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DARCY – WEISH BACH FRICTION FACTOR THROUGH POROUS RECTANGULER OPEN – CHANNELS – II A NUMERICAL (GALERKIN) SOLUTION

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ABSTRACT

Darcy-Weish Bach Friction Factor plays a notable role in the estimation of flow drop in open channels. Most of the studies on open channel flows are experimental due to geometric complexity of channels and their networks.The present paper deals with an estimation of Darcy-Weish Bach Friction factor through an open porous straight channel of rectangular cross-section. An approximate solution is obtained to the basic flow equation employing Galerkin Technique.The fluid is assumed to be incompressible homogenous and viscous.The momentum equation adopted takes care of the convective acceleration and viscous shears generated in the flow region in addition to classical Darcy friction force.An approximate solution is obtained employing Galerkin Technique.The mean velocity and Darcy-Weish Bach Friction factor have been computed and their variations are illustrated.

Introduction:

Fluid flows through pipes and channels have been a subject of intensive study by both experimental and analytical scientists. Hydraulic Engineers and Mathematicians for the last several centuries because of their wide applications. In a pipe flow, the pipe is completely filled with the fluids. As such it is subjected to hydraulic pressure only and is not influenced by the atmospheric pressure.

In contrast to this, the channel flow, quite often referred to as open channel flow has a free surface which is in contact with the atmosphere and so the atmospheric pressure influences the flow in the channel. Open channel flows are widely employed in the design of irrigation canal net work. Most of the investigations on open channel flows are experimental studies only. This may be due to geometric complexity of channels and their network. Treatises written by Bakhmeteff[1], Posey[2], Ven Te Chow[5] on the subject of Open Channels flows present many experimental studies on friction losses, the wall shears, seepage and evaporation in flows through channels of different non-circular cross-sections. The drop in the flow rate due to friction in channels is invariably expressed in terms of a friction factor which is referred in the literature as Darcy-Weish Bach friction factor (f).

An Empirical relation has been given to find the variation of the friction factor with equivalent hydraulic radius of the channels. The treatises cited above amply illustrate such a relation. Recently Prabhakar Reddy[2] studied analytically for the "Darcy's Weish Bach' friction factor for a wide spectrum of channel – cross sections Rectangular, Trapezoidal, Isosceles triangular, general triangle, semicircle, circular segments and parabola.

The present investigation is aimed at the estimation of Darcy's Weish Bach Friction Factor through open porous channels. An approximate solution is obtained to the basic flow equation, employing Galerkins technique.The fluid is assumed to be incompressible homogenous and the channel is of rectangular cross section and medium is porous. The momentum equation adopted is the one given by YAMAMOTO.K and YOSHIDA.K[7] and YAMAMOTO.K and IUMAMURA[8] which takes in to consideration of convective acceleration and the viscous stresses generated in the flow region. The channel walls are taken to be impermeable by allowing no cross seepage in to the channel and the flow is due to a constant pressure gradient down the channel length.

The analytical solution for this paper was given earlier by the present authors Pattabhi Ramacharyulu NCh , Venkateshwarlu. A [2].

MATHEMATICAL FORMULATION

Consider a uniform free surface flow of a homogenous incompressible fluid of constant density $\rho = \gamma g$ and constant viscosity μ through a straight open channel of depth. Consider a rectangular Cartesian frame O (XYZ) with the origin on the free surface, the axis of Z parallel to the flow direction, Y – axis vertically upwards (consequently the X and Y axis in the plane of the channel cross section – perpendicular to its length (i.e. the Z – direction).

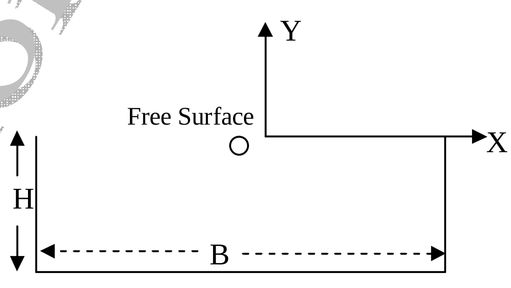

Fig 1: Channel Cross section

With reference to this frame of reference, the velocity can be taken as $\vec{V}$ = (o, o, W(x, y)) which evidently satisfies the continuity equation

$$\text{div}\ \overline{V} = 0 \qquad - \qquad (1)$$

where V is the fluid velocity

The momentum equation in the Z – direction reduces to

$$\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} - \frac{W}{K} = \frac{-C}{\mu} \quad - \quad (2)$$

where μ is the coefficient of viscosity of the fluid and K the porosity coefficient of the medium. Further C stands for the pressure gradient $C = -\frac{\partial P}{\partial Z}$, where P stands for the pizometric head in the channel.

Also the velocity W(x, y) satisfies the following boundary condition:

$w/_{Y=-H} = 0$ - (3.1)

$w/_{Y=\pm B/2} = 0$ - (3.2)

on the free surface which is stress free: $\frac{\partial W}{\partial Y}/_{Y=0} = 0$ -(3.3)

DARCY – WEISHBACH FRICTION FACTOR

The average velocity $\overline{W}$ in the channel defined by the equation

$$\overline{W} = \iint_A W dA \quad - \quad (7)$$

The integration being carried over the channel cross – section area A.

The Darcy – weishbach friction factor in the channel (p – 8 of [3]) is given by the definition equation $f = \frac{8Rgs}{(\overline{W})^2} = \frac{K_1}{R^*}$ - (8)

where R represents the Hydraulic radius.

$$R = \frac{\text{Channel Cross-section area}}{\text{Channel wetted perimeter}} = \frac{BH}{B+2H}, \quad s = \frac{\mu c}{\rho g}, R^* = \frac{\gamma s^2}{Rg\mu^2} \quad - \quad (9)$$

Further K_1, is the parameter characterstic of the Darcy – Weishbach Friction Factor associated with the channel cross – section .

We introduce non-dimensional quantities as per the following scheme:

$$\left.\begin{array}{lll} X = Bx & Y = By, & H = Bh \\ W = \frac{B^2 C}{\mu} w, & & K = \frac{B^2}{\sigma^2} \end{array}\right\} \quad - \quad (10)$$

Flow rate $Q = \iint W dX dY = \frac{B^2 C}{\mu} . B^2 \iint w dx dy$

$Q = \frac{B^4 C}{\mu} \iint w dx dy$, where $q = \iint w dx dy$

$$\overline{W} = \frac{Q}{BH} = \frac{Q}{B^2 h} = \frac{1}{B^2 h} \times \frac{B^4 C}{\mu} \iint w dx dy = \frac{B^2 C}{\mu h} \times \iint w dx dy = \frac{B^2 C}{\mu h} . q$$

$$\overline{W} = \frac{B^2 C}{\mu} . \overline{w} \qquad - \qquad (11)$$

Where $\overline{w} = \frac{1}{h} \iint w\,dx\,dy = \frac{q}{h}$

The basic equation and the boundary conditions in terms of the non – dimensional quantities introduced above is rewritten by

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - \sigma^2 w = -1 \text{ with the rectangle} \qquad - \qquad (12)$$

On the bottom $w/_{y=-h} = 0$ - (13)

On the sides $w/_{x=\pm\frac{1}{2}} = 0$ - (14)

and on the free surface

$$\frac{\partial w}{\partial y} /_{y=0} = 0 \qquad - \qquad (15)$$

Approximation Solutions Employing Galerkin's Technique:

I. FIRST GALERKIN APPROXIMATION

Consider the function $\phi(x, y) = (x^2 - 1/4)\, y^2 (y + h)$ - (16)
This satisfies all the boundary conditions (13, 14, 15)
Let $w^{(1)}(x, y) = A_0^{(1)} (x^2 - 1/4) y^2 (y + h)$ - (17)
Whether $A_0^{(1)}$ is suitable constant
This evidently satisfy all the boundary conditions (13, 14, 15)
The error in the momentum equation by assuming (16) as solution is

$$\in^{(1)} = \frac{\partial^2 w^{(1)}}{\partial x^2} + \frac{\partial^2 w^{(1)}}{\partial y^2} - \sigma^2 w^{(1)} + 1$$

$$= A_0^{(1)} [2(y^3 + hy^2) + (x^2 - 1/4)(6y + 2h) - \sigma^2 (y^3 + y^2 h)(x^2 - 1/4)] + 1 \quad - \quad (18)$$

The constant $A_0^{(1)}$ is so chosen that $< \in^{(1)} . \phi^{(1)} > \; = \; 0$

i.e. $$\int_{y=0}^{y=-h} \int_{x=-\frac{1}{2}}^{x=\frac{1}{2}} \in^{(1)} . \phi^{(1)} dx\,dy = 0 \qquad - \qquad (19)$$

This yields $$A_0^{(1)} = \frac{-175}{4h\,(14 + 10\,h^2 + \sigma^2 h^2)}$$

Using this, we get the flow rate $q^{(1)} = \int\int w^{(1)} dx\,dy$ Integrated over the channel Cross section

$$q^{(1)} = \left(\frac{h^4}{72}\right) A_0^{(1)}$$

Mean velocity $\overline{W} = \left(\frac{B^2 C}{\mu}\right)\frac{q}{h}$

$$\overline{W} = \left(\frac{B^2 C}{\mu}\right) \times \frac{1}{h}\left(\frac{h^4}{72}\right) A_0^{(1)}$$

Friction factor $f = \frac{8Rgs}{(\overline{W})^2}$ where $R = \frac{Bh}{2h+1}$ $s = \frac{\mu c}{\rho g}$

$$= \left(\frac{8^5 \times 9^2 \times \mu^3}{\rho C B^3}\right) \times \frac{1}{25(2h+1)h^3\left[\frac{1}{5} + \frac{h^2}{7} + \frac{h^2\sigma^2}{35}\right]^2}$$

$$F^* = \frac{f}{\left(\frac{8\mu^3}{\rho C B^3}\right)} = \left(\frac{8^4 \times 9^2}{25}\right) \times \frac{1}{(2h+1)h^3}\left[\frac{1}{5} + \frac{h^2}{7} + \frac{h^2\sigma^2}{35}\right]^{-2}$$

F* is computed for wide spectra of h & σ ;
h = 0.1, 0.2, …..1, 5, ….
σ = 0.1, 0.2, …..1, 5, ….
and the results are illustrated in fig 2 & 3

Gelerkin's 2nd approximation

Let $w^{(2)}(x, y) = \{A_0^{(2)} + A_1^{(2)} x + B_1^{(2)} y\}(x^2 - 1/4) y^2 (y + h)$ - (1)

where $A_0^{(2)}$, $A_1^{(2)}$, $B_1^{(2)}$ are constant to be decided upon its solution.
This evidently satisfy all the boundary conditions
The error in the momentum equation by assuming (1) as solution is

$$\in^{(2)} = \frac{\partial^2 w^{(2)}}{\partial x^2} + \frac{\partial^2 w^{(2)}}{\partial y^2} - \sigma^2 w^{(2)} + 1$$

$$\in^{(2)} = A_0^{(2)}[(2y^3 + 2y^2) + (6y + 2h)(x^2 - 1/4) - \sigma^2 (y^3 + y^2 h)(x^2 - 1/4)$$
$$+ A_1^{(2)} x[(6y^3 + 6y^2 h) + (6y + 2h)(x^2 - 1/4) - \sigma^2 (y^3 + y^2 h)(x^2 - 1/4)]$$
$$+ B_1^{(2)} y[(2y^3 + 2y^2 h) + (12y + 6h)(x^2 - 1/4) - \sigma^2 (x^2 - 1/4)(y^3 + y^2 h)] + 1$$

- (2)

The Constants $A_0^{(2)}, A_1^{(2)}, B_1^{(2)}$ are to be chosen so that

$$< \in^{(2)} . \phi^{(2)} > = 0 \quad - \quad (3)$$

$$< \in^{(2)} . x\phi^{(2)} > = 0 \quad - \quad (4)$$

$$< \in^{(2)} . y\phi^{(2)} > = 0 \quad - \quad (5)$$

From (3), (4), (5) we get

$$A_0^{(2)} = \frac{1}{40h}\left[\frac{5\left(\frac{1}{175}+\frac{h^2}{126\times 3}+\frac{h^2\sigma^2}{252\times 15}\right)-\frac{1}{2}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)}{\frac{1}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)\left(\frac{1}{175}+\frac{h^2}{126\times 3}+\frac{h^2\sigma^2}{252\times 15}\right)-\frac{1}{8}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)^2}\right]$$

$$A_1^{(2)} = 0$$

$$B_1^{(2)} = \frac{1}{120h}\left[\frac{\frac{5}{3}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)-\frac{2}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)}{\frac{1}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)\left(\frac{1}{125}+\frac{h^2}{126\times 3}+\frac{h^2\sigma^2}{252\times 15}\right)-\frac{1}{8}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)^2}\right]$$

Using these in [1], we get the flow rate

$$q^{(2)} = \iint w^{(2)}dxdy = \frac{h^4}{72}A_0^{(2)} - \frac{h^5}{120}B_1^{(2)}$$

$$\frac{h^4}{360}\left[5A_0^{(2)} - 3B_1^{(2)}h\right]$$

$$Q^{(2)} = \frac{B^4C}{\mu}\iint wdxdy = \frac{B^4C}{\mu}q^{(2)}$$

Mean velocity

$$\overline{W} = \frac{Q^{(2)}}{BH} = \frac{B^2C}{\mu h}q^{(2)}$$

$$\frac{B^2Ch^3}{360\mu}\left[5A_0^{(2)} - 3B_1^{(2)}h\right]$$

and friction factor $f = \frac{8Rgs}{\left(\overline{W}\right)^2}$, $R = \frac{Bh}{2h+1}$, $s = \frac{\mu c}{\rho g}$

$$f = \left(\frac{8^5\times 9^2\times \mu^3}{\rho C B^3}\right)\frac{5^2\times(75)^2}{(2h+1)h^3}\left[\frac{\frac{1}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)\left(\frac{1}{175}+\frac{h^2}{378}+\frac{h^2\sigma^2}{1260}\right)-\frac{1}{8}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)^2}{-\frac{17}{14}+\frac{13h^2}{36}+\frac{47h^2\sigma^2}{315}}\right]$$

$$F^* = \left(\frac{f}{\frac{8\mu^3}{\rho C B^3}}\right)$$

$$= \left(8^4\times 9^2\times 5^2\times(75)^2\right)\frac{1}{(2h+1)h^3}\left[\frac{\frac{1}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)\left(\frac{1}{175}+\frac{h^2}{378}+\frac{h^2\sigma^2}{1260}\right)-\frac{1}{8}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)^2}{-\frac{17}{14}+\frac{13h^2}{36}+\frac{47h^2\sigma^2}{315}}\right]^2$$

F* is computed for wide spectra of h & σ ;

Numerical computation have been carried out for the variation of the friction factor F* vrs a wide spectrum of the depth h and porosity parameter σ

h = 0.1, 0.2, …..1, 5, ….

σ = 0.1, 0.2, …..1, 5, ….

The results are illustrated in fig 4 & 5

<u>CONCLUSION :</u>

From the fig. 2, 3 and Fig 4,5 it is observed that the friction factor increases with porosity coefficient σ increases. The fig.2 & 4 shows that for h<1 not much variation of friction factor for different σ, the friction factor linearly falls. As channel depth h is increased. Beyond that it is asymptotic to the h axis raising as the porosity parameter increases. From fig.3 & 5 the channel depth has no significant influence as friction factor upto the porosity coefficient σ = 20.5 approximately. Beyond that the friction factor increases as the porosity σ increases and increases with the channel depth.

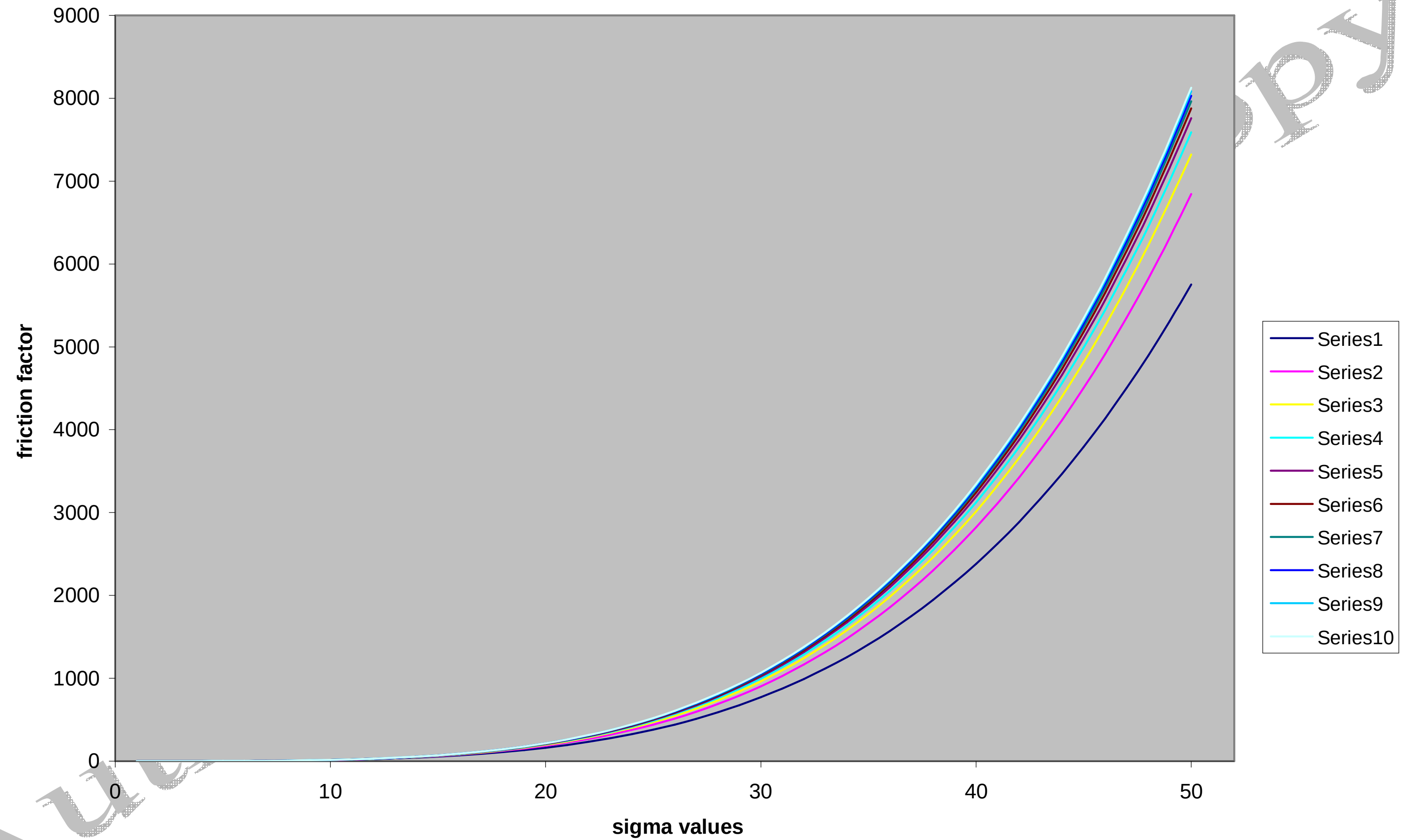

fig2.friction factor vs σ [h=1 to 10]
Rectangular channel - I Approx.

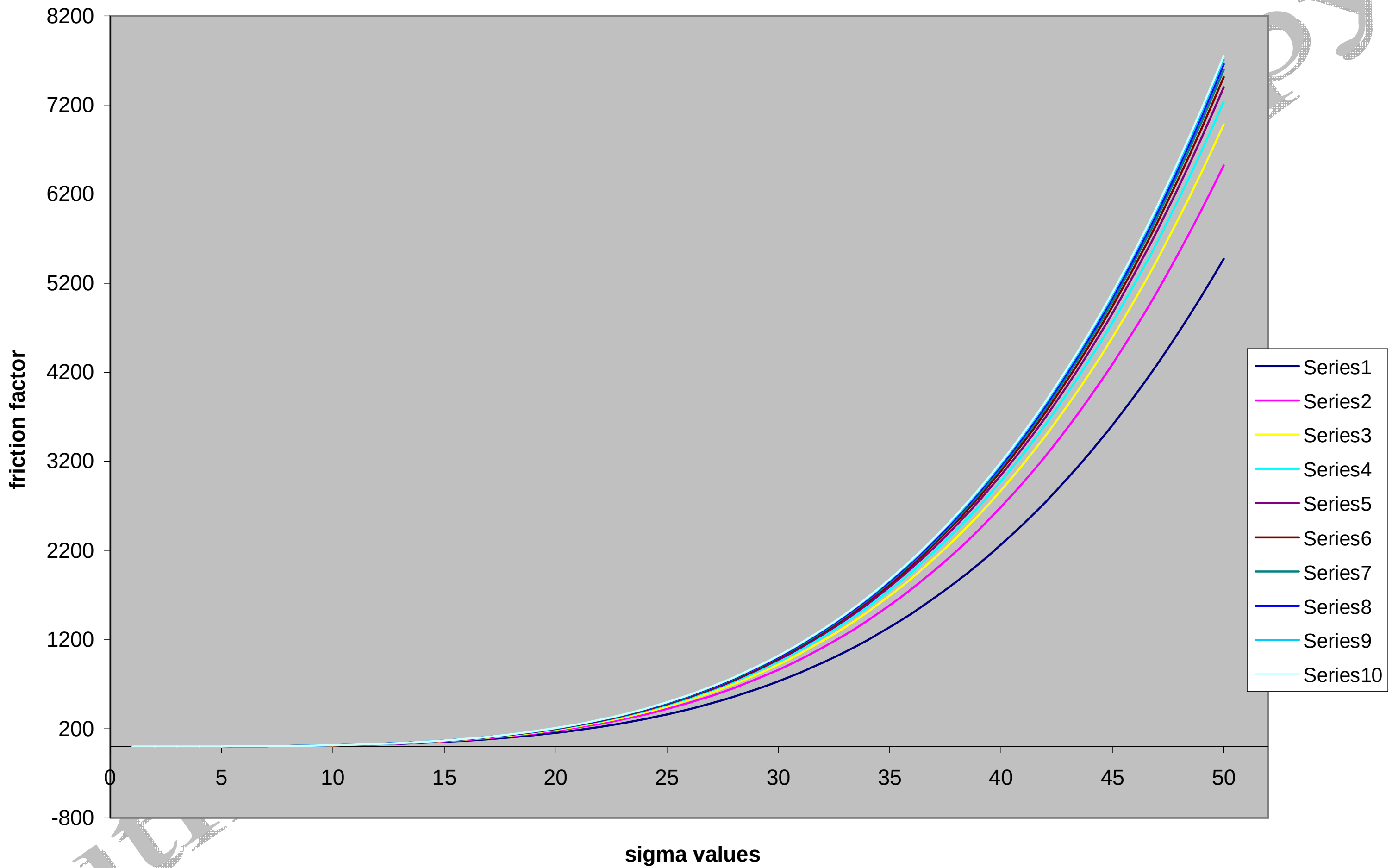

fig3.friction factor vs σ [h=1 to 10]
Rectangular channel - II Approx.

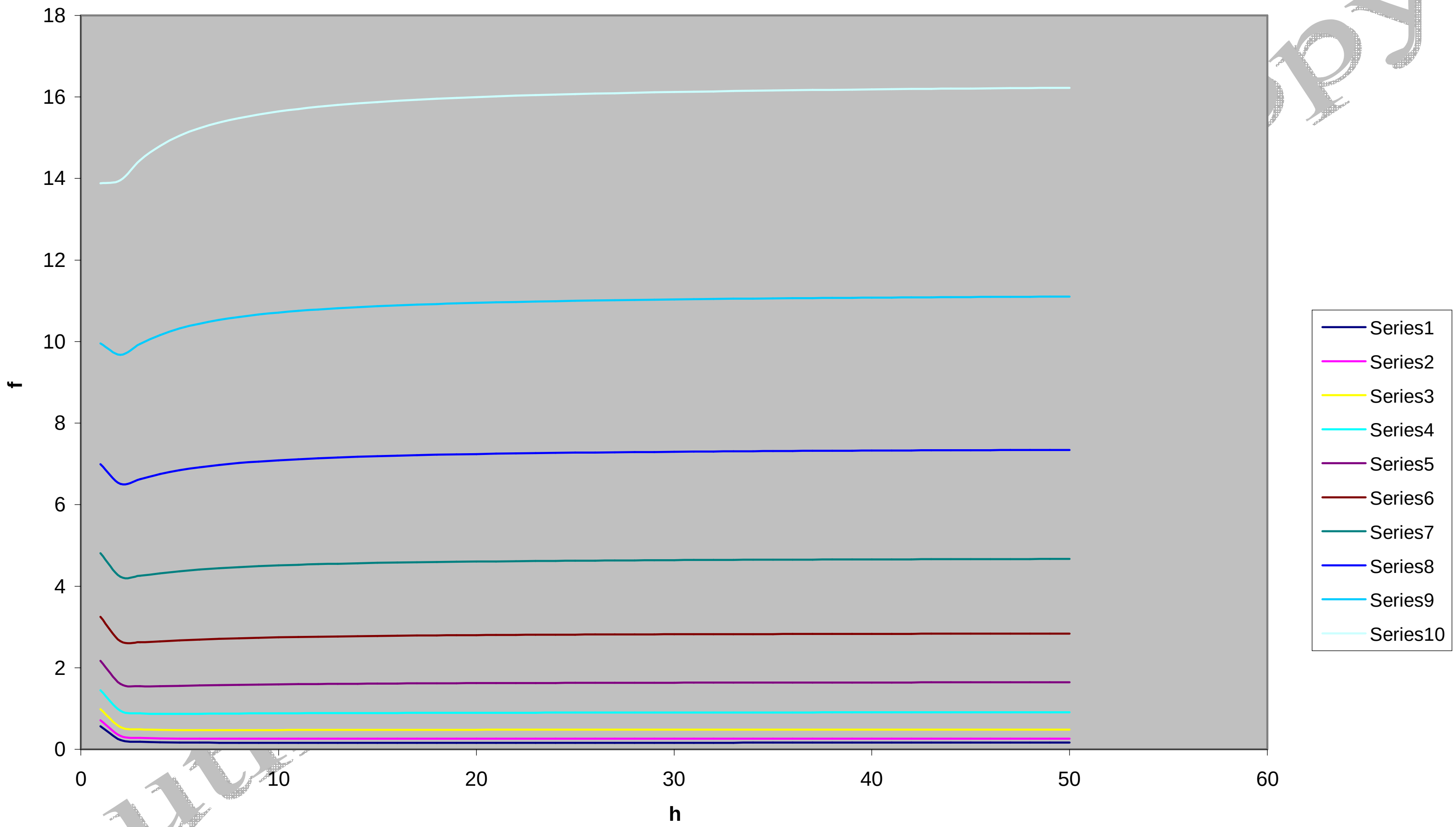

fig4. friction factor vs height [σ=1 to 10]
Rectangular channel - I Approx.

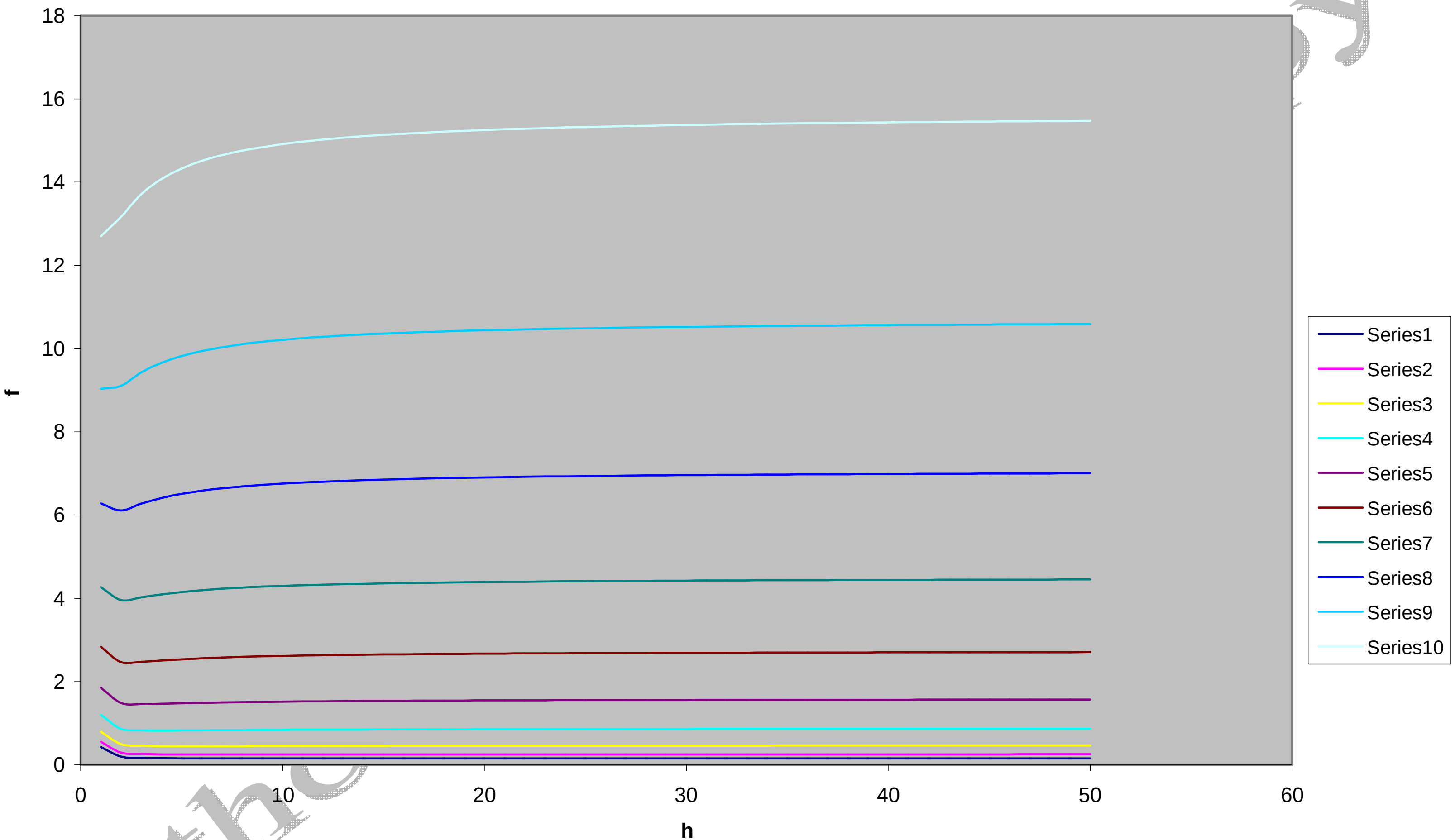

fig5.friction factor vs height [σ = 1 to 10]
Rectangular channel - II Approx.

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