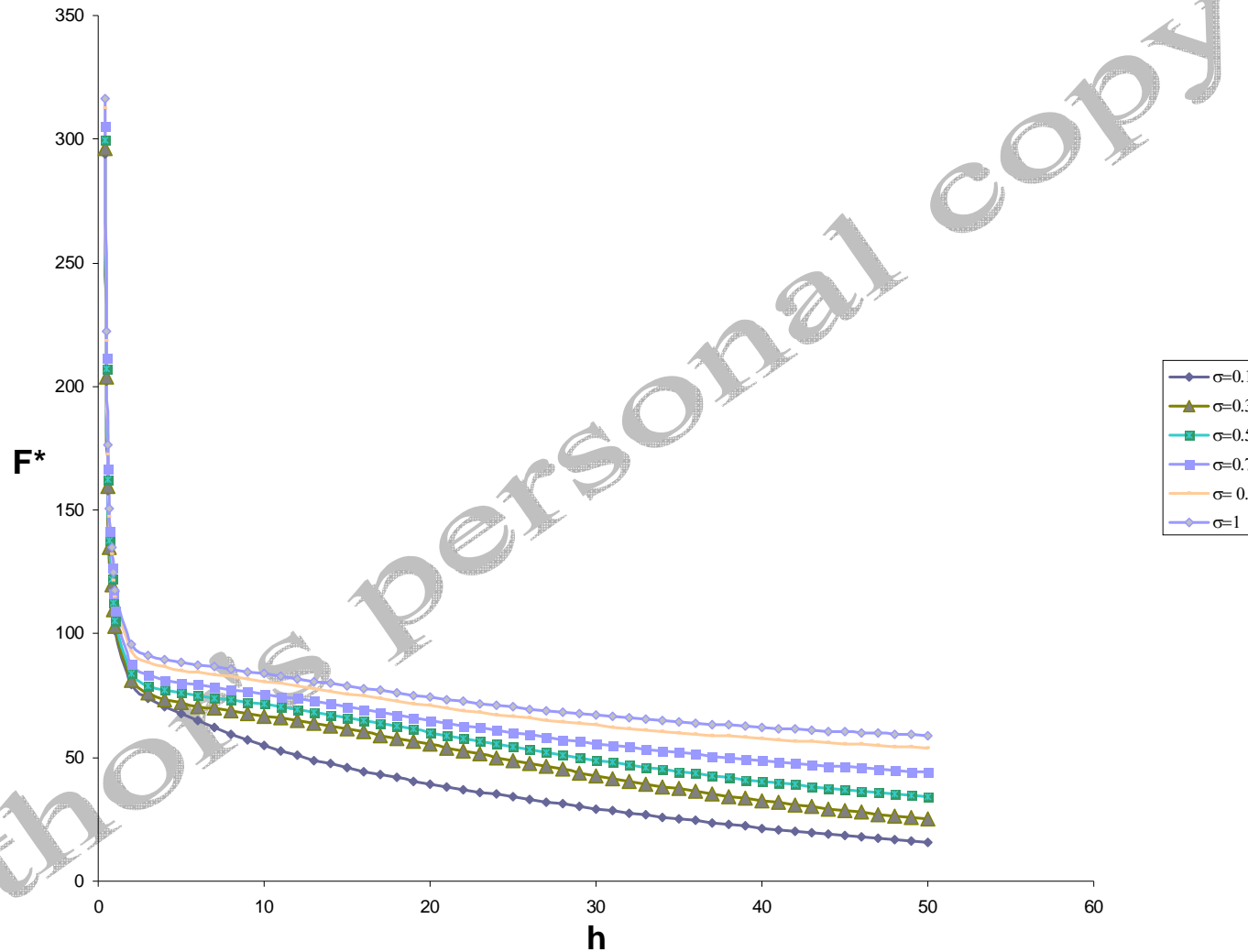


Fig.2 : Variation of the Friction Factor  $F^*$  with the channel depth  $h$  for different porosity coef.  $\sigma =0.1, 0.2$  etc.

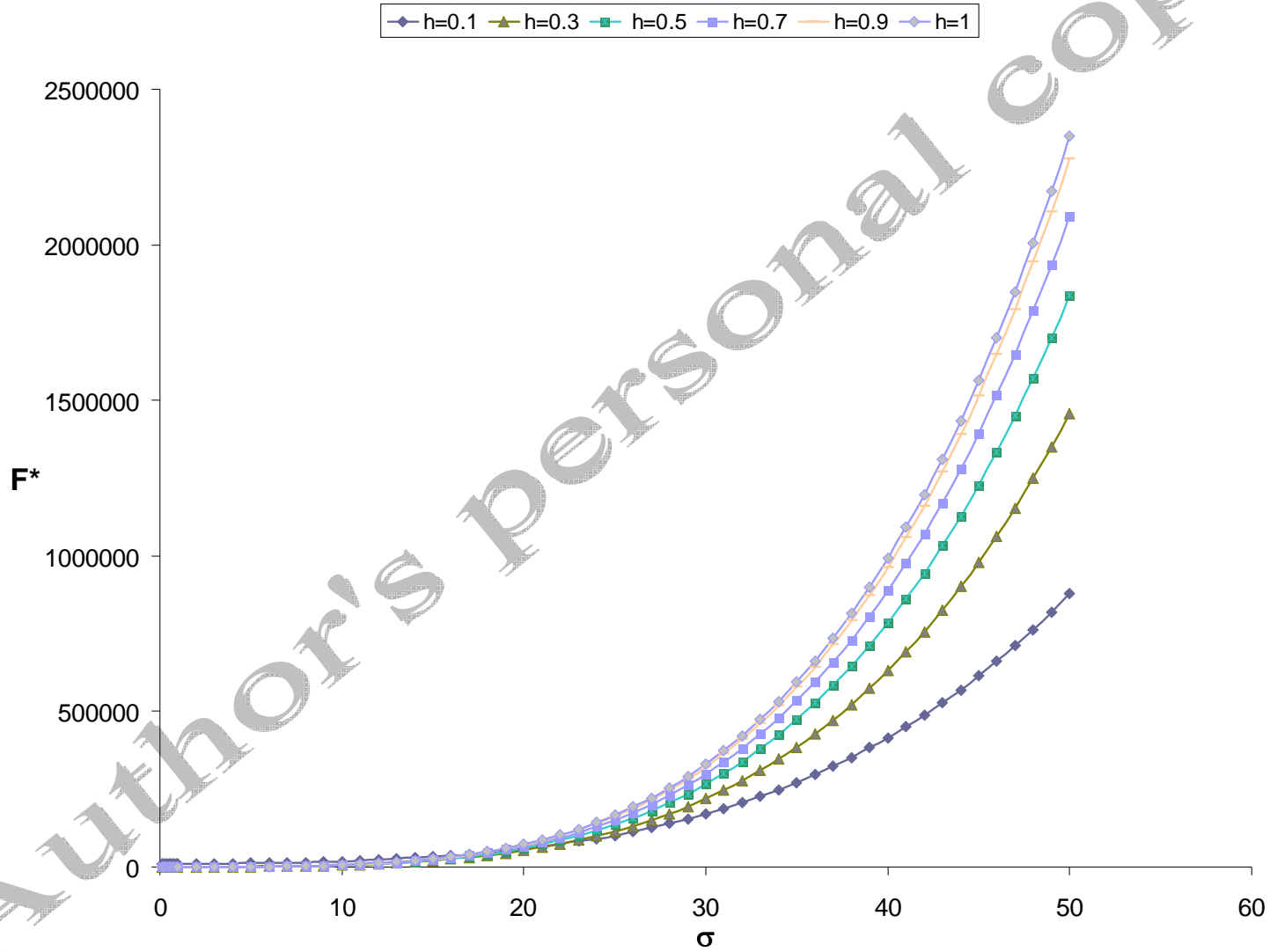


Fig.3 : Variation of the Friction Factor  $F^*$  vs the porosity  $\sigma$  for different channel depth  $h$ .