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RADIATION EFFECTS ON FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH UNIFORM HEAT FLUX

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Abstract

An analysis is performed to study the flow of an incompressible viscous fluid past an exponentially accelerated infinite vertical plate in the presence of thermal radiation with uniform heat flux. The dimensionless governing equations are solved using Laplace-transform technique. The velocity and temperature are studied for different physical parameters like radiation parameter, time and an accelerating parameter a. It is observed that velocity increases with increasing values of a or Gr but decreases with increasing radiation parameter N.

Key words: exponential, accelerated vertical plate, Radiation, heat flux

Nomenclature

A - constant

a' - accelerating parameter

a - dimensionless accelerating parameter

C_p - specific heat at constant pressure

g - acceleration due to gravity

Gr - thermal Grashof number

k - thermal conductivity of the fluid

N - radiation parameter

Pr - Prandtl number

p - pressure

q_r - radiaive heat flux in the y-direction

T - temperature of the fluid near the plate

T_w - temperature of the plate

T_∞ - temperature of the fluid far away from the plate

t' - time

t - dimensionless time

u' - velocity of the fluid in the x-direction

u_0 - velocity of the plate

u - dimensionless velocity

y - coordinate axis normal to the plate

y' - dimensionless coordinate axis normal to the plate

Greek symbols

α - thermal diffusivity

β - volumetric coefficient of thermal expansion

μ - coefficient of viscosity

ν - kinematic viscosity

ρ - density

τ - dimensionless skin-friction

θ - dimensionless temperature

η - similarity parameter

erfc - complementary error function

1. Introduction

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

The problem of unsteady free convection fluid flows are encountered in many industrial problems such as filtration processes, the drying of porous materials in textile industries and the saturation of porous materials by chemicals, nuclear reactors, spacecraft design and solar energy collectors. A few representative fields of interest in which combined heat and mass transfer plays an important role, are design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and pollution of the environment.

Sakiadis [1,2] studied the growth of the two dimensional velocity boundary layer over a continuously moving horizontal plate emerging from a wide slot at uniform velocity. Soundalgekar [3] was the first to present an exact solution for the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the usual Laplace transform technique and the effects of heating or cooling of the plate on the flow field were discussed through Gr. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [4]. The Skin-friction for accelerated vertical plate has been studied analytically by Hossian and Shayo [5].

The object of the present paper is to study the thermal radiation effects on flow past an exponentially accelerated infinite vertical plate with uniform heat flux. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. Mathematical Formulation

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate in the presence of thermal radiation with uniform heat flux is considered. The x'- axis is taken along the plate in the vertical direction and the y'- axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature in a stationary condition. At time t'>0, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a' t')$ in its own plane and the temperature of the plate is raised to T_w. At the same time, the heat is supplied from the plate to the fluid at uniform rate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\,\beta\,(T' - T'_\infty) + \nu\,\frac{\partial^2 u'}{\partial y'^2} \qquad (1)$$

$$\rho C_p\,\frac{\partial T'}{\partial t'} = k\,\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \qquad (2)$$

with the following initial and boundary conditions

$$t' \le 0: \quad u' = 0,\; T' = T'_\infty \qquad \text{for all } y'$$

$$t' > 0: \quad u' = u_0 \exp(at),\; \frac{\partial T'}{\partial y} = -\frac{q}{k} \qquad \text{at } y'=0 \qquad (3)$$

$$u' = 0,\; T' \to T'_\infty \qquad \text{as } y' \to \infty$$

The local radiant for the case of an optically thin gray gas is expressed by

$$q_r = -\frac{4\sigma}{3\kappa^*}\,\frac{\partial T'^4}{\partial y'} \qquad (4)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \qquad (5)$$

By using equations (4) and (5), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_\infty'^3}{3\kappa^*} \frac{\partial^2 T'}{\partial y'^2} \qquad (6)$$

On introducing the following dimensionless quantities

$$u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'}, \; a = \frac{a' \nu}{u_0^2}$$

$$Gr = \frac{g\beta\nu (T_w' - T_\infty')}{u_0^3}, \quad \Pr = \frac{\mu C_p}{k}, \quad N = \frac{\kappa^* k}{4\sigma T_\infty'^3} \qquad (7)$$

in equations (1) to (6), leads to

$$\frac{\partial u}{\partial t} = Gr\,\theta + \frac{\partial^2 u}{\partial y^2} \qquad (8)$$

$$3\,N\,\Pr \frac{\partial \theta}{\partial t} = (3N + \Pr) \frac{\partial^2 \theta}{\partial y^2} \qquad (9)$$

The initial and boundary conditions in non-dimensionless form are

$$u = 0, \; \theta = 0, \qquad \text{for all } y, \; t' \le 0$$

$$t' > 0: \; u = u = \exp(at), \; \frac{\partial \theta}{\partial y} = -1 \quad \text{at} \quad y = 0 \qquad (10)$$

$$u = 0, \; \theta \to 0, \qquad \text{as} \quad y \to \infty$$

The equations (8) and (9), subject to the boundary conditions (10), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$\theta = 2\sqrt{t}\left[\frac{\exp(-\eta^2 a)}{\sqrt{\pi}} - \eta\sqrt{a}\,\mathrm{e}rfc(\eta\sqrt{a})\right] \qquad (11)$$

$$u = \frac{e^{at}}{2}\left[\bar{e}^{2\eta\sqrt{at}} erfc(\eta - \sqrt{at}) + e^{2\eta\sqrt{at}} erfc\;(\eta + \sqrt{at})\right]$$
$$+ \frac{Gr\; t^{3/2}}{3\sqrt{a}(a-1)}\left[\begin{array}{l}\frac{4}{\sqrt{\pi}}(1+\eta^2)\;\mathrm{e}^{-\eta^2} - \eta\,(6+4\eta^2)\,erfc(\eta) \\ -\frac{4}{\sqrt{\pi}}(1+\eta^2 a)\;\mathrm{e}^{-\eta^2 a} + \eta\sqrt{a}\,(6+4\eta^2 a)\,erfc(\eta\sqrt{a})\end{array}\right] \qquad (12)$$

where $a = \dfrac{3N\,\mathrm{Pr}}{3N+4}$ and $\eta = \dfrac{y}{2\sqrt{t}}$

3. Results and Discussion

For physical understanding of the problem numerical computations are carried out for different parameters a, N and t upon the nature of the flow and transport. The values of Prandtl number Pr are chosen such that they represent air (Pr=0.71) and water (Pr=7.0). The numerical values of the velocity and temperature are computed for different physical parameters like a, Prandtl number, radiation parameter N and time.

The velocity profiles for different values of the radiation parameter (N = 0.2, 30) are shown in Fig.1. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.
The variations of velocity profiles for different values of time (t = 0.2, 0.4, 0.6), a=0.5, Gr=5 and Pr = 0.71 are shown in Fig.2. It is clear that the velocity increases with increasing values of the time.

The effects of velocity for different values of (a = 0.2, 0.5, 0.8), Gr=5, Pr = 0.71 at t=0.2 are studied and presented in Fig.3. It is observed that the velocity increases with increasing values of a.

The temperature profiles are calculated for different values of thermal radiation parameter (N = 2, 5, 30) from equation (11) and these are shown in Fig.4, for air (Pr = 0.71). The effect of thermal radiation parameter is important in temperature profiles. It is observed that temperature increases with decreasing radiation parameter.

In Fig.5, the temperature profiles are shown for different values of t. It is observed that temperature increases with increasing time.

The temperature profiles are calculated for different values of the Prandtl number Pr = 0.71 and Pr=7.0 are shown in Fig.6. It is observed that temperature increases in the presence of air than in water.

From the velocity field, the effect of heat transfer on the skin-friction is studied and is given in dimensionless form as

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0} \tag{10}$$

Hence, from the equations (10) and (12), wall shear stress is as follows.

$$\tau = \frac{1}{\sqrt{\pi t}}\left[e^{at}\;1+\sqrt{\pi a t}\,erf\sqrt{at}\; - \frac{Gr\,t^{3/2}\sqrt{\pi}}{\sqrt{a}\,(\sqrt{a}+1)}\right]$$

The numerical values of τ are presented in the Table 1. It is observed that from this Table, skin-friction increases with increasing values of the accelerating parameter. and time. It is also observed that the skin-friction decreases with increasing Grashof number.

Table 1: Values of the non-dimensional skin-friction

Gr	a	t	Pr=0.71	Pr=7.0
2	2	0.2	2.4883	2.4965
5	2	0.2	2.4552	2.4756
10	2	0.2	2.4001	2.4409
2	2	0.4	3.9510	3.9971
2	0	0.2	-0.0220	-0.1390
2	5	0.2	7.9899	7.9981

4. Conclusions

The theoretical solution of flow past an exponentially accelerated infinite vertical plate in the presence of thermal radiation is considered. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like radiation parameter, a and t are studied graphically. It is observed that the velocity increases with increasing values of a and t.

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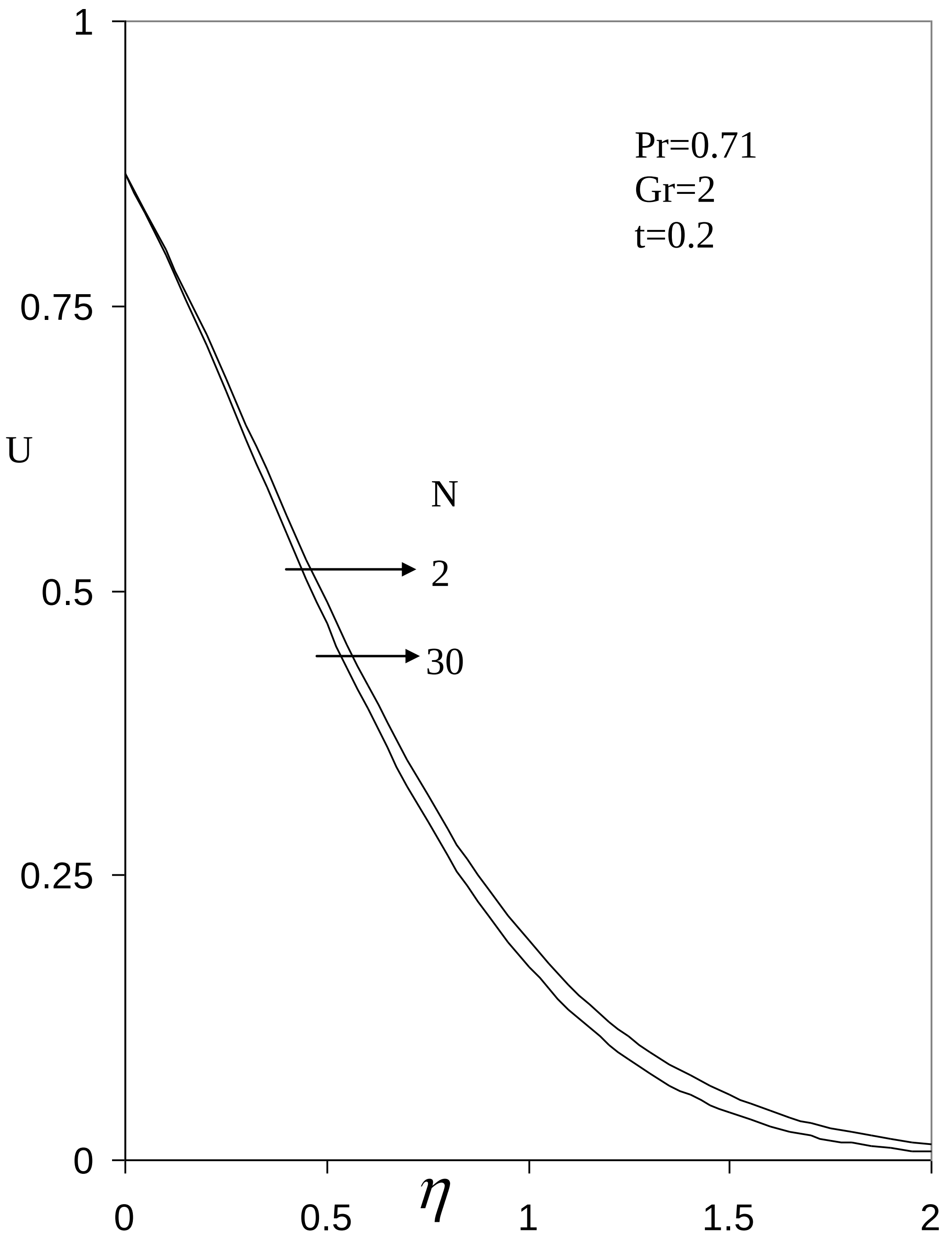

Figure 1: Velocity profiles for different N

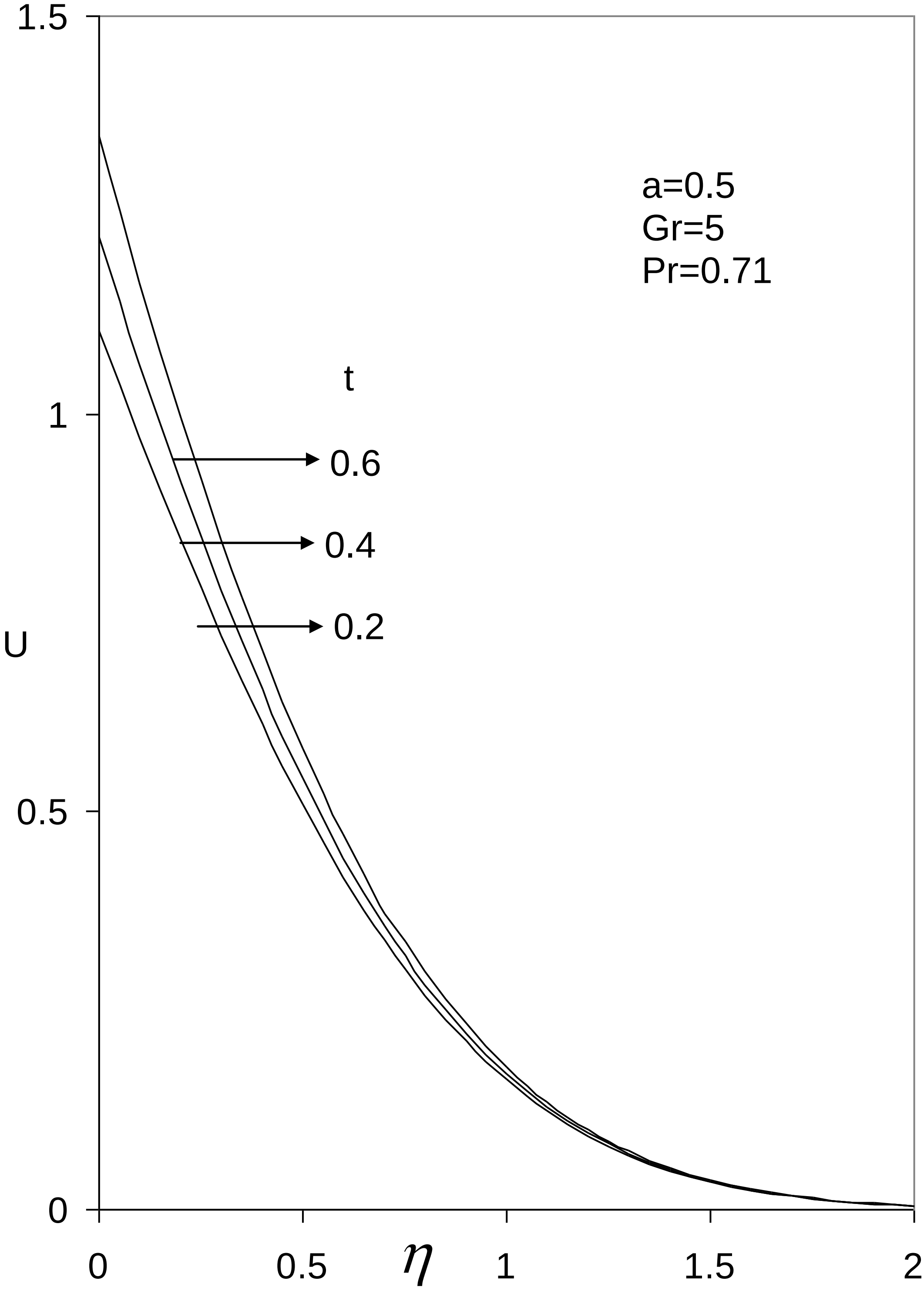

Figure 2: Velocity profiles for different t

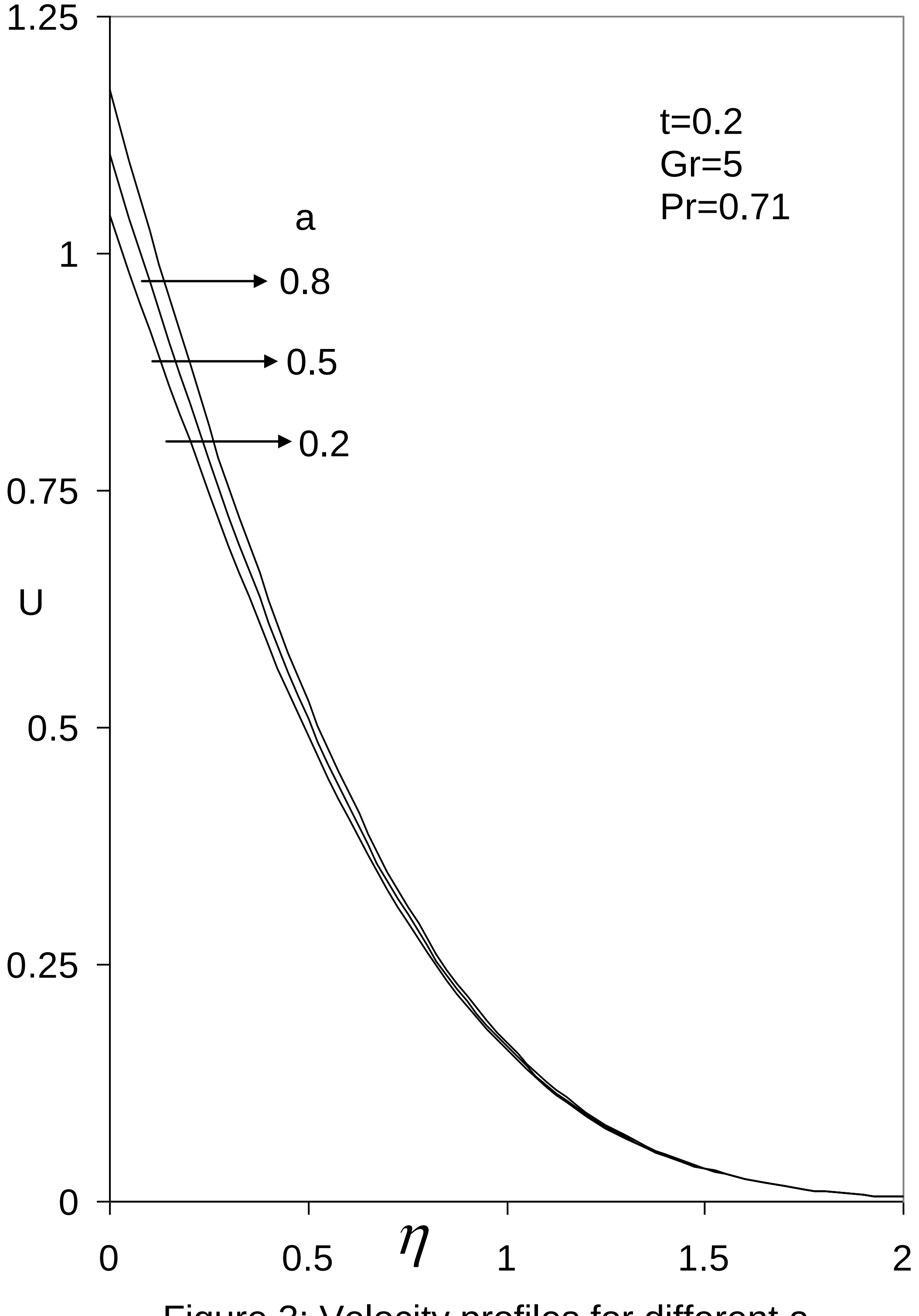

Figure 3: Velocity profiles for different a

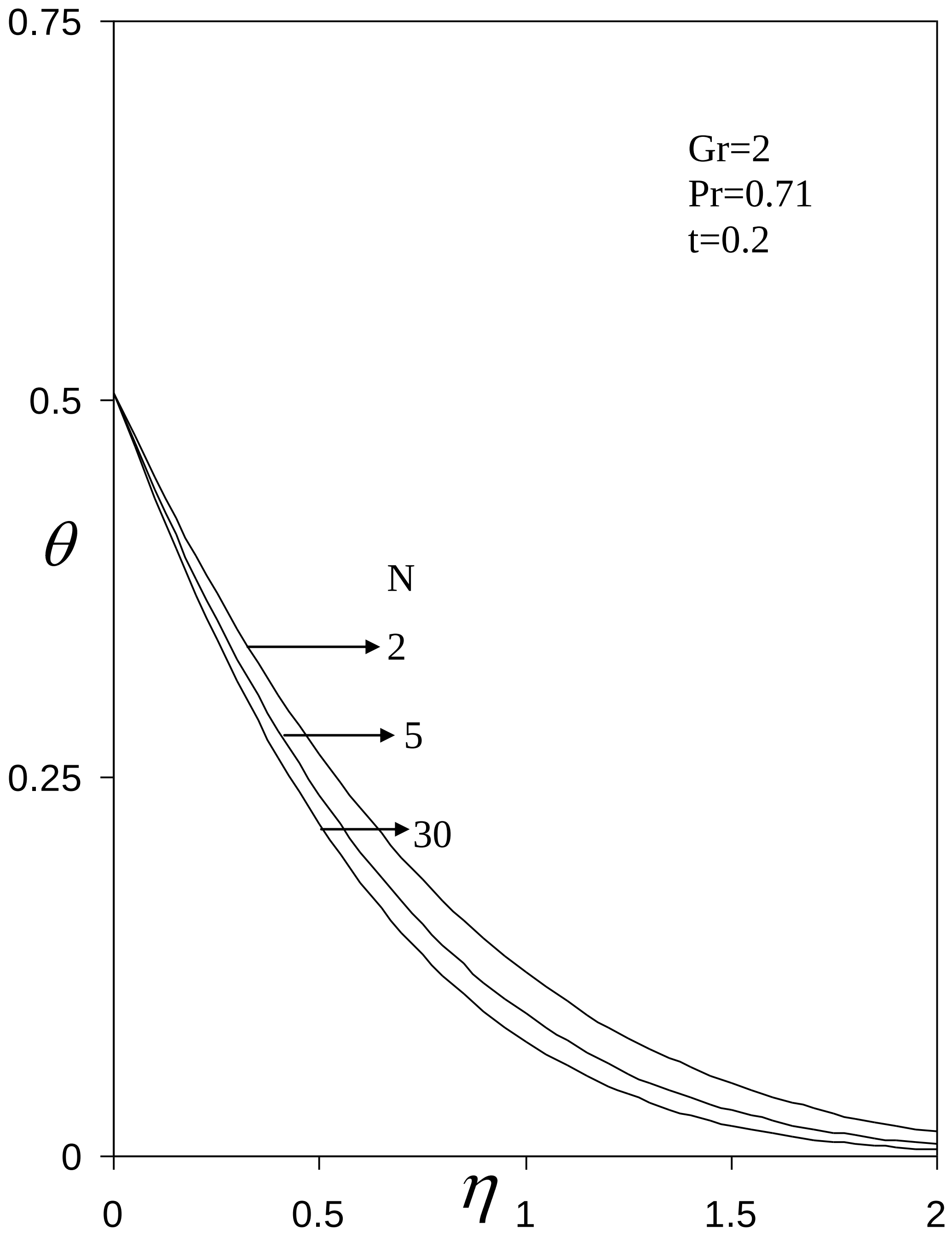

Figure 4: Temperature profiles for different N

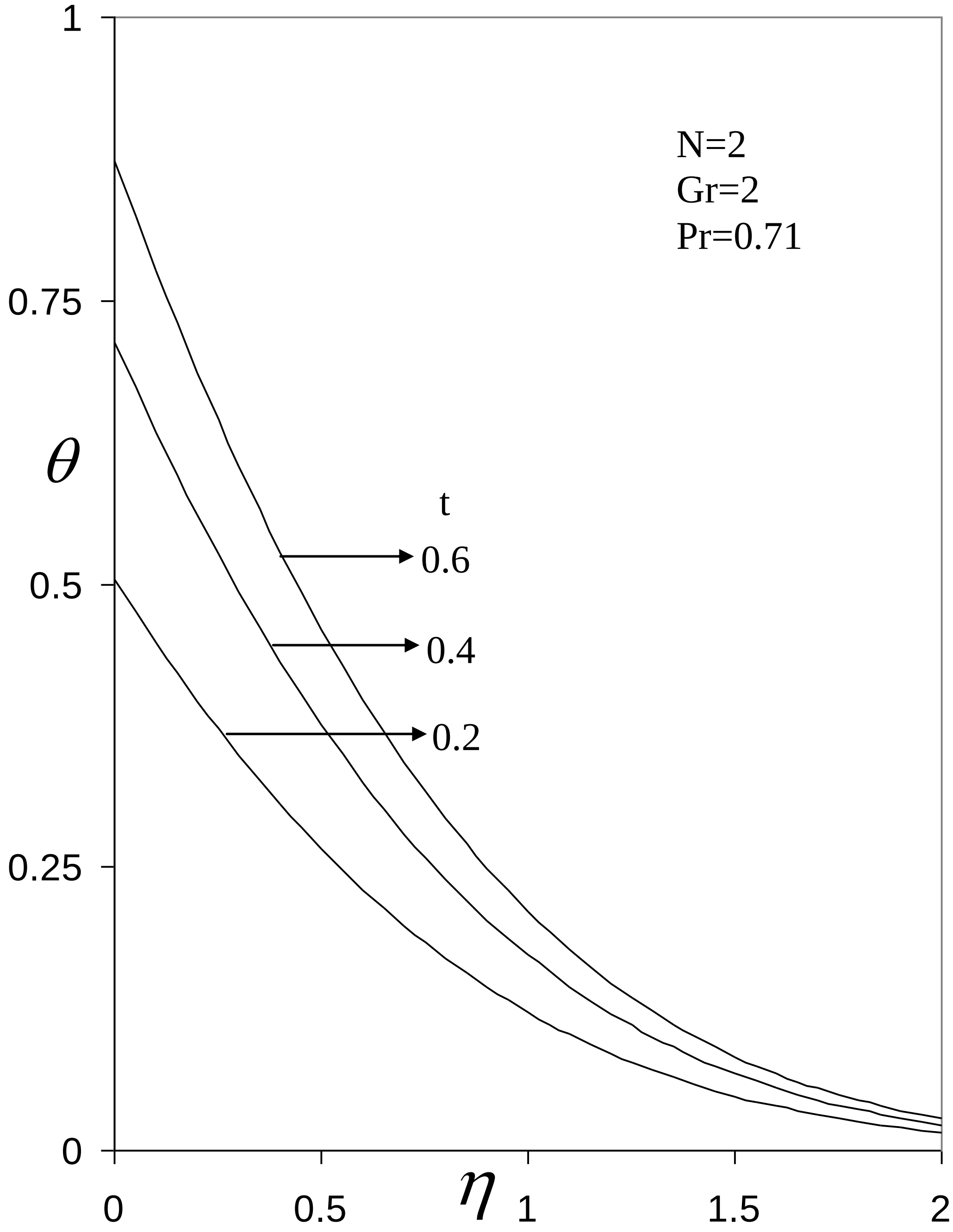

Figure 5: Temperature profiles for different t

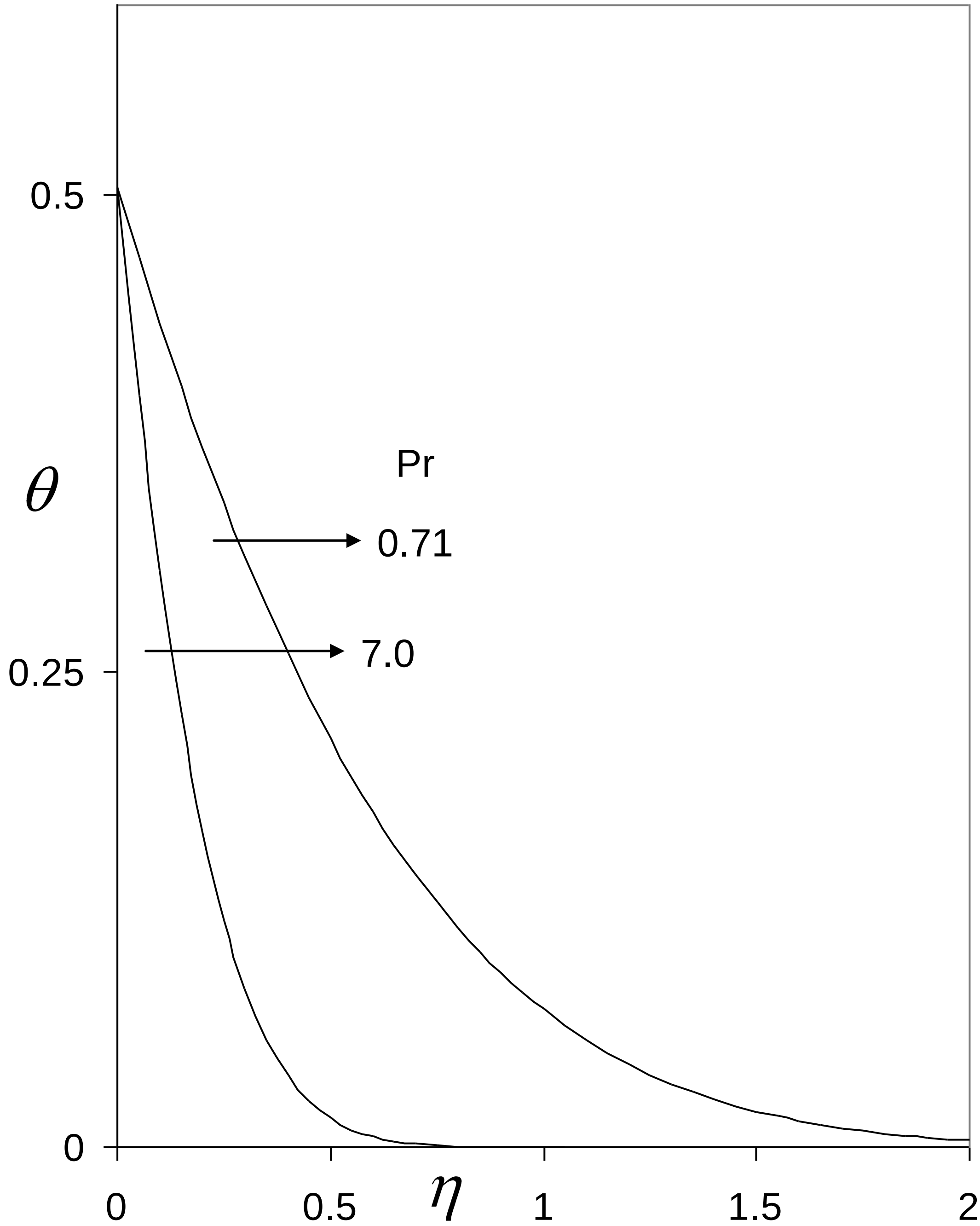

Figure 6: Temperature profiles for different Pr