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THERMAL RADIATION EFFECTS ON FLOW PAST A VERTICAL OSCILLATING PLATE WITH VARIABLE TEMPERATURE AND UNIFORM MASS DIFFUSION

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Abstract

Thermal radiation effects on flow past an infinite vertical oscillating plate in the presence of variable temperature and uniform mass diffusion is considered. The temperature near the plate is made to rise linearly with time. The fluid considered is a gray, absorbing-emitting radiation but a non-scattering medium. The dimensionless governing equations are solved using the Laplace-transform technique. The velocity, temperature and concentration are studied for different parameters like radiation parameter, phase angle and time. The variation of the skin-friction for different values of the parameters is also shown in a table.

Keywords : oscillating vertical plate, radiation , variable temperature

Nomenclature

A - constant

C' - Species concentration in the fluid

C'_w - concentration of the plate

C'_∞ - concentration in the fluid far away from the plate

C - dimensionless concentration

C_p - specific heat at constant pressure

g - acceleration due to gravity

Gr - thermal Grashof number

k - thermal conductivity of the fluid

Pr	- Prandtl number
p	- pressure
q_r	- radiative heat flux in the y-direction
N	- radiation parameter
T'	- temperature of the fluid near the plate
T_w'	- temperature of the plate
T_∞'	- temperature of the fluid far away from the plate
t'	- time
t	- dimensionless time
u'	- velocity of the fluid in the x-direction
u_0	- velocity of the plate
u	- dimensionless velocity
y	- coordinate axis normal to the plate
y'	- dimensionless coordinate axis normal to the plate
k^*	- mean absorption coefficient
β	- volumetric coefficient of thermal expansion
β^*	- volumetric coefficient of expansion with concentration
η	- similarity parameter
μ	- coefficient of viscosity
ν	- kinematic viscosity
ρ	- density
σ	- Stefan-Boltzmann constant
$\bar{\tau}$	- dimensionless average skin-friction
θ	- dimensionless temperature
η	- similarity parameter
ωt	- phase angle
erfc	- complementary error function

Introduction

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology.

The first exact solution of the Navier-stokes equation was given by Stokes (1851) which is concerned with the flow of a viscous incompressible fluid past an infinite horizontal plate oscillating in its own plane in an infinite mass of stationary fluid. Such a flow past an infinite vertical plate oscillating in its own plane was first studied by Soundalgekar (1979) in case of an isothermal plate. The effects of transversely applied uniform magnetic field on the flow past an infinite vertical oscillating isothermal plate were studied by Soundalgekar (1981). M.A.Mansour (1990) studied the interaction of free convection with thermal radiation of the oscillatory flow past a vertical plate. Free convection effects on MHD flow past an infinite vertical oscillating plate with constant heat flux were studied by Soundalgekar et al (1997). Chandrakala and Bhaskar (2009) studied the effects of thermal radiation on the flow past an infinite vertical oscillating isothermal plate in the presence of transversely applied magnetic field.

The unsteady flow past a moving infinite vertical oscillating plate in the presence of radiation with variable temperature and uniform mass diffusion has not received much attention from contemporary researchers. It is now proposed to study the thermal radiation effects on the oscillatory past a vertical plate with variable temperature and uniform mass diffusion.

Basic equations and analysis

Consider the flow of an incompressible viscous radiating fluid past an impulsively started infinite vertical plate. The x'- axis is taken along the plate in the vertical direction and the y'- axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration in a stationary condition. At time $t'>0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with constant velocity u_0, the plate temperature is made to rise linearly with time and the level of concentration near the plate is raised to C_w . The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\,\beta\,(T' - T'_\infty) + \nu\,\frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\rho C_p\,\frac{\partial T'}{\partial t'} = k\,\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\,\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

with the following initial and boundary conditions

$$t' \le 0: \quad u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \qquad \text{for all } y'$$

$$t' > 0: \quad u' = u_0 \cos\omega t,\ T' = T'_\infty + \left(T'_w - T'_\infty\right)At',\ C' = C'_w \qquad \text{at } y'=0 \tag{4}$$

$$u' = 0,\ T' \to T'_\infty,\ C' \to C'_\infty \qquad \text{as } y' \to \infty \quad \text{Where } A = \frac{u_0{}^2}{\nu}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$q_r = -\,\frac{4\sigma}{3\kappa^*}\,\frac{\partial T'^4}{\partial y'} \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{6}$$

By using equations (4) and (5), equation (2) reduces to

$$\rho C_p\,\frac{\partial T'}{\partial t'} = k\,\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T'^3_\infty}{3\kappa^*}\,\frac{\partial^2 T'}{\partial y'^2} \tag{7}$$

On introducing the following dimensionless quantities

$$u = \frac{u'}{u_0},\quad t = \frac{t' u_0{}^2}{\nu},\quad y = \frac{y' u_0}{\nu},\quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$Gr = \frac{g\,\beta\,\nu\,(T'_w - T'_\infty)}{u_0{}^3},\quad \Pr = \frac{\mu C_p}{k},\quad N = \frac{\kappa^* k}{4\sigma T'^3_\infty}, \tag{8}$$

$$Gc = \frac{g\,\beta\,\nu\,(C'_w - C'_\infty)}{u_0{}^3},\quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

in equations (1) to (7), leads to

$$\frac{\partial u}{\partial t} = Gr\,\theta + \frac{\partial^2 u}{\partial y^2} \tag{9}$$

$$3\,N\,\Pr\,\frac{\partial \theta}{\partial t} = (3N + \Pr)\,\frac{\partial^2 \theta}{\partial y^2} \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\,\frac{\partial^2 C}{\partial y^2} \tag{11}$$

The initial and boundary conditions in non-dimensionless form are

$$\begin{aligned} & u = 0,\ \theta = 0,\ C = 0 && \text{for all } y,\ t' \le 0 \\ t' > 0:\ & u = u_0 \cos \omega t,\ \theta = t,\ C = 1 && \text{at } y = 0 \\ & u = 0,\ \theta \to 0,\ C \to 0 && \text{as } y \to \infty \end{aligned} \tag{12}$$

The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$\theta = t\left[\left(1 + 2\eta^2 a\right) erfc(\eta\sqrt{a}) - \frac{2}{\sqrt{\pi}}\eta\sqrt{a}\exp(-\eta^2 a)\right] \tag{13}$$

$$C = erfc(\eta\sqrt{Sc}) \tag{14}$$

$$\begin{aligned} u = {} & \frac{\exp(-i\omega t)}{4}\left[\begin{aligned}&\exp(-2\eta\sqrt{(-i\omega)t}\ erfc(\eta - \sqrt{(-i\omega)t}) \\ &+ \exp(2\eta\sqrt{(-i\omega)t}\ erfc(\eta + \sqrt{(-i\omega)t})\end{aligned}\right] \\ & + \frac{\exp(i\omega t)}{4}\left[\begin{aligned}&\exp(-2\eta\sqrt{(i\omega)t}\ erfc(\eta - \sqrt{(i\omega)t}) \\ &+ \exp(2\eta\sqrt{(i\omega)t}\ erfc(\eta + \sqrt{(i\omega)t})\end{aligned}\right] \\ & - \frac{Gr\,\exp(bt)}{2b^2(1-a)}\left[\begin{aligned}&\exp(-2\eta\sqrt{abt}\ erfc(\eta - \sqrt{abt}) \\ &+ \exp(2\eta\sqrt{abt}\ erfc(\eta + \sqrt{abt})\end{aligned}\right] \\ & - \frac{Gr}{b^2(1-a)}\ erfc(\eta\sqrt{a}) \\ & - \frac{Gr\,\exp(ct)}{2c(1-Sc)}\left[\begin{aligned}&\exp(-2\eta\sqrt{c\,Sc\,t}\ erfc(\eta - \sqrt{c\,Sc\,t}) \\ &+ \exp(2\eta\sqrt{c\,Sc\,t}\ erfc(\eta + \sqrt{c\,Sc\,t})\end{aligned}\right] \\ & - \frac{Gr\,t}{b(1-a)}\left[\left(1 + 2\eta^2 a\right) erfc(\eta\sqrt{a}) - \frac{2}{\sqrt{\pi}}\eta\sqrt{a}\exp(-\eta^2 a)\right] \end{aligned}$$

$$+ \frac{Gr \exp(bt)}{2b^2(1-a)} \begin{bmatrix} \exp(-2\eta\sqrt{abt})\ erfc(\eta\sqrt{a} - \sqrt{bt}) \\ + \exp(2\eta\sqrt{abt})\ erfc(\eta\sqrt{a} - \sqrt{bt}) \end{bmatrix}$$

$$- \frac{Gr}{c(1-Sc)}\ erfc(\eta\ \sqrt{Sc})$$

$$+ \frac{Gr \exp(ct)}{2c(1-Sc)} \begin{bmatrix} \exp(-2\eta\sqrt{c\,Sc\,t}\ erfc(\eta\sqrt{Sc} - \sqrt{ct}) \\ + \exp(2\eta\sqrt{c\,Sc\,t}\ erfc(\eta\sqrt{Sc} + \sqrt{ct}) \end{bmatrix} \quad (15)$$

Where $a = \frac{3N\,\mathrm{Pr}}{3N+4}$, $b = \frac{1}{a-1}$, $c = \frac{1}{Sc-1}$ and $\eta = \frac{y}{2\sqrt{t}}$

Discussion of results

The numerical values of the velocity, temperature and wall concentration are computed for different parameters like radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number, time and Pr = 0.71. The purpose of the calculations given here is to assess the effects of the parameters N, t and ωt.

The velocity profiles for different values of the radiation parameter (N = 0.2, 30) are shown in Fig.1. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.

The velocity profiles for different values of the time (t = 0.2, 0.4, 0.6) are shown in Fig.2. This shows that the velocity decreases with increasing values of time t.

In Fig.3, the velocity profiles are shown for different values of ωt. It is observed that velocity increases with decreasing phase angle ωt.

The temperature profiles are calculated for different values of thermal radiation parameter (N = 2, 5, 30) from equation (13) and these are shown in Fig.4, for air (Pr = 0.71). The effect of thermal radiation parameter is important in temperature profiles. It is observed that temperature increases with decreasing radiation parameter. In Fig.5, the temperature profiles are shown for different values of t. It is observed that temperature increases with increasing time.

Figure 6 represents, the effect of concentration profiles for different Schmidt number (Sc = 0.16, 0.3, 0.6, 2.01). The effect of concentration is important in concentration field. It is observed that the wall concentration increases with decreasing values of the Schmidt parameter. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

From the velocity field, we now study the skin-friction. It is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0} \qquad (16)$$

Hence, from Eqs (15) and (16) , the wall shear stress in the presence of magnetic field is as follows.

$$\tau = \frac{1}{\sqrt{\pi t}}\left[\frac{\exp(-i\omega t)}{2}\left(1+\sqrt{(-i\omega)\pi t}\ erf\left(\sqrt{(-i\omega)t}\right)\right)\right.$$
$$+\frac{\exp(i\omega t)}{2}\left(1+\sqrt{(i\omega)\pi t}\ erf\left(\sqrt{(i\omega)t}\right)\right)+\frac{Gr\,(1+bt)}{b^2(1-a)}$$
$$-\frac{Gr\exp(bt)}{b(1-a)}\left(1+\sqrt{ab\pi t}\ erf\sqrt{abt}\right)+\frac{Gc}{c\,(1-Sc)}$$
$$-\frac{Gc\exp(ct)}{c\,(1-Sc)}\left(1+\sqrt{c\,Sc\pi t}\ erf\sqrt{c\,Sct}\right)-\frac{Gr\sqrt{a}}{b^2(1-a)}-\frac{2\,Gr\,t\sqrt{a}}{b(1-a)}$$
$$+\frac{Gr\exp(bt)}{b(1-a)}\left(\sqrt{a}+\sqrt{ab\pi t}\ erf\sqrt{bt}\right)$$
$$\left.-\frac{Gc\sqrt{Sc}}{c\,(1-Sc)}+\frac{Gc\exp(ct)}{c\,(1-Sc)}\left(\sqrt{Sc}+\sqrt{c\,Sc\pi t}\ erf\sqrt{c\,t}\right)\right] \qquad (17)$$

The numerical values of τ are presented in Table 1. It is observed from this table, that an increase in the Schmidt number parameter leads to fall in the value of the skin-friction but an increase in the radiation parameter leads to rise in the value of skin-friction. As time advances the value of skin-friction decreases.

Table 1. Values of the non-dimensional Skin-friction

N	Gr	Gc	Sc	t	$\omega t = 0$	$\omega t = \pi/6$	$\omega t = \pi/4$	$\omega t = \pi/3$
0.2	2	5	0.6	0.2	1.1534	-0.2049	-1.1718	-3.5467
2	2	5	0.6	0.2	1.2923	-0.0660	-1.5800	-3.4078
2	2	5	0.3	0.2	2.6025	1.2441	-0.2698	-2.0976
2	2	5	0.6	0.4	0.4237	-0.6370	-1.8449	-3.3289
5	2	5	0.6	0.2	1.9162	0.5578	-0.9561	-2.7839
10	2	5	0.6	0.2	2.5529	1.1941	-0.3195	-2.1473
0.2	2	5	0.6	0.2	3.4901	1.8974	0.0770	-2.1705
2	2	5	0.6	0.2	3.8325	2.2398	0.4194	-1.8280
0.2	2	5	0.6	0.2	8.3526	6.2493	3.8217	0.7807
2	2	5	0.6	0.2	8.5101	6.4069	3.9792	0.9383

Conclusions

An exact analysis is performed to study the thermal radiation effects on flow past an impulsively started infinite oscillating vertical plate with variable temperature and uniform mass diffusion. The dimensionless governing equations are solved by the usual Laplace-transform technique. The velocity, temperature and wall concentration are studied for different physical parameters are studied graphically. The conclusions of the study are as follows.

(i) The presence of radiation causes a fall in the velocity and temperature.

(ii) Velocity increases with decreasing phase angle ωt.

(iii) As time increases, it is found that there is a decrease in velocity.

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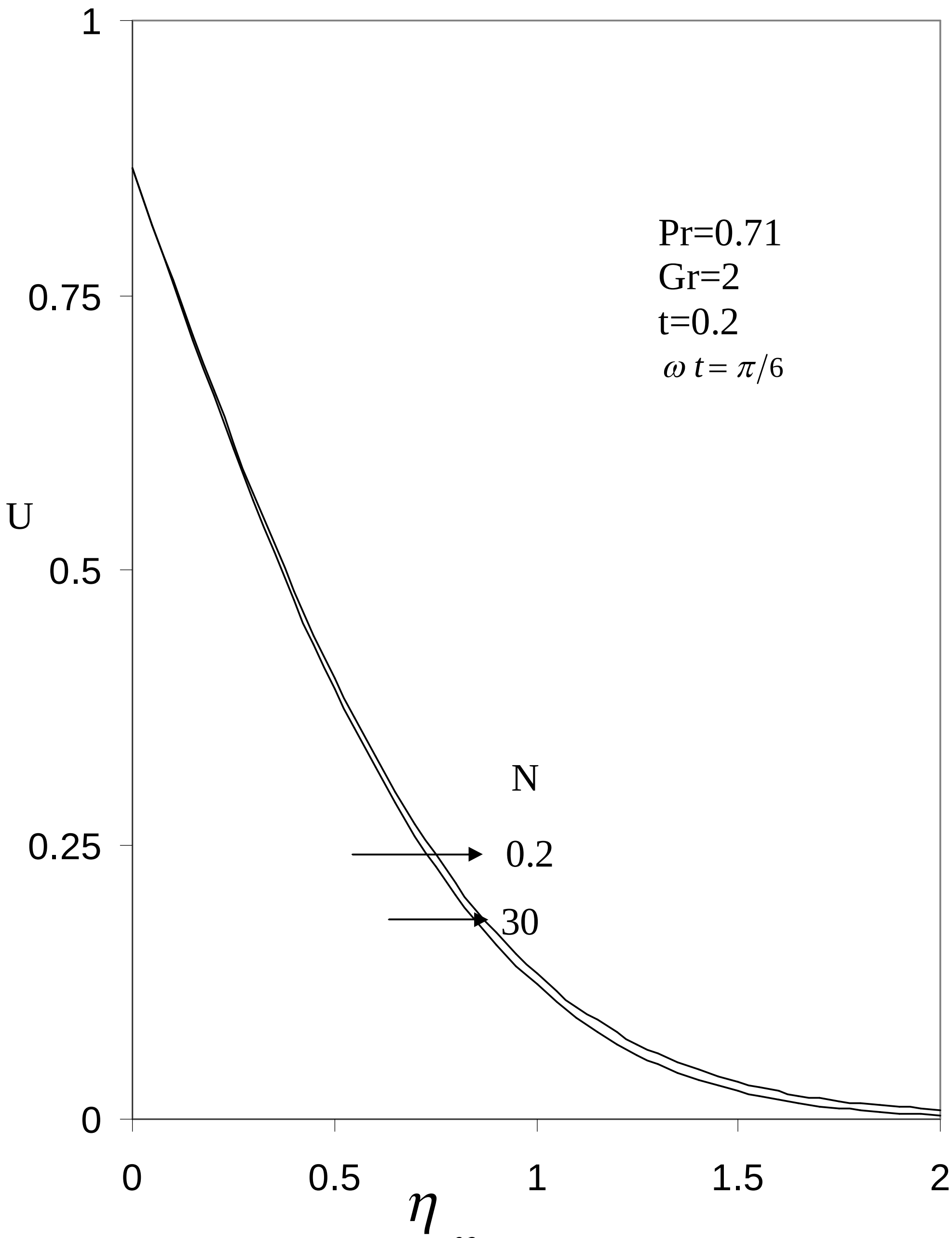

η

Fig.1. Velocity profiles for different N

Fig.1. Velocity profiles for different M

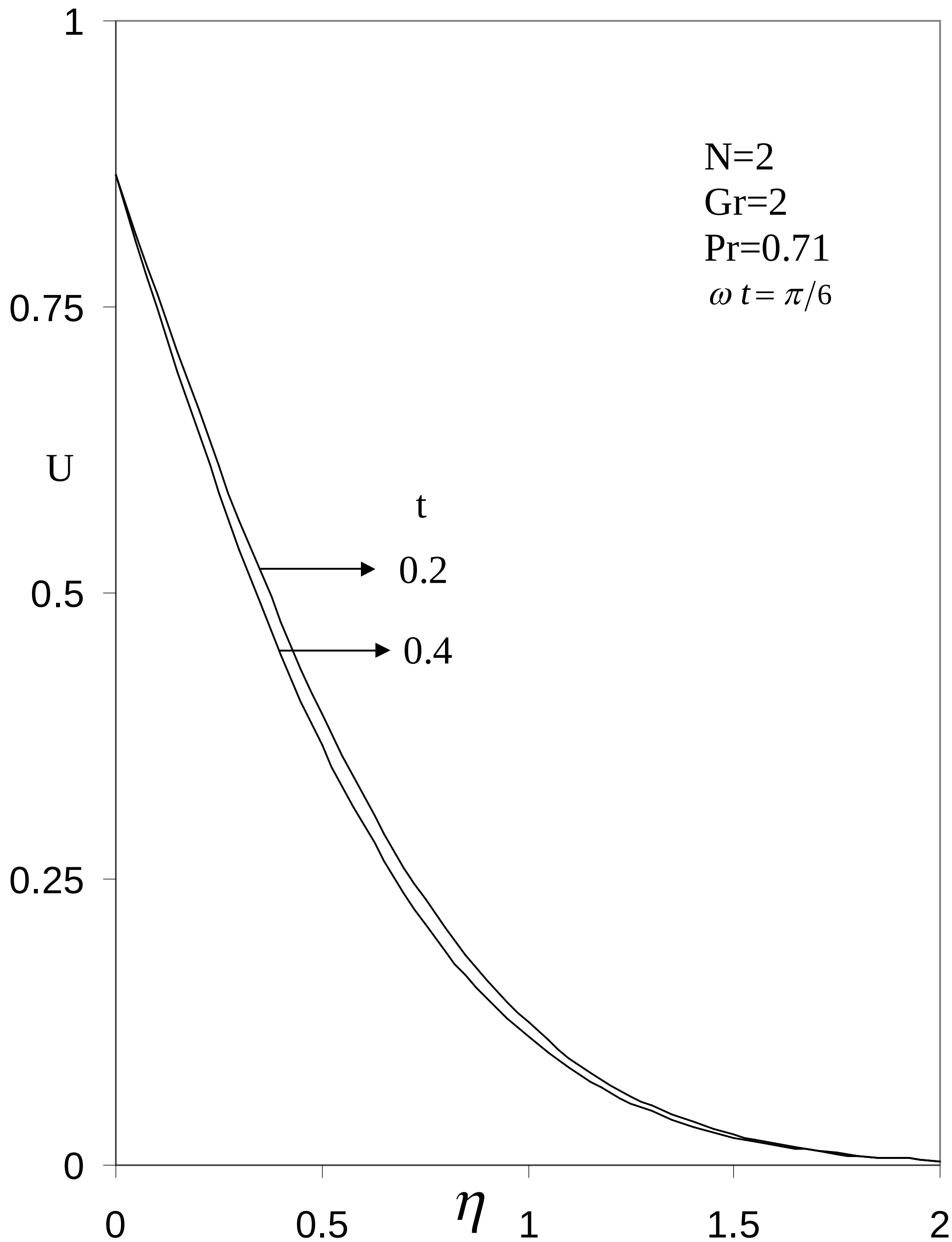

Fig.2. Velocity profiles for different t

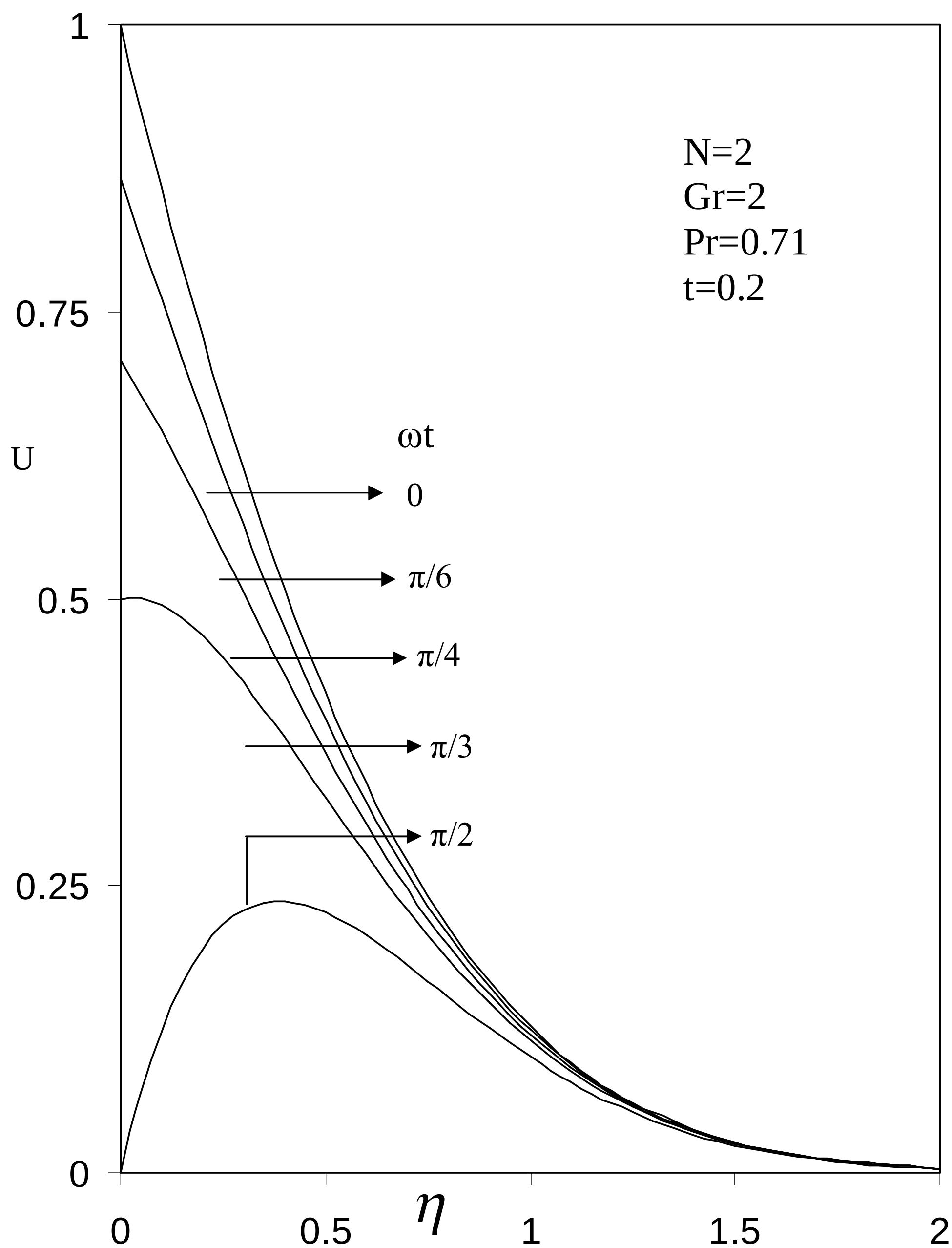

Fig.3. Velocity profiles for different ωt

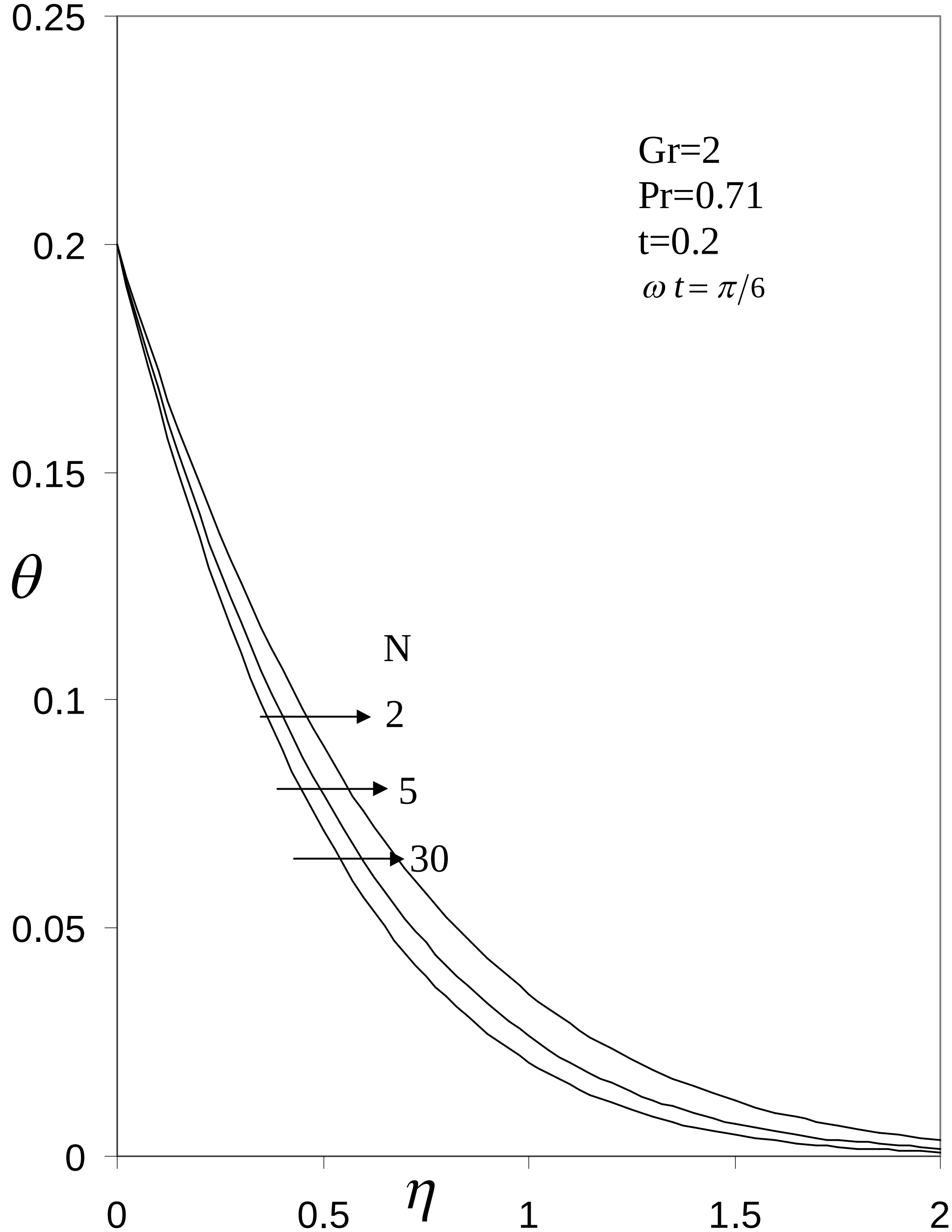

Fig.4. Temperature profiles for different N

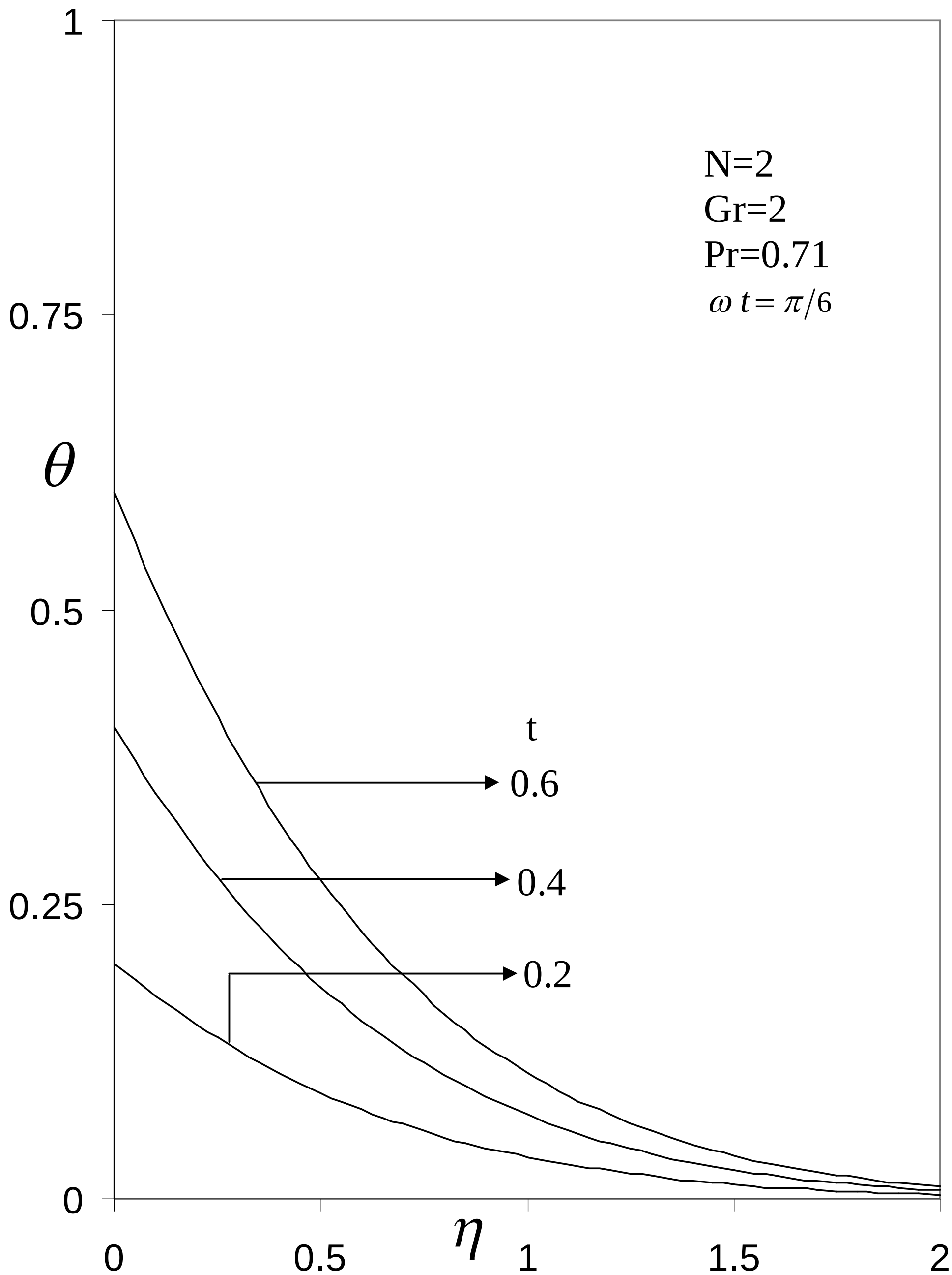

Fig.5. Temperature profiles for different t

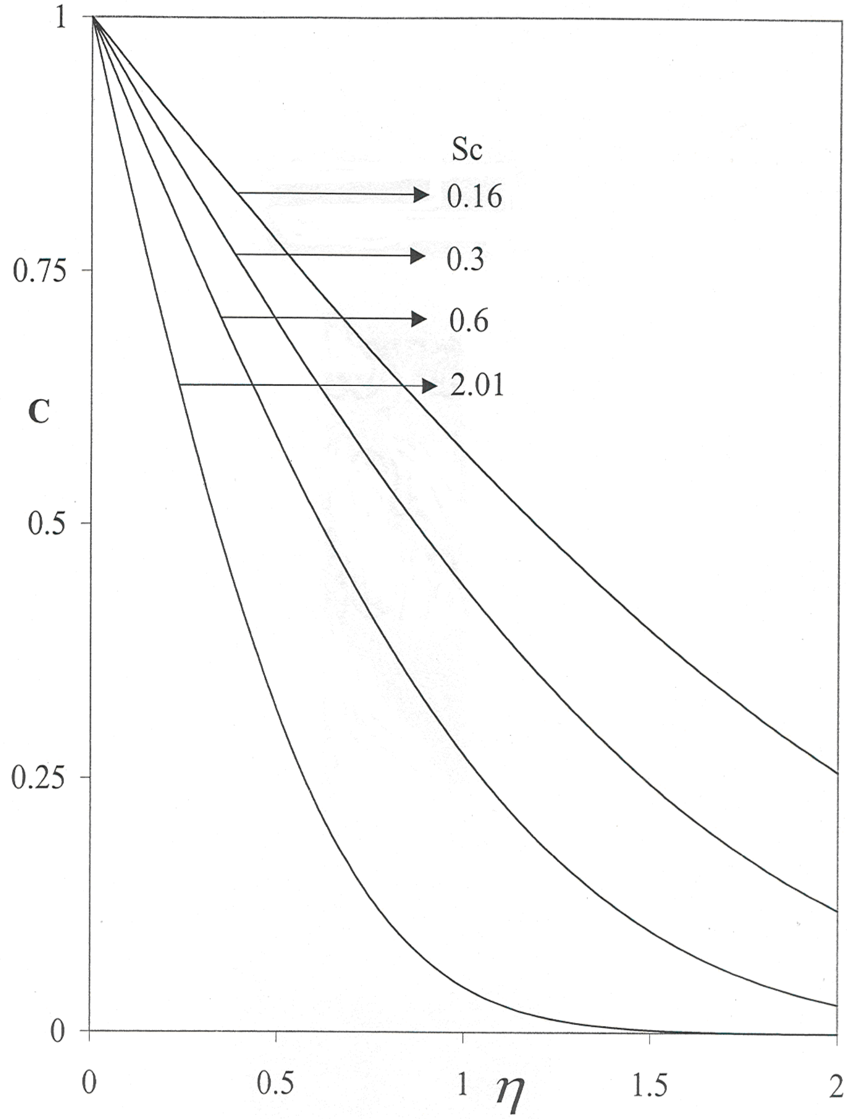

Fig.6. Concentration profiles for different Sc