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Chained Duoternary Semigroups

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ABSTRACT : In this paper, the terms chained ternary semigroup, cancellable clement , cancellative ternary semigroup, A-regular element, π- regular element, π- invertible element are introduced. It is proved that in a duo chained ternary semigroup T, i) if P is a prime ideal of T and $x \notin P$ then $\bigcap_{n=1}^{\infty} x^n PT$ = P for all odd natural numbers n . ii) T is a semiprimary ternary semigroup. iii) If $a \in T$ is a semisimple element of T, then $< a >^w \neq \phi$. iv) If $< a >^w = \phi$ for all $a \in T$, then T has no semisimple elements. v) T has no regular elements, then for any $a \in T$, $< a >^w = \phi$ or $< a >^w$ is a prime ideal. vi) If T is a duo chained cancellative ternary semigroup then for every non π-invertible element a, $< a >^w$ is either empty or a prime ideal of T. Further it is proved that if T is a chained ternary semigroup with $T \backslash T^3 = \{ x \}$ for some $x \in T$, then i) $T \backslash \{ x \}$ is an ideal of T. ii) $T = xT^1T^1 = T^1xT^1 = T^1T^1x$ and $T^3 = xTT = TxT = TTx$ is the unique maximal ideal of T. iii) If $a \in T$ and $a \notin < x >^w$ then $a = x^n$ for some odd natural number n > 1.iv) $T \backslash < x >^w = \{ x, x^3, x^5, \ldots \}$ or $T \backslash < x >^w = \{x, x^3, \ldots, x^r\}$ for some odd natural number r. v) If $a \in T$ and $a \in < x >^w$ then $a = x^r$ for some odd natural number r or $a = x^n s_n t_n$ and $s_n \in < x >^w$ or $t_n \in < x >^w$ for every odd natural number n. vi) If T contains cancellable elements then x is cancellable element and $< x >^w$ is either empty or a prime ideal of T. It is also prove that, in a duo chained ternary semigroup T, T is archemedian ternary semigroup without idempotent elements if and only if $< a >^w = \phi$ for every $a \in T$.

Mathematical subject classification (2010) : 20M07; 20M11; 20M12.

Keywords : - chained ternary semigroup, cancellable clement and cancellative ternary semigroup.

1. INTRODUCTION :

The algebraic theory of semigroups was widely studied by CLIFFORD and PRESTON [5], [6]; PETRICH [15]. The ideal theory in duo semigroups was developed by BOURNE [4], HARBANS LAL [10], SATYANARAYANA [19], [20], MANNEPALLI and NAGORE [14]. The ideal theory in duo semigroups was developed by ANJANEYULU [1], [2], HOEHNKE [11] and KAR.S and MAITY. B. K[12], [13]. SANTIAGO [18] developed the theory of ternary semigroups. SARALA. Y, ANJANEYULU. A and MADHUSUDHANA RAO.D [16], [17] introduced the ideal theory in ternary semigroups and characterize the properties of ideals.

GIRI and WAZALWAR[7] initiated the study of prime radicals in semigroups. ANJANEYULU. A[1], [2], [3] initiated the study of primary and semiprimary ideals in semigroups. He also introduced chained duo semigroups. HANUMANTHA RAO.G, ANJANEYULU. A and GANGADHARA RAO. A[8], [9] introduced the study of primary and semiprimary ideals in ternary semigroups. In this paper we introduce the notions of chained duo ternary semigroups, noetherian ternary semigroups and characterize chained duo ternary semigroups, noetherian ternary semigroups.

2. PRILIMINARIES :

DEFINITION 2.1 : Let T be a non-empty set. Then T is said to be a ***ternary semigroup*** if there exist a mapping from T×T×T to T which maps $(x_1, x_2, x_3) \rightarrow x_1x_2x_3$ satisfying the condition : $\left[x_1x_2x_3 \;\; x_4x_5 \right] = \left[x_1 \;\; x_2x_3x_4 \;\; x_5 \right] = \left[x_1x_2 \;\; x_3x_4x_5 \right] \; \forall \; x_i \in \mathrm{T}, 1 \leq i \leq 5$.

NOTE 2.2 : For the convenience we write $x_1x_2x_3$ instead of $x_1x_2x_3$

NOTE 2.3 : Let T be a ternary semigroup. If A,B and C are three subsets of T , we shall denote the set ABC = $abc : a \in A, b \in B, c \in C$.

DEFINITION 2.4 : A ternary semigroup T is said to be ***duo*** provided *abc* = *bca* = *cab* = *bac* = *cba* = *acb* for all *a,b,c*∈ T.

DEFINITION 2.5 : A nonempty subset A of a ternary semigroup T is said to be ***left ternary ideal*** or ***left ideal*** of T if *b, c* ∈ T, *a* ∈ A implies *bca* ∈ A.

NOTE 2.6 : A nonempty subset A of a ternary semigroup T is a left ideal of T if and only if TTA ⊆ A.

DEFINITION 2.7 : A nonempty subset of a ternary semigroup T is said to be a ***lateral ternary ideal*** or simply ***lateral ideal*** of T if *b, c* ∈ T , *a* ∈ A implies *bac* ∈ A.

NOTE 2.8 : A nonempty subset of A of a ternary semigroup T is a lateral ideal of T if and only if TAT ⊆ A.

DEFINITION 2.9: A nonempty subset A of a ternary semigroup T is a ***right ternary ideal*** or simply ***right ideal*** of T if *b, c* ∈ T , *a* ∈ A implies *abc* ∈ A

NOTE 2.10 : A nonempty subset A of a ternary semigroup T is a right ideal of T if and only if ATT ⊆ A.

DEFINITION 2.11 : A nonempty subset A of a ternary semigroup T is a ***two sided ternary ideal*** or simply ***two sided ideal*** of T if *b, c* ∈ T , *a* ∈ A implies *bca* ∈ A, *abc* ∈ A.

NOTE 2.12 : A nonempty subset A of a ternary semigroup T is a two sided ideal of T if and only if it is both a left ideal and a right ideal of T .

DEFINITION 2.13 : A nonempty subset A of a ternary semigroup T is said to be ***ternary ideal*** or simply an ***ideal*** of T if *b, c* ∈ T , *a* ∈ A implies *bca* ∈ A, *bac* ∈ A, *abc* ∈ A.

NOTE 2.14 : A nonempty subset A of a ternary semigroup T is an ideal of T if and only if it is left ideal, lateral ideal and right ideal of T .

DEFINITION 2.15 : An ideal A of a ternary semigroup T is said to be a ***proper ideal*** of T if A ≠ T.

DEFINITION 2.16 : An ideal A of a ternary semigroup T is said to be a ***trivial ideal*** provided T\ A is singleton.

DEFINITION 2.17 : An ideal A of a ternary semigroup T is said to be a ***maximal ideal*** provided A is a proper ideal of T and is not properly contained in any proper ideal of T.

DEFINITION 2.18 : An ideal A of a ternary semigroup T is said to be ***a principal ideal*** provided A is an ideal generated by a for some $a \in$ T. It is denoted by J (a) (or) < a >.

DEFINITION 2.19 : An ideal A of a ternary semigroup T is said to be a ***completely prime ideal*** of T provided x, y, $z \in$ T and $xyz \in$ A implies either $x \in$ A or $y \in$ A or $z \in$ A.

DEFINITION 2.20 : An ideal A of a ternary semigroup T is said to be a ***prime ideal*** of T provided X,Y,Z are ideals of T and XYZ $\subseteq$ A $\Rightarrow$ X $\subseteq$ A or Y $\subseteq$ A or Z $\subseteq$ A.

THEOREM 2.21 : Every completely prime ideal of a ternary semigroup T is a prime ideal of T.

THEOREM 2.22 : Let T be a duo ternary semigroup . An ideal P of T is a prime ideal if and only if P is a completely prime ideal.

DEFINITION 2.23 : An ideal A of a ternary semigroup T is said to be a ***completely semiprime ideal*** provided $x \in$ T, $x^n \in$ A for some odd natural number n >1 implies $x \in$ A.

THEOREM 2.24 : An ideal A of a ternary semigroup T is semiprime if and only if X is an ideal of T, $X^3 \subseteq A$ implies X ⊆ A.

THEOREM 2.25 : Every prime ideal of a ternary semigroup T is semiprime.

NOTATION 2.26 : If A is an ideal of a ternary semigroup T, then we associate the following four types of sets.

A_1 = The intersection of all completely prime ideals of T containing A.

$A_2 = \{x \in T : x^n \in A$ for some odd natural numbers $n\}$

A_3 = The intersection of all prime ideals of T containing A.

$A_4 = \{x \in T : \langle x \rangle^n \subseteq A$ for some odd natural number $n\}$

THEOREM 2.27 : If A is an ideal of a ternary semigroup T, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.

THEOREM 2.28 : If A is an ideal of a duo ternary semigroup T, then $A_1 = A_2 = A_3 = A_4$.

DEFINITION 2.29 : If A is an ideal of a ternary semigroup T , then the intersection of all prime ideals of T containing A is called ***prime radical*** or simply ***radical*** of A and it is denoted by $\sqrt{A}$ or *rad* A.

DEFINITION 2.30 : If A is an ideal of a ternary semigroup T , then the intersection of all completely prime ideals of T containing A is called ***completely prime radical*** or simply ***complete radical*** of A and it is denoted by *c.rad* A.

COROLLARY 2.31 : If $a \in \sqrt{A}$, then there exist an odd positive integer n such that $a^n \in$ A.

COROLLARY 2.32 : If A is an ideal of a duo ternary semigroup T, then *rad* A = *c.rad* A.

DEFINITION 2.33 : An element a of a ternary semigroup T is said to be ***regular*** if there exist x, $y \in$ T such that $axaya = a$.

DEFINITION 2.34 : A ternary semigroup T is said to be ***regular ternary semigroup*** provided every element is regular.

DEFINITION 2.35 : An element a of a ternary semigroup T is said to be ***left regular*** if there exist $x, y \in$ T such that $a = a^3xy$.

DEFINITION 2.36 : An element a of a ternary semigroup T is said to be ***lateral regular*** if there exist $x, y \in$ T such that $a = xa^3y$.

DEFINITION 2.37 : An element a of a ternary semigroup T is said to be ***right regular*** if there exist $x, y \in$ T such that $a = xya^3$.

DEFINITION 2.38 : An element a of a ternary semigroup T is said to be ***intra regular*** if there exist $x, y \in$ T such that $a = xa^5y$.

DEFINITION 2.39 : An element a of a ternary semigroup T is said to be ***semisimple*** if $a \in \langle a \rangle^3$ i.e. $\langle a \rangle^3 = \langle a \rangle$.

THEOREM 2.40 : An element a of a ternary semigroup T is said to be ***semisimple*** if $a \in \langle a \rangle^n$ i.e. $\langle a \rangle^n = \langle a \rangle$ for all odd natural number n.

DEFINITION 2.41 : A ternary semigroup T is called ***semisimple ternary semigroup*** provided every element in T is semisimple.

DEFINITION 2.42 : An element a of a ternary semigroup T is said to be an ***idempotent*** element provided $a^3 = a$.

THEOREM 2.43 : Let T be a ternary semigroup and $a \in$ T. If a is idempotent, then a is semisimple.

THEOREM 2.44 : Let T be a ternary semigroup. If T has no semisimple elements, then T has no idempotent elements.

DEFINITION 2.45 : A ternary semigroup T is said to be an ***idempotent ternary semigroup*** or ***ternary band*** provided every element of T is an idempotent.

THEOREM 2.46 : If T is a ternary semigroup with unity 1 then the union of all proper ideals of T is the unique maximal ideal of T.

THEOREM 2.47 : If T is a duo ternary semigroup and A is an ideal of T, then $abc \in$ A if and only if $\langle a \rangle \langle b \rangle \langle c \rangle \subseteq$ A.

COROLLARY 2.48 : If T is a duo ternary semigroup and $a, b, c \in$ T, then $\langle abc \rangle = \langle a \rangle \langle b \rangle \langle c \rangle$.

DEFINITION 2.49 : An ideal A of a ternary semigroup T is said to be a ***completely semiprime ideal*** provided $x \in$ T, $x^n \in$ A for some odd natural number $n > 1$ implies $x \in$ A.

DEFINITION 2.50 : An ideal A of a ternary semigroup T is said to be ***semiprime ideal*** provided X is an ideal of T and $X^n \subseteq$ A for some odd natural number n implies X ⊆ A.

DEFINITION 2.51 : A ternary semigroup T is said to be an ***archimedean ternary semigroup*** provided for any $a, b \in$ T there exists an odd natural number n such that $a^n \in$ TbT.

DEFINITION 2.52 : A ternary semigroup T is said to be a ***strongly archimedean ternary semigroup*** provided for any $a, b \in$ T, there exist an odd natural number n such that $\langle a \rangle^n \subseteq \langle b \rangle$.

THEOREM 2.53 : Every strongly archimedean ternary semigroup is an archimedean ternary semigroup.

THEOREM 2.54 : If T is a duo ternary semigroup, then the following are equivalent.

1) T is a strongly archimedean semigroup.

2) T is an archimedean semigroup.

3) T has no proper completely prime ideals.

4) T has no proper prime ideals.

THEOREM 2.55 : An ideal Q of ternary semigroup T is a semiprime ideal of T if and only if $\sqrt{Q}$ = Q.

DEFINITION 2.56 : An ideal A of a ternary semigroup T is said to be ***semiprimary*** if $\sqrt{A}$ is a prime ideal

DEFINITION 2.57 : A ternary semigroup T is said to be ***semiprimary ternary semigroup*** if every ideal of T is a semi primary ideal.

DEFINITION 2.58 : A ternary semigroup T is said to be ***simple ternary semigroup*** if T is its only ideal.

THEOREM 2.59 : If T is a left simple ternary semigroup (or) a lateral simple ternary semigroup (or) a right simple ternary semigroup then T is a simple ternary semigroup.

THEOREM 2.60 : If T is a duo ternary semigroup such that T^3 = T, then every maximal ideal of T is a prime ideal of T.

THEOREM 2.61 : T is a duo ternary semigroup such that T^3 = Tand T having maximal ideals then T contains regular elements.

3. CHAINED DUO TERNARY SEMIGROUPS

DEFINITION 3.1 : A ternary semigroup T is said to be a ***chained ternary semigroup*** if the ideals in T are linearly ordered by set inclusion.

NOTE 3.2 : An ideal P of a duo ternary semigroup T is prime if and only if it is completely prime. i.e., P is prime if and only if $x, y, z \in$ T , $xyz \in$ P ⇒ either $x \in$P or $y \in$ P or $z \in$ P.

NOTATION 3.3 : If A is any ideal of a ternary semigroup T, then denote $A^w = \bigcap_{n=1}^{\infty} A^n$ where n is odd natural number**.**

THEOREM 3.4 : Let T be a duo chained ternary semigroup and P is a prime ideal of T and $x \notin$ P then $\bigcap_{n=1}^{\infty} x^n$ PT ∪ x^nTP = P for all odd natural numbers *n* .

***Proof* :** Since $x \notin$ P and P is prime , $x^n \notin$P for all odd natural numbers n. Since $x^n \in$ T and P is an ideal of T, x^nPT ∪ x^nTP ⊆ P for all odd natural numbers n.

Therefore $\bigcap_{n=1}^{\infty} x^n$ PT ⊆ P for all $x \in$ T. Since T is a duo ternary semigroup, $x^n T^1 T^1$ is an ideal of T. Since $x^n \notin$ P, $x^n T^1 T^1 \nsubseteq$ P. Since T is a chained ternary semigroup, P ⊆ $x^n T^1 T^1$ for all odd natural numbers n. Let $y \in$ P. Then $y \in x^n T^1 T^1 \Rightarrow y = x^n st$ for some $s, t \in T^1$. Now $x^n st \in$ P, $x^n \notin$ P. Since P is prime, $s \in$P or $t \in$P. Therefore $y = x^n st \in x^n$PT∪ x^nTP for all odd natural number n and hence P ⊆ x^nPT ∪ x^nTP for all odd natural number n. Hence P ⊆ $\bigcap_{n=1}^{\infty} x^n$ PT ∪ x^nTP for all odd natural numbers $n \in$ N.

Therefore P = $\bigcap_{n=1}^{\infty} x^n$ PT ∪ x^nTP.

THEOREM 3.5 : If T is a duo chained ternary semigroup, then T is a semiprimary ternary semigroup.

Proof **:** Let A be an ideal of T. We have $\sqrt{A} = \bigcap_{n=1}^{\infty} P_\alpha$ = Intersection of all prime ideals of T containing A. Since T is duo chained terinary semigroup, we have $\{ P_\alpha : \alpha \in \Delta \}$ forms a chain. By Zorns Lemma, $\{ P_\alpha : \alpha \in \Delta \}$ has minimal element say P_β. Therefore $\sqrt{A} = P_\beta$ and P_β is a prime ideal of T, and hence $\sqrt{A}$ is prime. Therefore A is a semiprimary ideal of T and hence T is a semiprimary ternary semigroup.

THEOREM 3.6 : Let T be a duo chained ternary semigroup. If $a \in$ T is a semisimple element of T, then $< a >^w \neq \phi$.

Proof **:** Suppose that a is a semisimple element of T. Therefore $a \in < a >^3$, implies that $< a > = < a >^3$. Therefore $a \in < a > = < a >^n$ for all odd natural numbers n and hence $a \in \bigcap_{n=1}^{\infty} < a >^n = < a >^w$ and hence $< a >^w \neq \phi$.

COROLLARY 3.7 : Let T be a duo chained ternary semigroup. If $< a >^w = \phi$ for all $a \in$ T, then T has no semisimple elements.

Proof **:** Suppose that $< a >^w = \phi$ for all $a \in$ T. Suppose if possible T has a semisimple element x. By theorem 3.6, $< x >^w \neq \phi$. It is a contradiction. Therefore T has no semisimple elements.

COROLLARY 3.8 : Let T be a duo chained ternary semigroup. If $< a >^w = \phi$ for all $a \in$ T, then T has no idempotent elements.

Proof **:** Suppose that $< a >^w = \phi$ for all $a \in$ T. By theorem 3.7, T has no semisimple elements. By theorem 2.44, T has no idempotent elements.

THEOREM 3.9 : Let T be a duo ternary semigroup and $a \in$ T. Then a is semisimple if and only if a is left, right, lateral regular and regular.

Proof **:** Suppose that a is semisimple in T. Therefore $a \in < a >^3$. Since T is duo, $a \in < a >^3 = < a^3 >$. Therefore $a = a^3st$ for some $s, t \in$ T. Hence a is left regular. Since T is duo, $a = a^3st = pa^3q = xy\, a^3 = alama$ for some $l, m, p, q, x, z \in$ T. Therefore a is left, right, lateral regular and regular.

Conversely suppose that a is left, right, lateral regular and regular.
Therefore $a = a^3st = sa^3t = sta^3 = asata$ for some $s, t \in$ T. Now $a = a^3st \in < a^3 >= < a >^3$. Hence a is semisimple.

THEOREM 3.10 : Let T be a chained duo ternary semigroup. If T has no regular elements, then for any $a \in$ T, $< a >^w = \phi$ or $< a >^w$ is a prime ideal.

Proof **:** Suppose that T has no idempotent elements and $a \in$ T.
We have $< a >^w = \bigcap_{n=1}^{\infty} < a >^n$.

Assume that $< a >^w \neq \phi$. If possible, suppose that $< a >^w$ is not prime. Then there exist $x, y, z \in$ T such that $xyz \in < a >^w$ and $x, y, z \notin < a >^w$. By theorem 2.47, $< x > < y >< z > = < xyz > \subseteq < a >^w$.
Now $x, y, z \notin < a >^w$, implies that there exists odd natural numbers n, m, p such that $x \notin < a >^n$, $y \notin < a >^m$ and $z \notin < a >^p$.
Consider $k = \min \{n, m, p\}$. Then $x, y, z \notin < a >^k$. Since T is duo chained ternary semigroup,
we have $< a >^k \subseteq < x >$, $< a >^k \subseteq < y >$ and $< a >^k \subseteq < z >$.
Therefore $< a >^{3k} = < a >^k< a >^k< a >^k \subseteq < x > < y >< z > = < xyz > \subseteq < a >^w \subseteq < a >^{9k}$.
Then $< a >^{3k} \subseteq < a >^{9k} = < a >^{3k} < a >^{3k} < a >^{3k}$ and hence $a^{3k} \in < a^{3k} >^3$.

Therefore a^{3k} is a semisimple element of T. By theorem 3.9, a^{3k} is a regular element of T.
It is a contradiction. Hence $< a >^w$ is a prime ideal of T.

DEFINITION 3.11 : Let T be ternary semigroup and $a \in$ T. Then a is said to be a
left cancellable clement if $aax = aay \Rightarrow x = y$,
lateral cancellable clement if $axa = aya \Rightarrow x = y$,
right cancellable clement if $xaa = yaa \Rightarrow x = y$ holds for all $x, y \in$ T.

DEFINITION 3.12 : Let T be ternary semigroup and $a \in$ T. Then a is said to be ***cancellable clement*** if it is left, lateral and right cancellable element.

DEFINITION 3.13 : A ternary semigroup T is said to be a
left cancellative if $abx = aby \Rightarrow x = y$ for all $a, b \in$ T
lateral cancellative if $axb = ayb \Rightarrow$ x = y for all $a, b \in$ T
right cancellative if $xab = yab \Rightarrow$ x = y for all $a, b \in$ T.

DEFINITION 3.14 : A ternary semigroup T is said to be ***cancellative ternary semigroup*** if T is left, lateral and right cancellative.

THEOREM 3.15 : In a ternary semigroup T, the following are equivalent.
1. T is lateral cancellative.
2. T is left and right cancellative.
3. T is cancellative

Proof : (1) ⇒ (2) : Suppose that ternary semigroup T is lateral cancellative. Therefore $axb = ayb \Rightarrow x = y$.
Let $a, b, x, y \in$ T such that $xab = yab$.
Now $ab[xab] = ab[yab]] \Rightarrow a[bxa]b = a[bya]b \Rightarrow bxa = bya \Rightarrow x = y$.
Thus T is right cancellative. Similarly we can prove that T is left cancellative.

(2) ⇒ (3) : Suppose that ternary semigroup T is left and right cancellative.
Let $a, b, x, y \in$ T such that $axb = ayb$.
Now $axb = ayb \Rightarrow a[axb]b = a[ayb]b \Rightarrow [aax]bb = [aay]bb \Rightarrow aa[x]bb = aa[y]bb$. Since T is left and right cancellative, we get $x = y$. Thus T is lateral cancellative.

(3) ⇒ (1) : Suppose that ternary semigroup T is cancellative. By the definition 3.13, T is lateral cancellative.

DEFINITION 3.16 : Let T be a ternary semigroup and $a \in$ T. Then a is said to be ***strongly regular element*** if there exists $x \in$ T such that $axaxa = a$.

THEOREM 3.17: Let T be a ternary semigroup and *a* ∈ T. Then *a* regular element in T if and only if *a* is *strongly* regular element in T.

Proof **:** Suppose that a regular element in T. Therefore there exists $x, y \in$ T such that $axaya = a$. Now $axayaxaya = axaya = a \Rightarrow a(xay)a(xay)a = a$. That is $asasa = a$ where $(xay) = s$. Hence a is strongly regular.
Conversely, suppose that a is strongly regular element in T. Therefore there exists $x \in$ T such that $axaxa = a$. Hence a is regular in T.

DEFINITION 3.18 : Let T be a ternary semigroup and $a \in$ T. Then a is said to be ***π- regular*** if there exists $x \in$ T such that $a^n x a^n x a^n = a^n$ for some odd natural number n.

DEFINITION 3.19 : Let T be a ternary semigroup and $a \in$ T. Then a is said to be ***π- invertible element*** if there exists $x \in$ T such that $a^n x a^n x a^n = a^n$ and $x a^n x a^n x = x$ for some odd natural number n.

THEOREM 3.20 : If T is a duo chained cancellative ternary semigroup then for every non π-invertible element *a*, $< a >^w$ is either empty or a prime ideal of T.

***Proof* :** Suppose that a is a non π-invertible element in T. If $< a >^w = \phi$ then theorem is trivial. Let $< a >^w \neq \phi$. If possible, suppose that $< a >^w$ is not prime.
Then there exist $x, y, z \in$ T such that $xyz \in < a >^w$ and $x, y, z \notin < a >^w$.
By theorem 2.48, $< x > < y > < z > = < xyz >$. Now $x, y, z \notin < a >^w$, implies that there exists odd natural numbers n, m, p such that $x \notin < a >^n$, $y \notin < a >^m$ and $z \notin < a >^p$.
Consider $k = \min \{ n, m, p \}$. Then $x, y, z \notin < a >^k$. Since T is chained ternary semigroup, we have $< a >^k \subseteq < x >$, $< a >^k \subseteq < y >$ and $< a >^k \subseteq < z >$.
Therefore $< a >^{3k} = < a >^k < a >^k < a >^k \subseteq < x > < y > < z > = < xyz > \subseteq < a >^{9k} \subseteq < a >^w$. Then $< a >^{3k} \subseteq < a >^{9k} = < a >^{3k} < a >^{3k} < a >^{3k}$ and hence $a^{3k} \in < a^{3k} >^3$. Therefore a^{3k} is a semisimple element of T. By theorem 3.9, a^{3k} is a regular element of T. By theorem 3.17, a^{3k} is a strogly regular element of T. Therefore $a^{3k} = a^{3k} x a^{3k} x a^{3k}$ for some $x \in$ T.
Now $a^{3k} x a^{3k} x a^{3k} x a^{3k} = a^{3k} x a^{3k}$. Since T is cancellative, $x a^{3k} x a^{3k} x = x$. Hence a is a π-invertible element in T. It is a contradiction. Thus $< a >^w$ is a prime ideal of T.
Hence $< a >^w = \phi$ or $< a >^w$ is prime ideal of T.

THEOREM 3.21 : Let T be a chained ternary semigroup. If $T \neq T^3$ then $T \setminus T^3 = \{ x \}$ for some $x \in$ T.

***Proof* :** Suppose if possible $x, y \in T \setminus T^3$ and $x \neq y$. Since T is a chained ternary semigroup, $< x > \subseteq < y >$ or $< y > \subseteq < x >$. If $< x > \subseteq < y >$, then $x \in < y >$ and hence $x = yst$ for some $s, t \in$ T. Therefore $x \notin T^3$, which is not true. If $< y > \subseteq < x >$, then $y \in < x >$ and hence $y = xpq$ for some $p, q \in$ T. Therefore $y \in T^3$, which is not true. It is a contradiction . Therefore $x = y$. So there exists unique $x \in$ T such that $x \notin T^3$. Therefore $T \setminus T^3 = \{ x \}$.

THEOREM 3.22 : Let T be a chained ternary semigroup with $T \setminus T^3 = \{ x \}$ for some $x \in$ T. Then $T \setminus \{ x \}$ is an ideal of T.

***Proof* :** Let $a \in T \setminus \{x\}$ and $s, t \in$ T. we have $ast \in T^3$. Since $x \notin T^3$, we have $ast \neq x$ and hence $ast \in T \setminus \{ x \}$. Hence $T \setminus \{ x \}$ is a right ideal of T. similarly, we can get $sta, sat \in T \setminus \{ x \}$. Therefore $T \setminus \{ x \}$ is an ideal of T.

THEOREM 3.23 : Let T be a duo chained ternary semigroup. If $T \neq T^3$ such that $T \setminus T^3 = \{ x \}$ for some $x \in$ T, then $T = xT^1T^1 = T^1xT^1 = T^1T^1x$ and $T^3 = xTT = TxT = TTx$ is the unique maximal ideal of T.

***Proof* :** Since $T \setminus T^3 = \{ x \}$, $T^3 = T \setminus \{x\}$. Now xT^1T^1 is an ideal of T and T^3 is an ideal of T. Since $x \notin T^3$ and T is a chained ternary semigroup, $T^3 \subseteq xT^1T^1$. Clearly, $xTT \subseteq T^3$. Hence $T^3 = TTx = TxT = xTT$.
Since T^3 is trivial, $T^3 = xTT = TxT = TTx$ is the unique maximal ideal of T.

THEOREM 3.24 : Let T be a duo chained ternary semigroup with $T \neq T^3$ such that $T \setminus T^3 = \{ x \}$ for some $x \in$ T. If $a \in$ T and $a \notin < x >^w$ then $a = x^n$ for some odd natural number $n > 1$

***Proof* :** Since T is a duo chained ternary semigroup with $T \neq T^3$ such that $T \setminus T^3 \neq \{ x \}$. By theorem 3.23, $T^3 = TTx = xTT = T \setminus \{ x \}$. Since $a \notin < x >^w$, there exists a odd natural number k such that $a \notin < x >^k$. Let n be the least odd positive integer such that $a \in < x >^{n-2}$ and $a \notin < x >^n$. Therefore $a \in x^{n-2}TT \setminus x^nTT$ and hence $a = x^{n-2}st$ for some $s, t \in$ T.
If $s, t \in x$ TT then $a = x^n s_n s^l_n t_n t^l_n \in x^n TT = < x^n > = < x >^n$. It is a contradiction.

Hence $s, t \notin x$ TT. Therefore $s = x$ and $t = x$. Thus $a = x^n$ for some odd natural number n. If $n = 1$ then $a = x \in < x >$. It is a contradiction. Therefore $n > 1$.

THEOREM 3.25 : Let T be a duo chained ternary semigroup with T\T ^{3} = { x }. Then T\ $< x >^w$ = { $x, x^3, x^5, \ldots$ } or T\ $< x >^w$ ={ $x, x^3, \ldots, x^r$ } for some odd natural number r.

***Proof* :** By theorem 3.24, T\ $< x >^w \subseteq \{ x, x^3, x^5, \ldots \}$. If $x^n \in$ T\ $< x >^w$ for all odd natural number n, then T \ $< x >^w = \{ x, x^3, x^5, \ldots \}$. If $x^n \notin$ T \ $< x >^w$ for some odd natural number n, then we can choose the least odd positive integer r is such that $x^{r+2} \notin$ T \ $< x >^w$. Therefore $x, x^3, \ldots x^r \in$ T \ $< x >^w$ for all $n > r$. Therefore T \ $< x >^w$ ={ $x, x^3, x^5, \ldots, x^r$ }.

THEOREM 3.26 : Let T be a duo chained ternary semigroup with T ≠ T^3 such that T\T ^{3} = { x }. If $a \in$ T and $a \in < x >^w$ then $a = x^r$ for some odd natural number r or $a = x^n s_n t_n$ and $s_n \in < x >^w$ and $t_n \in < x >^w$ for every odd natural number n or $a = x^m z$ *where* $z \in < x >^w$ for some even natural number m.

***Proof* :** Since T is a duo chained ternary semigroup with T ≠ T ^{3} such that $x \in$ T\T ^{3}. By theorem 3.23, T^3 = TTx = xTT = T\ { x }.
Let $a \in$ T. Suppose that $a \in < x >^w$. Now $a \in < x >^w$ implies that $a \in \bigcap_{n=1}^{\infty} < x >^n$.

Therefore $a \in < x >^n = < x^n >$ for every odd natural number n . Therefore $a = x^n s_n t_n$ for some $s_n, t_n \in$ T for every odd natural number n.

Case 1 : If $s_n, t_n \notin < x >^w$ for some odd natural number n. By theorem 3.24, $s_n = x^r$, $t_n = x^p$ for some odd natural number $r, p > 1$ and hence $a = x^{n+r+p}$ for some odd natural number $n + r + p$.

Case 2 : If $s_n, t_n \in < x >^w$, then $a = x^n s_n t_n$ where $s_n , t_n \in < x >^w$.

Case 3 : If only one of the s_n or $t_n \in < x >^w$. Suppose that $s_n \in < x >^w$ and $t_n \notin < x >^w$ then $t_n = x^p$ for some odd natural number p. Therefore $a = x^n s_n t_n = a = x^{n+p} s_n$ where $n + p$ is even.

Hence $a = x^m z$ *where* $z \in < x >^w$ for some even natural number m.

THEOREM 3.27 : Let T be a duo chained ternary semigroup. Then T is archemedian ternary semigroup without idempotent elements if and only if $< a >^w = \phi$ for every $a \in$ T.

***Proof* :** Suppose that T is an archemedian ternary semigroup without idempotents. If possible, suppose that $< a >^w \neq \phi$ for some $a \in$ T. By theorem 3.10, $< a >^w$ is a prime ideal of T. Since T is an archemedian duo ternary semigroup, by theorem 2.54, T has no proper prime ideals. Therefore $< a >^w$ = T. Now $a \in < a >^w \subseteq < a >^3$ and hence a is semisimple. By theorem 3.9, a is regular. So T has idempotent elements. It is a contradiction. Hence $< a >^w = \phi$ for every $a \in$S. Conversely suppose that $< a >^w = \phi$ for every $a \in$S. Since $< a >^w = \phi$ for every $a \in$ T, By corollary 3.7, T has no semisimple elements. By theorem 2.44, T has no idempotent elements. If possible, suppose that P is proper prime ideal of T.

Let $x \in$ T such that $x \notin$ P. Since $x \notin$ P, by theorem 3.3, P = $\bigcap_{n=1}^{\infty} x^n$PT $\subseteq < x >^w$. Therefore P $\subseteq < x >^w = \phi$. It is a contradiction. Hence T has no proper prime ideals. By theorem 2.54, T is an archemedian ternary semigroup.

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