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NUMERICAL SOLUTIONS OF MAGNETOHYDRODYNAMIC UNSTEADY FREE CONVECTION FLOW PAST AN INFINITE VERTICAL PLATE WITH CONSTANT SUCTION AND HEAT SINK

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ABSTRACT

The unsteady hydromagnetic free convective flow of viscous, incompressible and electrically conducting fluid past an infinite vertical porous plate with constant suction and heat absorbing sink in the presence of uniform transverse magnetic field has been studied numerically by finite difference method. The effect of Grashof number, Prandtl number, sink strength, Hartmann number and Eckert number on velocity field, temperature distribution, skin-friction and rate of heat transfer have been studied and shown graphically.

Key words: Unsteady; Thermal energy; Porous plate; Constant suction; Sink;

1. INTRODUCTION

The study of unsteady free convection flows past a porous plate has applications in the fields of engineering, space science, biophysics, astrophysics and so on. At present, it is usual to use an engineering process in which, the fluid supports an exothermic chemical or nuclear reaction. For economic use, an appropriate process design requires accurate correlation for the heat transfer coefficient at the boundary surface. Therefore, due to increasing importance in technological and physical problems, free convection flows of viscous fluids have received attention of the workers.

Sparrow and Cess [10] have studied free convection flow, past an infinite vertical plate in the presence of magnetic field. Soundalgekar [9] studied free convection effects on MHD flow past a vertical porous plate. Malashetty and Lela [2] have investigated a similar case for two-phase flow under some changed boundary conditions. Shende and Soundalgekar [7] have studied unsteady forced and free convection MHD flow of an incompressible, viscous conducting fluid past an infinite vertical porous plate with variable suction and oscillatory wall temperature. Sacheti *et al* [3] have obtained an exact solution for the unsteady MHD problem. Sattar and Alam[5] have studied MHD free convective flow with Hall current in a porous medium for electrolytic solution (viz. salt water). Shah and Verma [6] have discussed MHD free convection flow by finite difference method. Sreekanth *et al* [11] have studied unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate under the influence of uniform transverse magnetic field. But they have neither considered the effect of constant suction nor included the heat absorbing sink and viscous dissipation. The propagation of thermal energy through mercury and electrolytic solution in the presence of external magnetic field and heat absorbing sinks has wide range of applications in chemical and aeronautical engineering, atomic propulsion, space science etc. Our objective in the present paper is to study the heat transfer in mercury (P_r = 0.025), air (P_r = 0.71), electrolytic solution (P_r = 1.0) and water (P_r = 7.0) past an infinite porous plate with constant suction in the presence of uniform transverse magnetic field and heat sink by numerically using finite difference method, which is more economical from computational view point.

2. MATHEMATICAL FORMULATION

The x'-axis is taken in the vertically upward direction along the infinite vertical plate and y'-axis is taken normal to it. Neglecting the induced magnetic field and applying Boussinesq's approximation, the governing equations of the flow in dimensional form are[4]:

Continuity equation

$$\frac{\partial V'}{\partial y'} = 0 \qquad \ldots (2.1)$$

i.e. $V' = -V_0$ … (2.2)

Momentum equation

$$\frac{\partial u'}{\partial t'} - V'\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \qquad \ldots(2.3)$$

Energy equation

$$\frac{\partial T'}{\partial t'} - V'\frac{\partial T'}{\partial y'} = K\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T'_\infty) + \frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \qquad \ldots (2.4)$$

where u' is the velocity of the fluid, g is the acceleration due to gravity, K is the thermal conductivity of the fluid, ν is the kinematic coefficient of viscosity, β is the volumetric coefficient of thermal expansion, T'w is the temperature of the plate, T'∞ is the temperature of the fluid far away from the plate, B_0 is the magnetic permeability, σ is the electrical conductivity of the fluid, Cp is the specific heat at constant pressure and other symbols have their usual meaning.

The boundary conditions are:

$$u' = 0,\ V' = -V_0',\ T' = T_w' + \varepsilon(T_w' - T_\infty')e^{iw't'} \quad \text{at } y' = 0$$
$$u' \to 0, T' \to T_\infty' \text{ as } y' \to \infty \qquad \ldots (2.5)$$

Introducing the following non-dimensional quantities,

$$y = \frac{y'V_0'}{\nu},\ t = \frac{t'V_0'^2}{4\nu},\ w = \frac{4\nu w'}{V_0'^2},\ u = \frac{u'}{V_0'},\ P_r = \frac{\nu}{K},$$
$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ G_r = \frac{\nu g\beta (T'_w - T'_\infty)}{V_0'^3},\ S = \frac{4S'\nu}{V_0'^2}, \quad \ldots (2.6)$$
$$E_c = \frac{V_0'^2}{C_p (T'_w - T'_\infty)},\ M = \frac{\sigma B_0^2 \nu}{\rho V_0'^2},\ K = \frac{K_0}{\rho C_p}$$

where P_r, G_r, S, E_c and M are the Prandtl number, Grashof number, Sink strength, Eckert number and Hartmann number respectively. With the help of non-dimensional quantities esq. (2.3) and (2.4) becomes

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G_r T + \frac{\partial^2 u}{\partial y^2} - Mu \quad \ldots (2.7)$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + \frac{ST}{4} + E_c\left(\frac{\partial u}{\partial y}\right)^2 \quad \ldots (2.8)$$

The modified boundary conditions are

$$\left.\begin{array}{l} u = 0,\ T = 1 + \varepsilon e^{i\omega t} \ \text{ at } \ y = 0 \\ u \to 0,\ T \to 0 \ \text{ as } \ y \to \infty \end{array}\right\} \quad \ldots (2.9)$$

3. SOLUTION OF THE PROBLEM

Applying Crank-Nicolson formula for (2.7) and (2.8), we get[8]

$$C.u_{i-1}^{j+1} + B.u_i^{j+1} + C.u_{i+1}^{j+1} = D_i^j \quad \ldots (3.1)$$
$$C_1.T_{i-1}^{j+1} + B_1.T_i^{j+1} + C_1.T_{i+1}^{j+1} = D_{1i}^j \quad \ldots (3.2)$$

where

$$C = -r/2,\ B = r + 1/4,\ C_1 = -r/2,\ B_1 = r + P_r/4$$
$$D_i^j = (r/2)u_{i-1}^j + (1/4 - rh - r - kM)u_i^j + (rh + r/2)u_{i+1}^j + kG_r T_i^j$$
$$D_{1i}^j = (r/2)T_{i-1}^j + (P_r/4 - P_r h - r + kS/4)T_i^j + (P_r h + r/2)T_{i+1}^j + rE_c\left(u_{i+1}^j - u_i^j\right)^2$$

where h, k are mesh sizes along space direction and time direction respectively and $r = k/h^2$. In esq. (3.1) and (3.2), taking i = 1 (1) n and using the boundary conditions (2.9), we get the following tri diagonal system of equations.

EU = A … (3.3)
FT = D … (3.4)

Where E and F are tri diagonal matrices of order n whose elements are defined by

$$E_{i,i} = B,\ F_{i,i} = B_1,\ \ i = 1(1)n$$
$$E_{i-1,i} = C,\ F_{i-1,i} = C_1,\ \ i = 2(1)n$$
$$E_{i,i-1} = C,\ F_{i,i-1} = C_1,\ \ i = 2(1)n$$

and U, A, T, D are column matrices having n-components, they are $u_i^{j+1},\ D_i^j,\ T_i^{j+1},\ D_{1i}^j \quad i=1(1)\,n$ respectively.

The following procedure[1] is adopted for the last boundary condition (2.9) to determine the number of space steps required at t = 1, so that u (N,1) = 0.

We write esq.(3.1) as

$$-B(I).u_{I-1}^1 + A(I).u_I^1 - C(I).u_{I+1}^1 = D_I^0 \qquad \dots (3.5)$$

when $B(I) = r/2 > 0,\ A(I) = r + 1/4 > 0,\ C(I) = r/2 > 0,\ I = 0,1,2,\cdots N.$

and also $A(I) > B(I) + C(I)$

suppose

$$\begin{aligned} u_I^1 &= w(I).u_{I+1}^1 + g(I), \quad 0 \le I \le N \\ u_{I-1}^1 &= w(I-1).u_I^1 + g(I-1), \quad 0 \le I \le N \end{aligned} \qquad \dots (3.6)$$

Eliminating u_{I-1}^1 from above equations, we get

$$u_I^1 = \frac{C(I).u_{I+1}^1}{A(I) - B(I).w(I-1)} + \frac{D_I^0 + B(I).g(I-1)}{A(I) - B(I).w(I-1)}$$

Thus $$w(I) = \frac{C(I)}{A(I) - B(I).w(I-1)},\ 1 \le I \le N \qquad \dots (3.7)$$

$$g(I) = \frac{D_I^0 + B(I).g(I-1)}{A(I) - B(I).w(I-1)},\ 1 \le I \le N \qquad \dots (3.8)$$

where $u_0^1 = 0,\ w(0) = 0,\ g(0) = 0$

Esq.(3.7) gives $u_2^1, u_3^1, u_4^1, \cdots$ *as functions of* u_1^1

Suppose $u_{N+1}^1 = 0$ … (3.9)

where N has to be determined. Thus approximate values of $u_1^1, u_2^1, u_3^1, \cdots, u_N^1$ are determined using (3.9). Increasing N by N+1 we get the value of $\bar{u}_1^1, \bar{u}_2^1, \bar{u}_3^1, \cdots, \bar{u}_{N+1}^1$ *and* $\bar{u}_{N+2}^1 = 0.$ Thus N is determined, so that $\left|\bar{u}_I^1 - u_I^1\right| < \varepsilon,\ 1 \le I \le N$ for given ε using (3.5).

$$\begin{aligned} \bar{u}_I^1 - u_I^1 &= w(I)(\bar{u}_{I+1}^1 - u_{I+1}^1),\ 1 \le I \le N \\ &= w(I).w(I+1).\cdots.w(N-1).w(N)(\bar{u}_{N+1}^1 - u_{N+1}^1),\ 1 \le I \le N \\ &= w(I).w(I+1).\cdots.w(N).\bar{u}_{N+1}^1,\ 1 \le I \le N+1 \\ &= w(I).w(I+1).\cdots.w(N).g(N+1),\ 1 \le I \le N+1 \end{aligned} \qquad \dots (3.9)$$

using (3.6), $\bar{u}_{N+1}^1 = g(N+1)$ … (3.10)

To determine N to a desired accuracy first test (3.10) $\bar{u}^1_{N+1}$ for the given accuracy $u^1_{N+1} \cong 0$, when this is met, test (3.9) for the required accuracy. It is found that N = 19 gives the accuracy of 10^{-4} for u at t = 1. The same procedure can be adopted for any other time step t. At t = 3, we get stable values for velocity field and temperature distribution.

SKIN-FRICTION AND RATE OF HEAT TRANSFER

The skin-friction at the plate is given by $\tau_\omega = \left(\frac{\partial u}{\partial y}\right)_{y=0}$

The rate of heat transfer at the plate is given by $q_\omega = \left(\frac{\partial T}{\partial y}\right)_{y=o}$

5. DISCUSSIONS AND CONCLUSION

In order to get physical insight into the problem, the velocity field, temperature distribution, skin- friction and rate of heat transfer are discussed taking different numerical values of the parameters encountered into the problem under consideration. For this purpose the values of Prandtl number (P_r) have been chosen for mercury (P_r = 0.025), air (P_r = 0.71), electrolytic solution (P_r = 1.0) and water (P_r = 7.0). The numerical values of the remaining parameters are chosen arbitrarily and for all these calculations we considered real parts only.

In figures 1 to 6, the velocity field **u** has been plotted versus y for different values of magnetic parameter (Hartmann number) (M), Prandtl number (P_r), Sink-strength (S), Grashof number (G_r) and Eckert number (E_c) respectively. From these figures we observed commonly that the velocity increases rapidly near the plate and after attaining a maximum value, it decreases slowly and steadily as y increases. In addition, we observed that an increase in magnetic parameter and Prandtl number decreases the velocity field while an increase in Sink-strength, Grashof number and Eckert number increases the velocity field.

In figures 7 to 10, the temperature distribution **T** has been plotted versus y for various values of Prandtl number, Sink-strength and Eckert number respectively. From these figures we observed that an increase in Prandtl number decreases temperature distribution while an increase in Sink-strength and Eckert number increases temperature distribution. It is interesting to note that temperature distribution decreases more rapidly for water (P_r = 7.0) when compared to air (P_r = 0.71) and the temperature field curve is almost linear for mercury, which is more sensible towards change in temperature. From this observation we conclude that mercury is most effective for maintaining temperature differences and can be efficiently in laboratory purposes. Air can replace mercury, the effectiveness of maintaining temperature changes is much less than mercury. If temperatures are maintained, air can be better and cheap replacement for industrial purposes.

Table-I represents the skin-friction for various values of M, P_r, S, G_r and E_c respectively. We observed that an increase in M, P_r and S leads to a decrease in skin-friction while an increase in G_r and E_c leads to an increase in skin-friction.

Table-II represents the rate of heat transfer for various values of M, P_r, S, G_r and E_c respectively. We observed that an increase in M, P_r and S leads to a decrease in rate of heat transfer while an increase in G_r and E_c leads to an increase in rate of heat transfer.

TABLE – I

Effect of M, P_r, S, G_r and E_c on skin-friction (h = 0.225, k = 0.001, $\varepsilon = 0.005$ and ωt = π/2)

M	Pr	S	Gr	Ec	τ
10.0	0.71	-0.10	10.0	0.10	1.706868
20.0	0.71	-0.10	10.0	0.10	1.089242
10.0	1.00	-0.10	10.0	0.10	1.554499
10.0	0.71	-0.20	10.0	0.10	1.693790
10.0	0.71	-0.10	20.0	0.10	3.453219
10.0	0.71	-0.10	10.0	0.20	1.713329

TABLE – II

Effect of M, P_r, S, G_r and E_c on rate of heat transfer (h = 0.1,k = 0.001, $\varepsilon = 0.005$ and ωt =π/2)

M	Pr	S	Gr	Ec	q
10.0	0.71	-0.10	10.0	0.10	-0.879712
20.0	0.71	-0.10	10.0	0.10	-0.891527
10.0	1.00	-0.10	10.0	0.10	-1.059118
10.0	0.71	-0.20	10.0	0.10	-0.889397
10.0	0.71	-0.10	20.0	0.10	-0.823096
10.0	0.71	-0.10	10.0	0.20	-0.861255

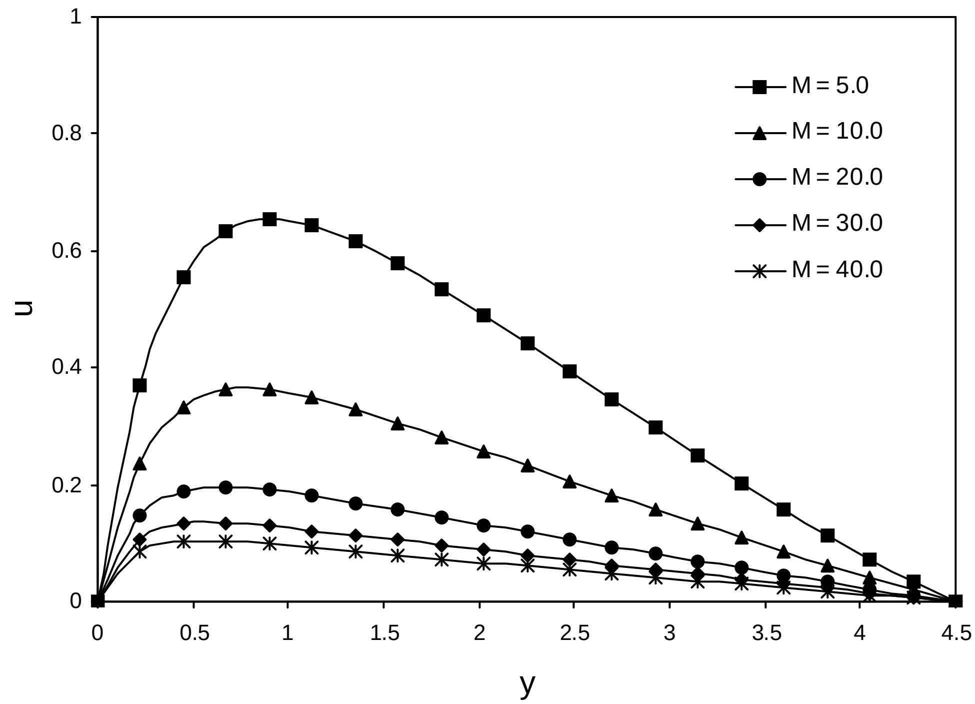

Fig.1. Effect of magnetic parameter M on velocity u
(G_r = 5.0, P_r = 0.025, S = -0.05, E_c =0.001, $\varepsilon = 0.005$, t = 3 and ωt = π/2)

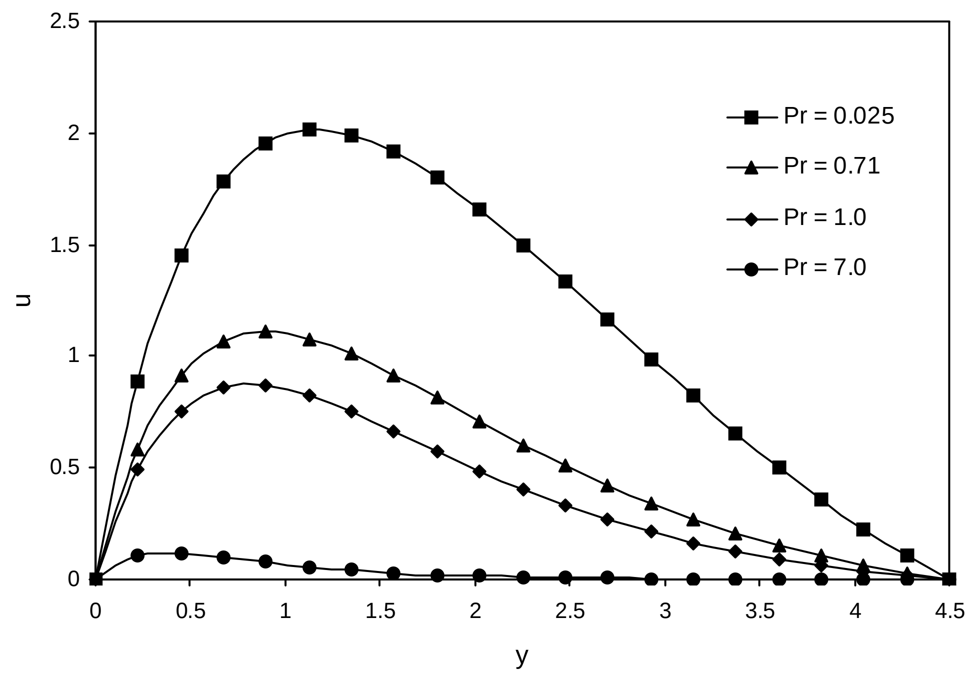

Fig.2. Effect of Prandtl number P_r on velocity u
(G_r = 5.0, M = 1.0, S = -0.05, Ec =0.001, $\varepsilon = 0.005$, t = 3 and ωt = π/2)

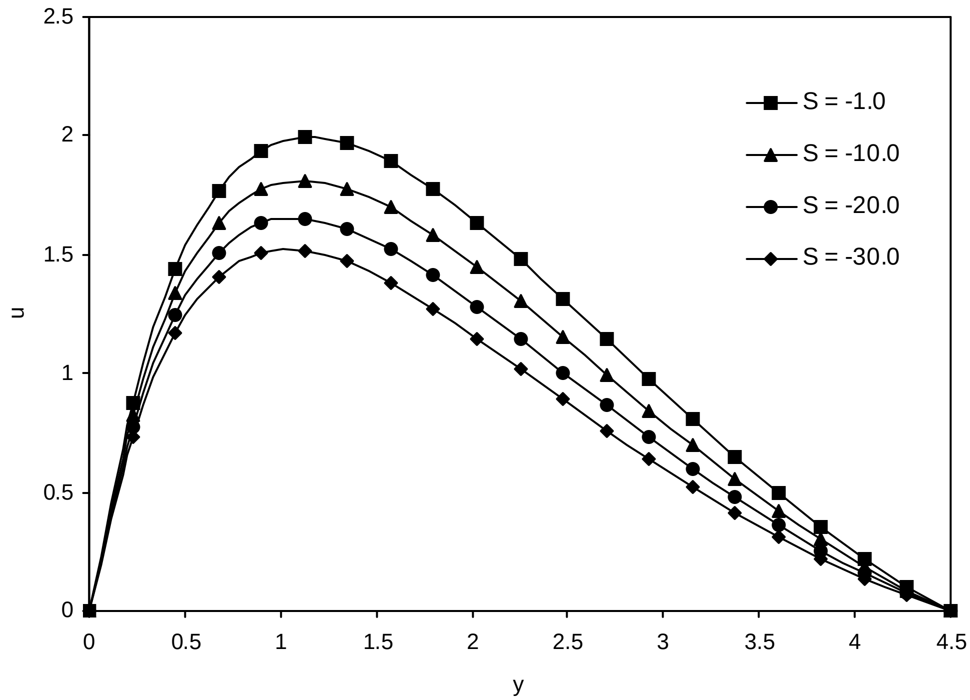

Fig.3. Effect of Sink-strength S on velocity u
(G_r = 5.0, P_r = 0.025, M = 1.0, E_c =0.001, $\varepsilon = 0.005$, t = 3 and ωt = π/2)

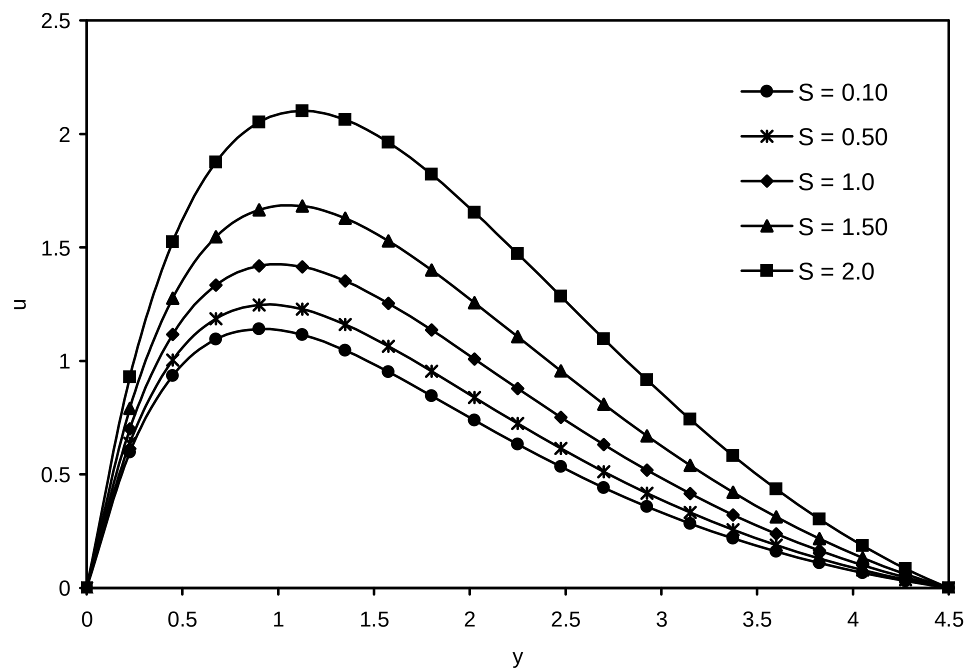

Fig.4. Effect of Sink-strength S on velocity u
(G_r = 5.0, P_r = 0.71, M = 1.0, E_c =0.001, $\varepsilon = 0.005$, t = 3 and ωt = π/2)

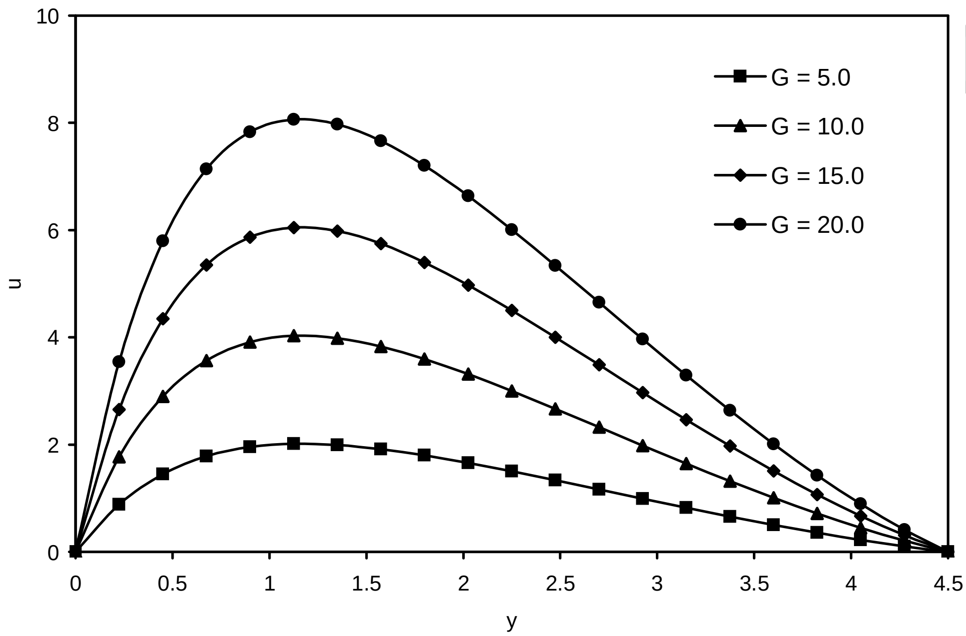

Fig.5. Effect of Grashof number G_r on velocity u
(M = 1.0, P_r = 0.025, S = -0.05, E_c =0.001, $\varepsilon = 0.005$, t = 3 and $\omega t = \pi/2$)

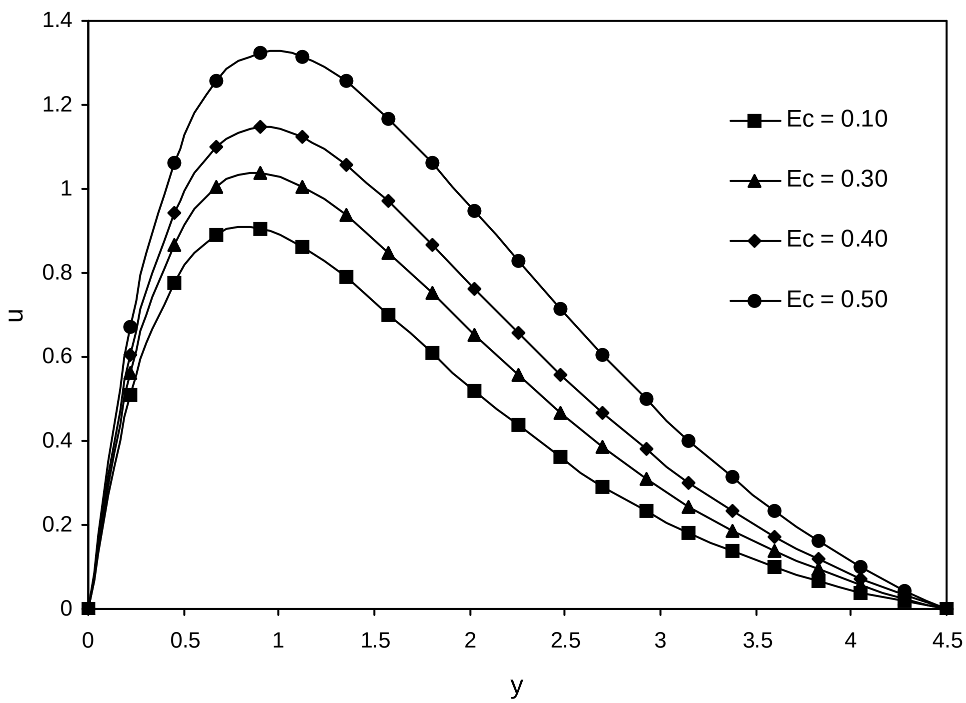

Fig.6. Effect of Eckert number E_c on velocity u
(G_r = 5.0, M = 1.0, P_r = 1.0, S = -0.10, $\varepsilon = 0.005$, t = 3 and $\omega t = \pi/2$)

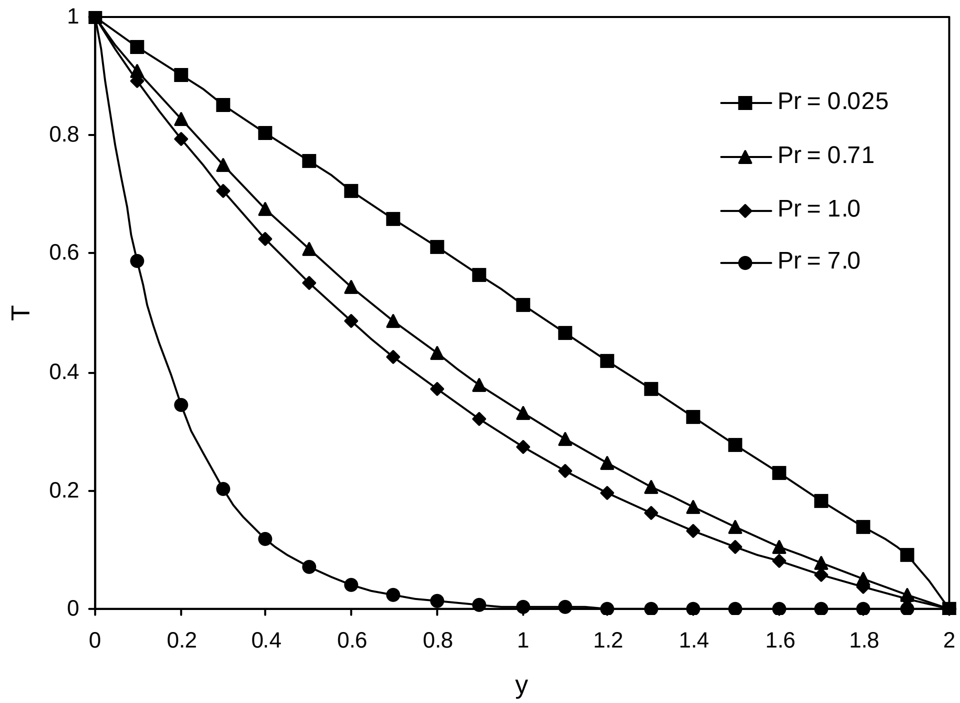

Fig.7. Effect of Prandtl number P_r on temperature T
(G_r = 5.0, M = 1.0, S = -0.05, E_c =0.001, $\varepsilon = 0.005$, t = 3 and $\omega t = \pi/2$)

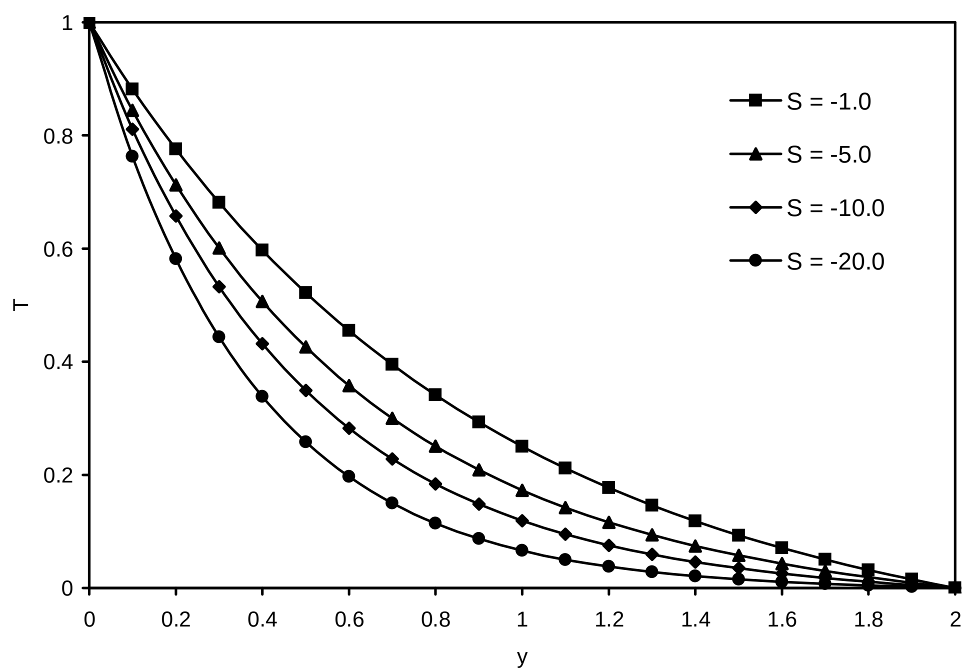

Fig.8. Effect of Sink-strength S on temperature T
(G_r = 5.0, M = 1.0, P_r = 1.0, E_c =0.001, $\varepsilon = 0.005$, t = 3 and $\omega t = \pi/2$)

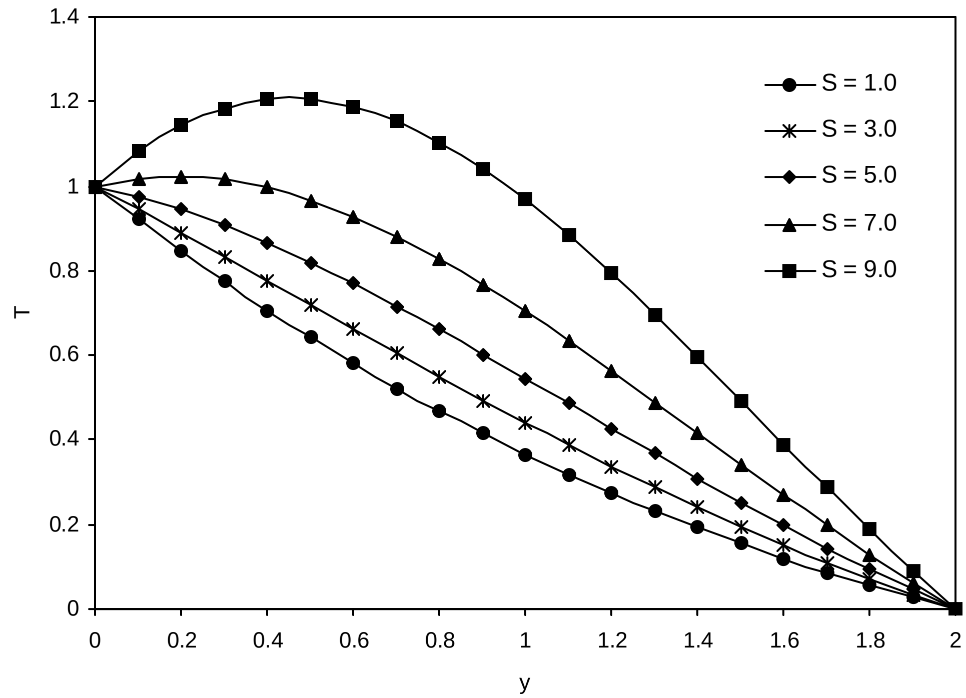

Fig.9. Effect of Sink-strength S on temperature T
(G_r = 5.0, M = 1.0, P_r = 1.0, Ec =0.001, $\varepsilon = 0.005$, t = 3 and ωt = π/2)

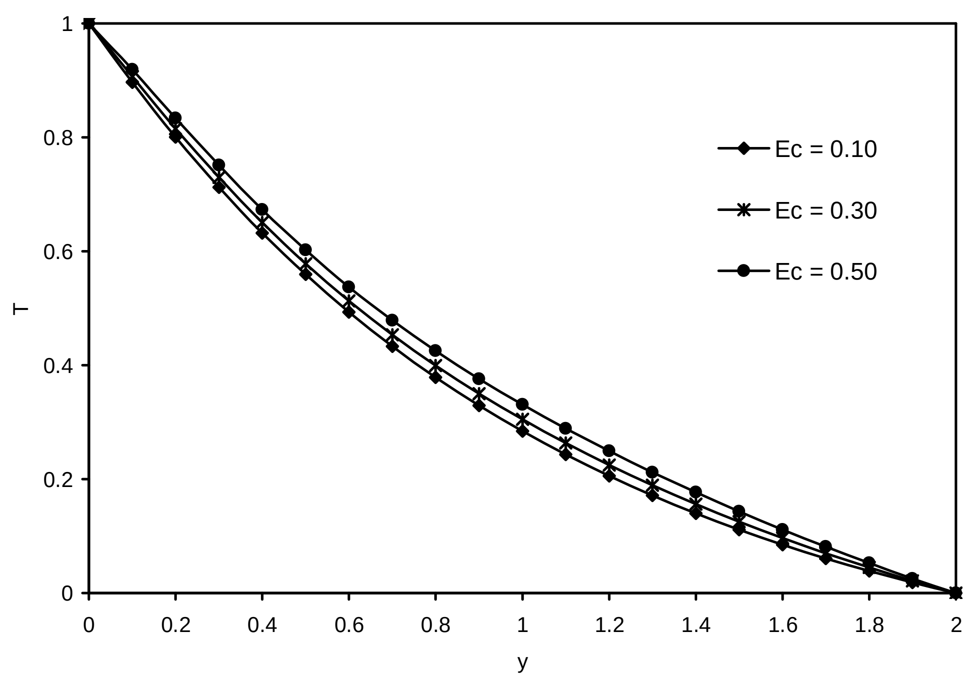

Fig.10. Effect of Eckert number E_c on temperature T
(G_r = 5.0, M = 1.0, P_r = 1.0, S = -0.10, $\varepsilon = 0.005$, t = 3 and ωt = π/2)

REFERENCES

1. M.K. JAIN, *Numerical solution of differential equations, 2 nd edition*, (Wiley Eastern Limited, New Delhi) 1984.
2. M.S. MALASHETTY AND V. LELLA, Magneto hydrodynamic heat transfer in two phase flow. *Int J Engg Sci*, **30** (1992) 371-377.
3. P. SACHETY, P. CHANDRAN AND A.K. SINGH, An exact solution for unsteady magnetohydrodynamic free convection flow with constant heat flux. *Int Comm Heat Mass Transfer*, **21** (1994) 131-142.
4. P.K. SAHOO, N. DATTA AND S. BISWAL, *Magnetohydrodynamic unsteady free convection flow past an infinite vertical plate with constant suction and heat sink. Indian J Pure Appl Math*, **34** (2003) 145-155.
5. A. SATTAR AND M. ALAM, MHD free convective heat and mass transfer flow with hall current and constant heat flux through a porous medium. *Indian J Pure Appl Math***. 26** (1995) 157.
6. R.C. SHAH AND A.P. VARMA, Multiphase flow through slightly dipping porous media with magnetic fluid. *Indian J Pure Appl Phys*, **38** (2000) 166-169.
7. S.R. SHENDE AND V.M. SOUNDALGEKAR, On unsteady free convection flow with mass transfer past an infinite vertical porous plate with variable suction. *Proc Math Soc*, **9** (1993) 101.
8. G.D. SMITH, *Numerical solutions of partial differential equations*, (Oxford University press, London) 1965.
9. V.M. SOUNDALGEKAR, Free convection effects on steady MHD flow past a vertical porous plate. *J Fluid Mech*, **66** (1974) 541-55.
10. E.M. SPARROW AND R.D. CESS, The effect of a magnetic field on free convection heat transfer. *Int J Heat Mass Transfer*, **3** (1961) 267-274.
11. S. SREEKANTH, A.S. NAGRAJAN AND S.V. RAMANA, Transient MHD free convection flow of an incompressible viscous dissipative fluid. *Indian J Pure Appl Math*, **32** (2001) 1051-1058.

NOMENCLATURE

u Velocity of the fluid
K Thermal conductivity of the fluid
β Volumetric coefficient
T∞ Temperature of the fluid far away from the plate
σ Electrical conductivity of the fluid
Pr Prandtl number
S Sink strength
M Hartmann number
qw Rate of heat transfer at the plate
g Acceleration due to gravity
ν Kinematic coefficient of viscosity
Tw Temperature of the plate
B0 Magnetic permeability
Cp Specific heat at constant pressure
Gr Grashof number
Ec Eckert number
τw Skin friction at the plate