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# RADIAL VIBRATIONS IN MICRO ELASTIC HOLLOW SPHERE

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**Abstract :** The frequency equations are derived for the radial vibrations in micropolar elastic hollow sphere. It is interesting to observe that a new type of wave is propagated which is not found in classical theory of elasticity. The frequency equation of the classical case is obtained as a particular case of this paper.

[**Key Words:** Radial Vibrations – Micro elastic Sphere]

## Introduction

The mechanical behaviour of elastic materials with micro-structure has been the subject of intensive studies in recent years. The theory of micropolar elasticity is formulated by Erigen [1] and stems from the non-linear theory of micro-elastic solids which was formulated by Erigen and Suhubi [2].

In the micropolar theory, a volume element □v is assumed to be a collection of micro-elements $\Delta v^{(\alpha)}\left(\alpha = 1, 2, \ldots N\right)$. In addition to the classical deformation it considers the rotation of micro elements about the centre of mass of □v.

The problems of radial vibrations of isotropic elastic sphere and hollow sphere are discussed by Ghosh [3], Love's [4] treatise contains an account of the forced vibrations of a sphere due to body forces derivable from a potental. Love [5] considered the sphere problem in connection with the problems of geodynamics. Grey and Eringen [6] obtained the complete solution of sphere subject to dynamic surface tractions and computed the natural frequencies of the free oscillations.

In this paper, we discussed the radial vibration in micropolar elastic hollow sphere and obtained the frequency equations. It is interesting to observe that an additional frequency equation is obtained which is not encounted in the classical elasticity. Further, the result of classical case is obtained as a particular case of it.

**Basic equations**

The fundamental equations for the motion of a micropolar elastic solid are given by the following.

i) The balance of momentum equation:

$$(\lambda+\mu)\,u_{l,lk}+(\mu+k)\,u_{k,ll}+k\in_{klm}\phi_{m,l}+\rho\left(f_k-\ddot{u}_k\right)=0 \tag{1}$$

ii) The balance of the stress moment equation:

$$(\alpha+\beta)\,\phi_{l,lk}+\gamma\phi_{k,ll}+k\in_{klm}u_{m,l}-2k\phi_k+\rho\left(l_k-j\phi_k\right)=0 \tag{2}$$

in the above equations, $\vec{u}$ is the displacement vector, $\vec{\phi}$ is the micro rotation vector, $\vec{f}$ is the body force, $\vec{l}$ is the body couple vector, $\rho$ is the density, $j$ is the micro-inertia, an index (say k) following a comma indicates differentiation with respect to the coordinate $(x_k)$, dot superposed on a symbol denotes differentiation with respect to time t and □, □, *k*, □, □, □ are the material coefficients which satisfy the following in equalities.,

$$\begin{array}{lll} 3\lambda+2\mu+k\geq 0, & 2\mu+k\geq 0, & k\geq 0 \\ 3\alpha+\beta+\gamma\geq 0, & -\gamma\leq\beta\leq\gamma, & \gamma\geq 0 \end{array} \tag{3}$$

The stress tensor $t_{kl}$ and couple stress tensor $m_{kl}$ are given by

$$t_{kl}=\lambda u_{r,r}\,\delta_{kl}+\mu\left(u_{k,l}+u_{l,k}\right)+k\left(u_{l,k}-\in_{klr}\phi_r\right) \tag{4}$$

$$m_{kl}=\lambda\phi_{r,r}\,\delta_{kl}+\beta\phi_{k,l}+\gamma\phi_{l,k} \tag{5}$$

where $\delta_{kl}$ is the Kronecker delta and $\in_{klm}$ is the permutation symbol.

**Formulation and solution of the problem:**

Consider a hollow sphere having a as radius of inner sphere (hollow sphere) and b as radius of outer sphere. We are interested only in radial vibrations (i.e., radial displacement and radial micro-rotation) and therefore, we shall take the displacement and micro-rotation vectors as

$$\vec{u}=u(r,t)\,\hat{e}_r \tag{6}$$

$$\vec{\phi}=\phi(r,t)\,\hat{e}_r \tag{7}$$

Under the absence of body forces and body couples, the equations of motion (1) in this case would reduce to

$$\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}-\frac{2}{r^2}u=\frac{\rho}{\lambda+2\mu+k}\frac{\partial^2 u}{\partial t^2} \tag{8}$$

We suppose

$$u = A' \cos(pt + \in) u_r \tag{9}$$

where $u_r$ is function of r only, $\in$ is the phase and p is the angular frequency.

Substituting (9) in (8) we get

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{2}{r}\frac{\partial u_r}{\partial r} - \frac{2}{r^2} u_r + h^2 u_r = 0 \tag{10}$$

where

$$h^2 = \frac{p^2 \rho}{\lambda + 2\mu + k} \tag{11}$$

The solution of (10) is given by

$$u_r = \frac{\partial}{\partial q}\left[\frac{A \sin q + B \cos q}{q}\right] \tag{12}$$

where

$$q = hr \tag{13}$$

and A, B are arbitrary constants

Thus

$$u(r,t) = \frac{\partial}{\partial q}\left[\frac{A \sin q + B \cos q}{q}\right]\cos(pt + \in) \tag{14}$$

The radial components of force stress and couples stress can be obtained from (4) and (5) and are given by

$$t_{rr} = (\lambda + 2\mu + k)\frac{\partial u}{\partial r} + \frac{2\lambda}{r} u \tag{15}$$

$$m_{rr} = (\alpha + \beta + \gamma)\frac{\partial \psi}{\partial r} + \frac{2\alpha}{r}\phi \tag{16}$$

Substituting (14) in (15) we get

$$\begin{aligned} t_{rr} = &\left[(2 - q^2)\sin q - 2q\cos q + \frac{2\lambda}{\lambda + 2\mu + k}\{q\cos q - \sin q\}\right]A \\ &+ \left[(2 - q^2)\cos q + 2q\sin q - \frac{2\lambda}{\lambda + 2\mu + k}(q\sin q + \cos q)\right]B \end{aligned} \tag{17}$$

Suppose

$$s = \frac{4\mu + 2k}{\lambda + 2\mu + k} \tag{18}$$

so that

$$2 - s = \frac{2\lambda}{\lambda + 2\mu + k}$$

Now the equation (17) reduces to

$$t_{rr} = \left[\left(2 - q^2\right)\sin q - 2q\cos q + \left(2 - s\right)\left\{q\cos q - \sin q\right\}\right]A$$
$$+\left[\left(2 - q^2\right)\cos q + 2q\sin q - \left(2 - s\right)\left\{q\sin q + \cos q\right\}\right]B = 0$$

For free vibrations radial stress on the boundary must be zero.

Therefore, we have

$$t_{rr} = 0 \text{ at } r = a \text{ and } r = b \tag{19}$$

In view of (19), we have

$$\left[\left(s - h^2a^2\right)\tanh a - sha\right]A + \left[sha\tanh a + s - h^2a^2\right]B = 0 \tag{20}$$

$$\left[\left(s - h^2b^2\right)\tanh b - shb\right]A + \left[shb\tanh b + s - h^2b^2\right]B = 0 \tag{21}$$

Eliminating A and B from equations (20) and (21), we get

$$\frac{\left(h^2a^2 - s\right)\tan\left(ha\right) + sha}{\left(h^2a^2 - s\right) - has\tan\left(ha\right)} = \frac{\left(h^2b^2 - s\right)\tanh b + shb}{h^2b^2 - s - hbs\tanh b} \tag{22}$$

which is the frequency equation for radial vibrations corresponding to macro displacement Allowing $k \to 0$ the classical result [3] can be obtained from (22).

Under the absence of body forces and body couples, the equations of motion (2), in the present case reduce to

$$\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} - \frac{2}{r^2}\phi - \frac{2k}{\alpha + \beta + \gamma}\phi = \frac{\rho j}{\alpha + \beta + \gamma}\frac{\partial^2\phi}{\partial t^2} \tag{23}$$

we suppose

$$\phi = B'\cos\left(pt + \in\right)\phi_r \tag{24}$$

where $\phi_r$ is a function of r only, $\in$ is the phase and p is angular frequency.

Substituting (24) in (23) we get

$$\frac{\partial^2\phi_r}{\partial r} + \frac{2}{r}\frac{\partial\phi_r}{\partial r} - \frac{2}{r^2}\phi_r + h_1{}^2\phi_r = 0 \tag{25}$$

where

$$h_1^2 = \frac{p^2\rho j}{\alpha + \beta + \gamma}$$

The solution of (25) is given by

$$\phi_r = \frac{\partial}{\partial q_1}\left[\frac{C\sin q_1 + D\cos q_1}{q_1}\right] \tag{26}$$

where $q_1 = h_1 r$ and C, D are constants. (27)

Thus

$$\phi = \frac{\partial}{\partial q_1}\left[\frac{C\sin q_1 + D\cos q_1}{q_1}\right].\cos(pt+\in) \tag{28}$$

Substituting (28) in (16), we get

$$\begin{aligned}&\left[\left(2-q_1^2\right)\sin q_1 - 2q_1\cos q_1 + \frac{2\alpha}{\alpha+\beta+\gamma}\left(q\cos q_1 - \sin q_1\right)\right]C\\ &+\left[\left(2-q_1^2\right)\cos q_1 + 2q_1\cos q_1 - \frac{2\alpha}{\alpha+\beta+\gamma}\left(q\sin q_1 + \cos q_1\right)\right]D = 0\end{aligned} \tag{29}$$

Suppose

$$s_1 = \frac{2(\beta+\alpha)}{\alpha+\beta+\gamma} \tag{30}$$

so that

$$2 - s_1 = \frac{2\alpha}{\alpha+\beta+\gamma}$$

In view of (30), the equation (29) reduces to

$$\begin{aligned}&\left[\left(2-q_1^2\right)\sin q_1 - 2q_1 m q_1\right] + \left(2-s_1\right)\left(q_1\cos q_1 - \sin q_1\right)C\\ &+\left[\left(2-q_1^2\right)\cos q_1 + 2q_1\sin q_1\right] - \left(2-s_1\right)\left(q_1\sin q_1 + \cos q_1\right)D = 0\end{aligned}$$

For free vibration radial couple stress on the boundary must be zero. Therefore, we have

$$m_{rr} = 0 \text{ at } r = a \text{ and } r = b \tag{31}$$

In view of (31), we have

$$\left[\left(s_1 - h_1^2 a^2\right)\tanh_1 a - s_1 h_1 a\right]C + \left[s_1 h_1 a\tanh_1 a + s_1 - h_1^2 a^2\right]D = 0 \tag{32}$$

$$\left[\left(s_1 - h^2 b^2\right)\tanh_1 b - s_1 h_1 b\right]C + \left[s_1 h_1 b\tanh_1 b + s_1 - h_1^2 b^2\right]D = 0 \tag{33}$$

Eliminating C and D from (32) and (33), we get

$$\frac{\left(h_1^2 a^2 - s_1\right)\tanh_1 a + s_1 h_1 a}{h_1^2 a^2 - s_1 - h_1 a s_1\tanh_1 a} = \frac{\left(h_1^2 b^2 - s_1\right)\tanh_1 b + s_1 h_1 b}{h_1^2 b^2 - s_1 - h_1 b s_1\tanh_1 b} \tag{34}$$

which is the frequency equation of radial vibration. It is interesting to observe that the frequency equation (34) is additional and it is due to the effect of micro rotation. Further it is not encountered in classical case.

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