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RADIATION EFFECTS ON UNSTEADY FLOW PAST AN IMPULSIVELY STARTED VERTICAL PLATE WITH HEAT FLUX IN A ROTATING FLUID

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Abstract

Theoretical study of thermal radiation effects on unsteady free convective flow over a moving vertical plate with uniform heat flux in a rotating fluid is considered. An exact solution is obtained for the axial and transverse components of the velocity by defining a complex velocity. The effects of rotation, radiation, free convection parameters and the skin friction components on the plate are discussed.

Keywords: Free-convection; thermal radiation; rotating fluid; heat flux; vertical plate.

List of symbols

Latine symbols

a* absorption coefficient
C_p specific heat at constant pressure
g acceleration due to gravity
Gr thermal Grashof number
k thermal conductivity of the fluid
Pr Prandtl number
q_r radiative heat flux in the y-direction
N radiation parameter
T' temperature of the fluid near the plate
T_w' temperature of the plate
T_∞' temperature of the fluid far away from the plate

t' time
t dimensionless time
u' velocity of the fluid in the x'-direction
u_0 velocity of the plate
u dimensionless velocity
v' velocity of the fluid in the y'-direction
v dimensionless velocity
y' coordinate axis normal to x'-axis
z' coordinate axis normal to the plate
z dimensionless coordinate axis normal to the plate

Greek symbols

β volumetric coefficient of thermal expansion
μ coefficient of viscosity
ν kinematic viscosity
Ω' rotation parameter
Ω dimensionless rotation parameter
ρ density
σ stefan-Boltzmann constant
τ dimensionless skin-friction
θ dimensionless temperature
erfc complementary error function

1 Introduction

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

Arpaci [2] studied the interaction between thermal radiation and laminar convection of heated vertical plate in a stagnant radiating gas. England and Emery [3] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. In all above studies, the stationary plate was considered. Singh [4] studied the effects of coriolis as well as magnetic force on the flow field of an electrically conducting fluid past an impulsively started infinite vertical plate. Bestman and Adjepong [5] studied the magnetohydrodynamic free convection flow, with radiative heat transfer, past an infinite moving plate in a rotating incompressible viscous and optically transparent medium. Das et al [6] have analyzed radiation effects on flow past an impulsively started infinite isothermal

vertical plate. Raptis and Perdikis [7] have studied the effects of thermal radiation and free convection flow past a moving infinite vertical plate. The governing equations were solved analytically.
However the effect of thermal radiation on moving infinite vertical plate with uniform heat flux, in a rotating fluid is not studied in the literature. It is proposed to study the thermal radiation effects on flow past an impulsively started infinite vertical plate with uniform heat flux, in a rotating fluid. The dimensionless governing equations are solved by Laplace transform technique.

2 Basic equations and analysis

Consider the three dimensional flow of a viscous incompressible fluid past an impulsively started infinite vertical plate with uniform heat flux in a rotating fluid. On this plate, x' - axis is taken along the plate in the vertically upward direction and the y' - axis is taken normal to x' - axis in the plane of the plate and z' - axis is normal to it. Both the fluid and the plate are in a state of rigid rotation with uniform angular velocity Ω' about the z' - axis. Initially, the plate and fluid are at the same temperature. At time $t'>0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with constant velocity u_0. At the same time, the heat is supplied from the plate to the fluid at uniform rate. Since the plate occupying the plane $z'=0$ is of infinite extent, all the physical quantities depend only on z' and t'. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = g\,\beta\left(T' - T'_\infty\right) + \nu\,\frac{\partial^2 u'}{\partial z'^2}, \tag{1}$$

$$\frac{\partial v'}{\partial t'} - 2\Omega' u' = \nu\,\frac{\partial^2 v'}{\partial z'^2}, \tag{2}$$

$$\rho C_p\,\frac{\partial T'}{\partial t'} = k\,\frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \tag{3}$$

The term $\frac{\partial q_r}{\partial z'}$ represents the change in the radiative flux with distance normal to the plate with the following initial and boundary conditions:

$$t' \le 0: \quad u' = 0,\ T' = T'_\infty \qquad \text{for all } z'$$

$$t' > 0: \quad u' = u_0,\ \frac{\partial T'}{\partial z} = -\frac{q}{k} \qquad \text{at } z' = 0 \tag{4}$$

$$u' = 0, \quad T' \to T'_\infty \qquad \text{as } z' \to \infty$$

By Rosseland approximation, radiative heat flux of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z'} = -\, 4a^* \sigma (T'^4_\infty - T'^4) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4\, T'^3_\infty T' - 3T'^4_\infty \tag{6}$$

By using equations (5) and (6), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + 16 a^* \sigma T'^3_\infty \left(T'_\infty - T' \right) \tag{7}$$

On introducing the following non-dimensional quantities

$$(u, v) = \frac{(u', v')}{u_0}, \quad t = \frac{t' u'^2_0}{\nu}, \quad z = \frac{z' u_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$
$$Gr = \frac{g \beta \nu (T'_w - T'_\infty)}{u'^3_0}, \quad \Pr = \frac{\mu C_p}{k}, \quad \Omega = \frac{\Omega' \nu}{u'^2_0}, \quad N = \frac{16 a^* \nu^2 \sigma T'^3_\infty}{k u'^2_0} \tag{8}$$

and the complex velocity $q = u + iv,\ i = \sqrt{-1}$ in equations (1) to (4), the equations relevant to the problem reduces to

$$\frac{\partial q}{\partial t} + 2i\Omega = Gr\, \theta + \frac{\partial^2 q}{\partial z^2} \tag{9}$$

$$3\, N \Pr \frac{\partial \theta}{\partial t} = (3N + \Pr) \frac{\partial^2 \theta}{\partial z^2} \tag{10}$$

The initial and boundary conditions in non-dimensional form are

$$q(z,t) = 0, \ \ \theta(z,0) = 0, \qquad \text{for all } z,\ t \le 0$$

$$t > 0: \ q(z,t) = 1, \quad \frac{\partial \theta}{\partial z} = -1, \qquad \text{at } z = 0 \tag{11}$$

$$q(z,t) = 0, \ \ \theta(z,t) \to 0, \qquad \text{as } z \to \infty$$

All the physical variables are defined in the nomenclature. The solutions are obtained for the equations (9) to (10), subject to the boundary conditions (11), by Laplace-transform technique and the solutions are derived as follows.

$$\theta = 2\sqrt{t}\left[\frac{\exp(-\eta^2 a)}{\sqrt{\pi}} - \eta\sqrt{a}\,erfc(\eta\sqrt{a})\right] \tag{12}$$

$$\begin{aligned}
u = {} & \frac{1}{2}\left[\exp(-z\sqrt{m})\,erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{mt}\right)+\exp(z\sqrt{m})\,erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{mt}\right)\right] \\
& + \frac{Gr\,(1+bt)}{2b^2(1-a)}\left[\exp(-z\sqrt{m})\,erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{mt}\right)+\exp(z\sqrt{m})\,erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{mt}\right)\right] \\
& - \frac{Gr\,z}{4b\,(1-a)\sqrt{m}}\left[\exp(-z\sqrt{m})\,erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{mt}\right)-\exp(z\sqrt{m})\,erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{mt}\right)\right] \\
& - \frac{Gr\exp(bt)}{2b^2(1-a)}\left[\exp(-z\sqrt{ab})\,erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{abt}\right)+\exp(z\sqrt{ab})\,erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{abt}\right)\right] \\
& - \frac{Gr}{b^2(1-a)}erfc\left(\frac{z\sqrt{a}}{2\sqrt{t}}\right) - \frac{Gr\,t}{b(1-a)}\left[\left(1+\frac{z^2a}{2t}\right)erfc\left(\frac{z\sqrt{a}}{2\sqrt{t}}\right)-\frac{z\sqrt{a}}{\sqrt{\pi t}}\exp\left(-\frac{z^2a}{4t}\right)\right] \\
& + \frac{Gr\exp(bt)}{2b^2(1-a)}\left[\exp(-z\sqrt{ab})\,erfc\left(\frac{z\sqrt{a}}{2\sqrt{t}}-\sqrt{bt}\right)+\exp(z\sqrt{ab})\,erfc\left(\frac{z\sqrt{a}}{2\sqrt{t}}+\sqrt{bt}\right)\right]
\end{aligned} \tag{13}$$

where $a = \dfrac{3N\ \Pr}{3N+4}$, $b = \dfrac{m}{a-1}$ and $m = 2i\Omega$

Using equation (13) we get the following expression for skin-friction components τ_X and τ_Y.

$$\tau_X + i\,\tau_Y = -\left(\frac{\partial q}{\partial z}\right)_{z=0}$$

$$\begin{aligned}
\tau_X + i\,\tau_Y = {} & \frac{1}{\sqrt{\pi t}}\left(1+\frac{Gr(1+bt)}{(1-a)b^2}\right)\left(1+\sqrt{m\pi t}\,erf(\sqrt{mt})\right) \\
& + \frac{Gr}{2b(1-a)\sqrt{m\pi t}}erf(\sqrt{mt}) - \frac{Gr\sqrt{a}\,(1+2bt)}{(1-a)b^2\sqrt{\pi t}} \\
& - \frac{Gr\,e^{bt}}{(1-a)b^2\sqrt{\pi t}}\left(1+\sqrt{\pi abt}\,erf(\sqrt{abt})\right) \\
& + \frac{Gr\,e^{bt}}{(1-a)b^2\sqrt{\pi t}}\left(\sqrt{a}+\sqrt{\pi abt}\,erf(\sqrt{abt})\right)
\end{aligned} \tag{14}$$

In equations (13) and (14), the argument of the complementary error function and error function is complex. Hence in order to obtain the u and v components of the velocity and skin-friction, we have used the following formula due to Abramowitz and Stegun [1].

$$erf\,(a+ib) = erf\,(a) + \frac{\exp(-a^2)}{2a\pi}\,[1-\cos(2ab)+i\sin(2ab)]$$
$$+\frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}\,[\,f_n(a,b)+ig_n(a,b)]$$
$$+\,\varepsilon(a,b)$$

Where

$$f_n = 2a - 2a\cosh(nb)\cos(2ab) + n\,\sinh(nb)\,\sin(2ab)$$
$$g_n = 2a\cosh(nb)\sin(2ab) + n\,\sinh(nb)\,\cos(2ab)$$
$$and\ \ |\varepsilon(a,b)| \approx 10^{-16}\ |erf\,(a+ib)|$$

3 Discussion of results

Using the above formula, expressions for u, v and τ_x and τ_y are obtained but they are omitted here to save the space. In order to get a physical view of the problem, these expressions are used to obtain the numerical values of u, v, τ_x and τ_y for different values of the various parameter like rotation, radiation and thermal Grashof number.

The primary velocity profiles of air for different values of the radiation parameter are shown in Fig.1. It is observed that the primary velocity increases with decreasing radiation parameter N in cooling of the plate. This shows that primary velocity decreases in the presence of high thermal radiation. It is also observed that greater cooling of the plate, due to free convection currents, increases the primary velocity of the plate.

The primary velocity profiles of air for different values of the rotation parameter are shown in Fig.2. It depicts that the primary velocity increases with decreasing rotation parameter Ω. It is also observed that greater cooling of the plate, due to free convection currents, increases the primary velocity of the plate in this case too.

The secondary velocity profiles of air for different values of the radiation parameter are shown in Fig.3, the effect of radiation increases the secondary velocity v.

Fig.4. shows the effect of rotation on v which is just reverse to that of radiation parameter. Further, greater cooling of the plate, due to free-convection currents, decreases the secondary velocity of the plate.

The temperature profiles for air (Pr = 0.71) are calculated for different values of thermal radiation parameter (N = 2, 5, 30) from equation (12) and these are shown in Fig.5. The effect of thermal radiation parameter is important in temperature profiles. It is observed that temperature increases with decreasing radiation parameter.

In Fig.6, the temperature profiles are shown for different values of t. It is observed that temperature increases with increasing time.

The skin-friction components for air (Pr = 0.71) at the wall for different values of Gr, t, N and Ω are shown in Table 1.

Table 1 Skin-friction τ

Gr	Ω	t	N	τ_x	τ_y
5	0.5	0.2	0.2	0.6905	-0.3349
			2.0	0.7655	0.1033
	2.0	0.2	0.2	1.2054	1.8277
			2.0	1.2836	1.9307
	0.5	0.4	0.2	-0.7041	-0.3136
10	0.5	0.2	0.2	0.0858	-1.1724
			2.0	0.2359	-0.2959

The effect of radiation increases the value of both the components of skin-friction. It is observed that the effect of rotation on skin-friction increases both the components τ_x and τ_y. As time advances the component τ_x decreases, but reverse phenomenon occurs for the component τ_y. Greater cooling of the plate, due to free-convection currents, lowers τ_x and τ_y.

4 Conclusions

An exact analysis is performed to study the thermal radiation effects on flow past an impulsively started infinite vertical plate with uniform heat flux in a rotating fluid. The dimensionless governing equations are solved by Laplace-transform technique. The conclusions of the study are as follows.

(i) The influence of the radiation or rotation parameter on primary flow has a retarding effect for cooling of the plate.

(ii) The secondary velocity is enhanced with the raise in thermal radiation and opposite phenomenon occurs with the rotation parameter.

(iii) The skin-friction components increases with increasing radiation parameter. The effect of rotation on τ_x and τ_y increases as rotation parameter increases.

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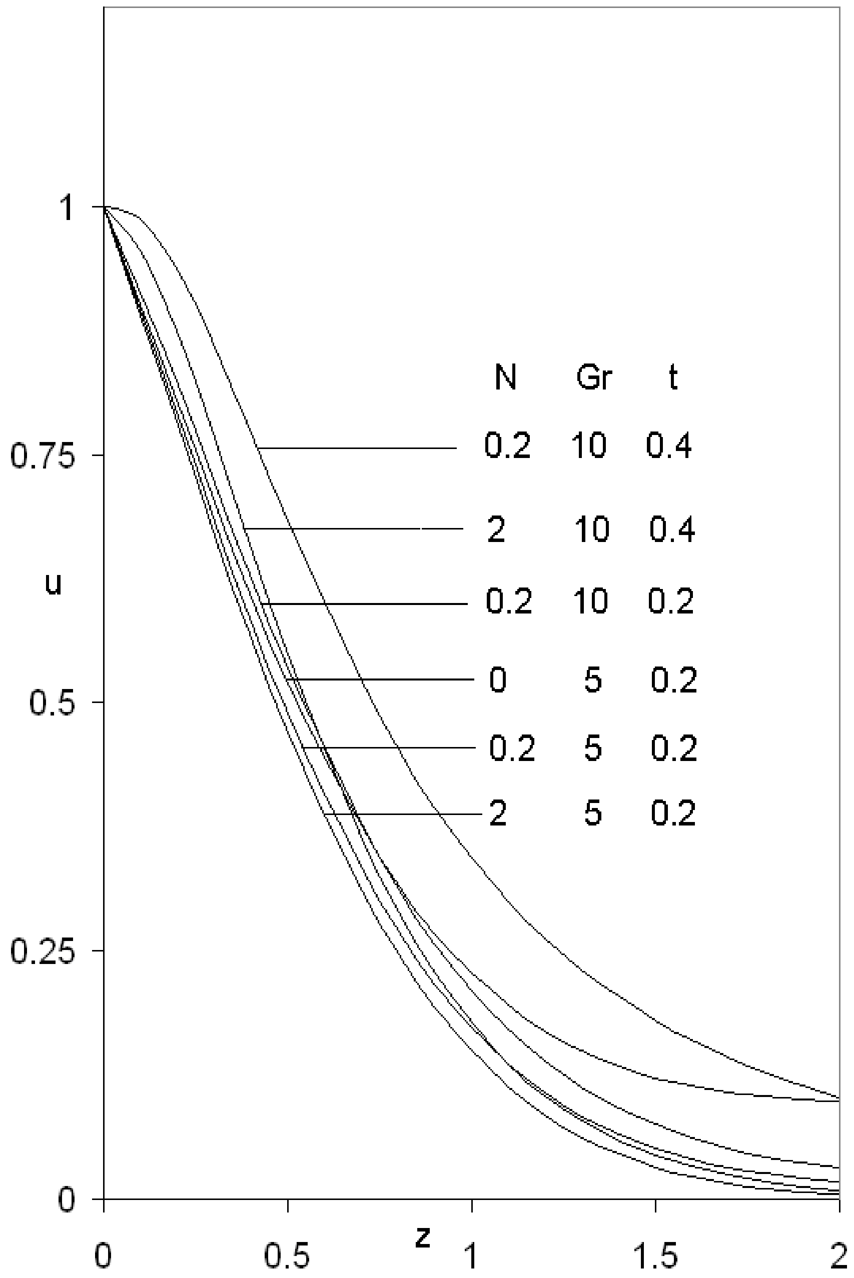

Figure.1: Primary velocity profiles for different N

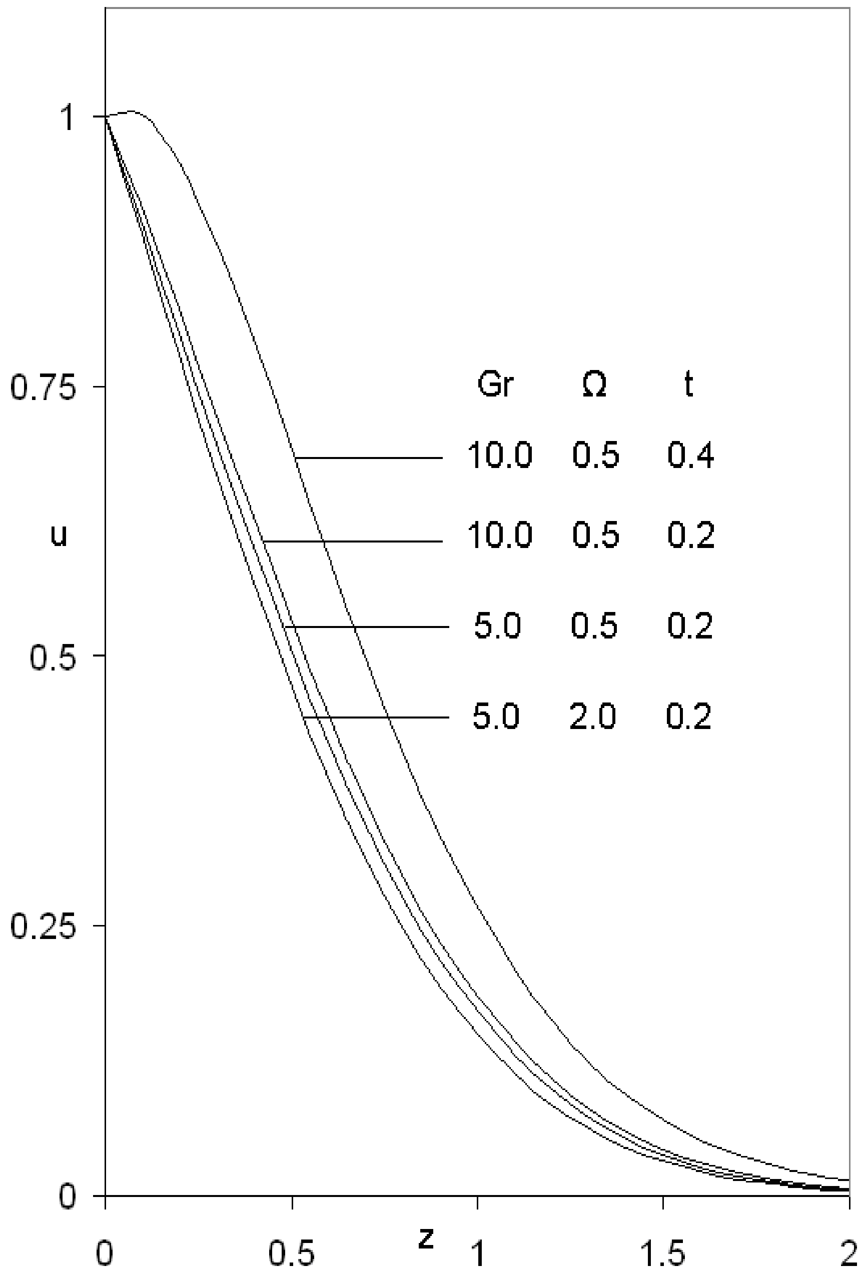

Figure.2: Primary velocity profiles for different Ω

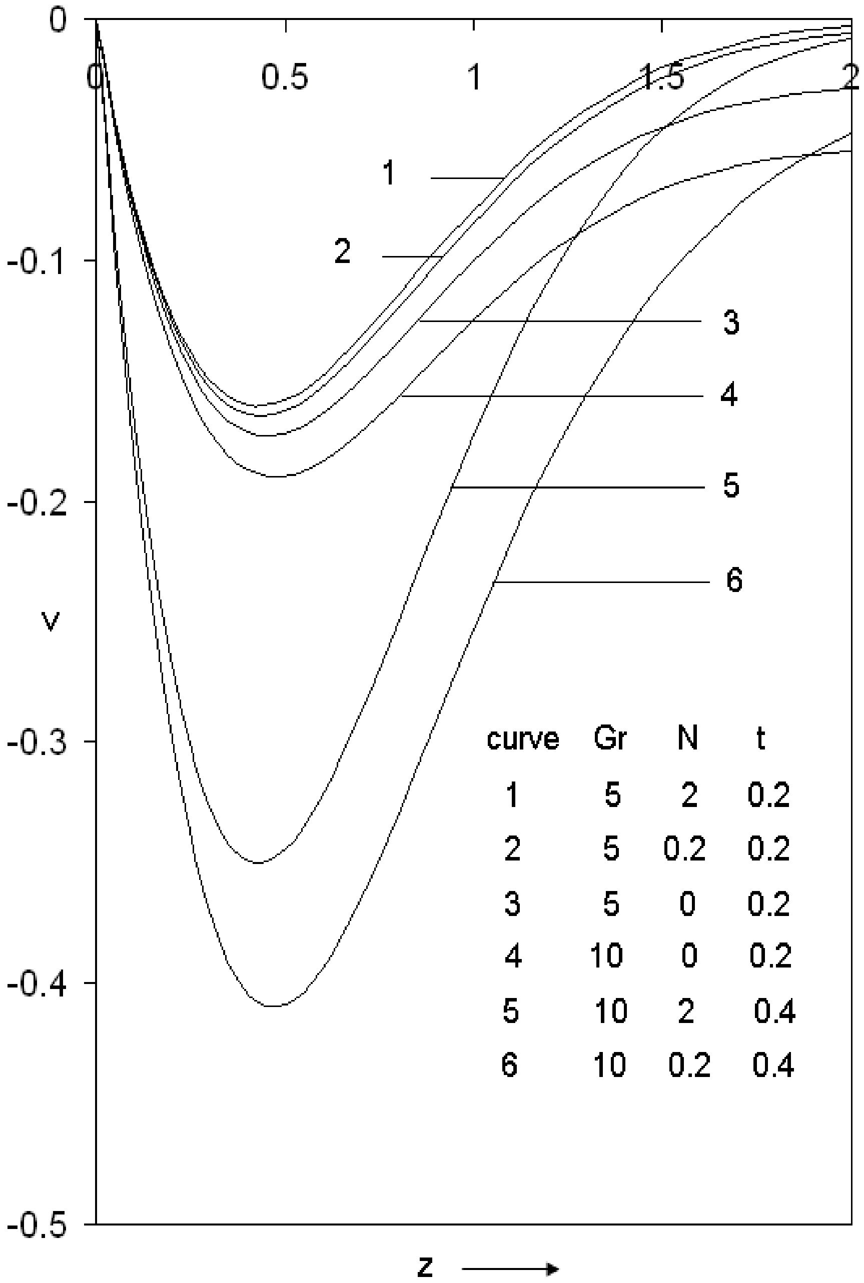

Figure.3: Secondary velocity profiles for different R

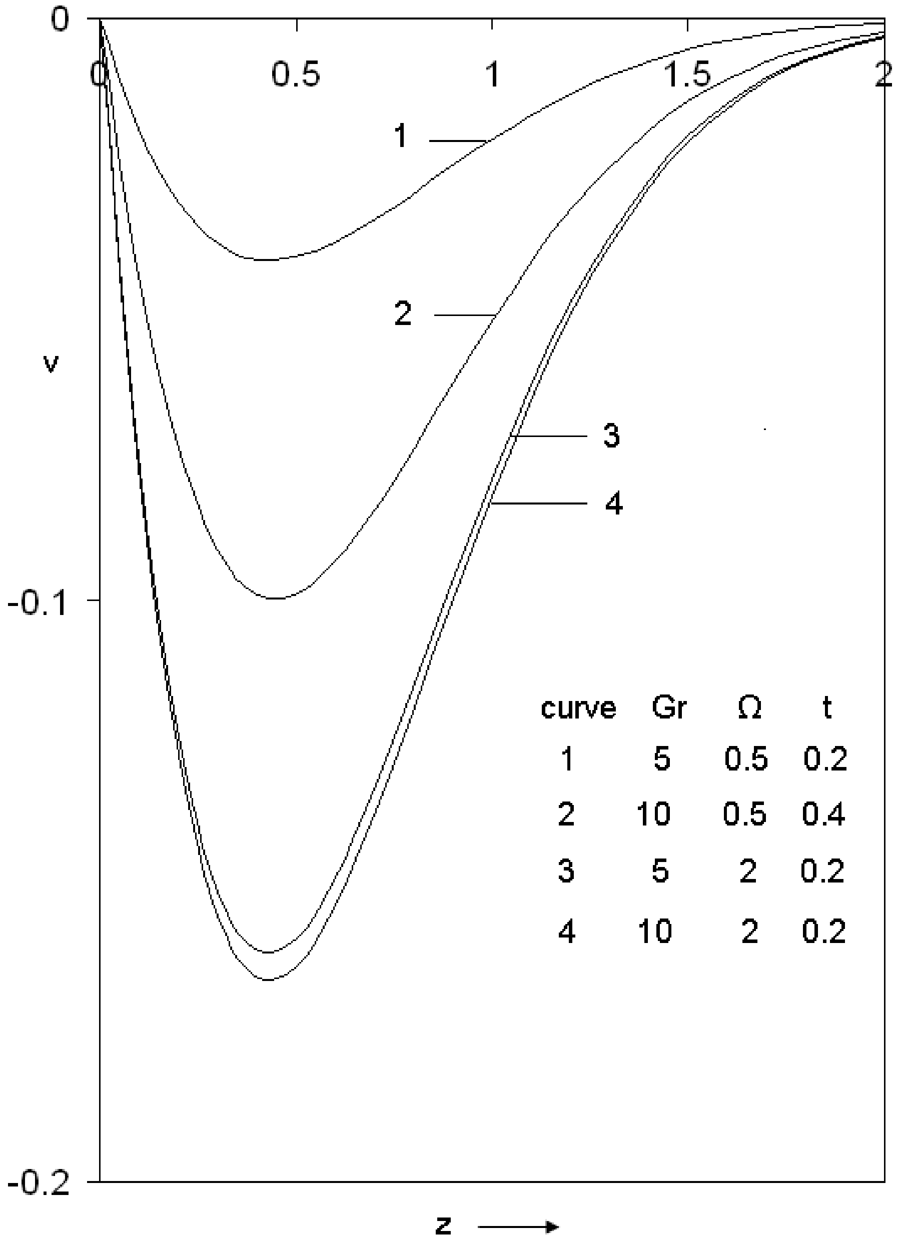

Figure.4: Secondary velocity profiles for different Ω

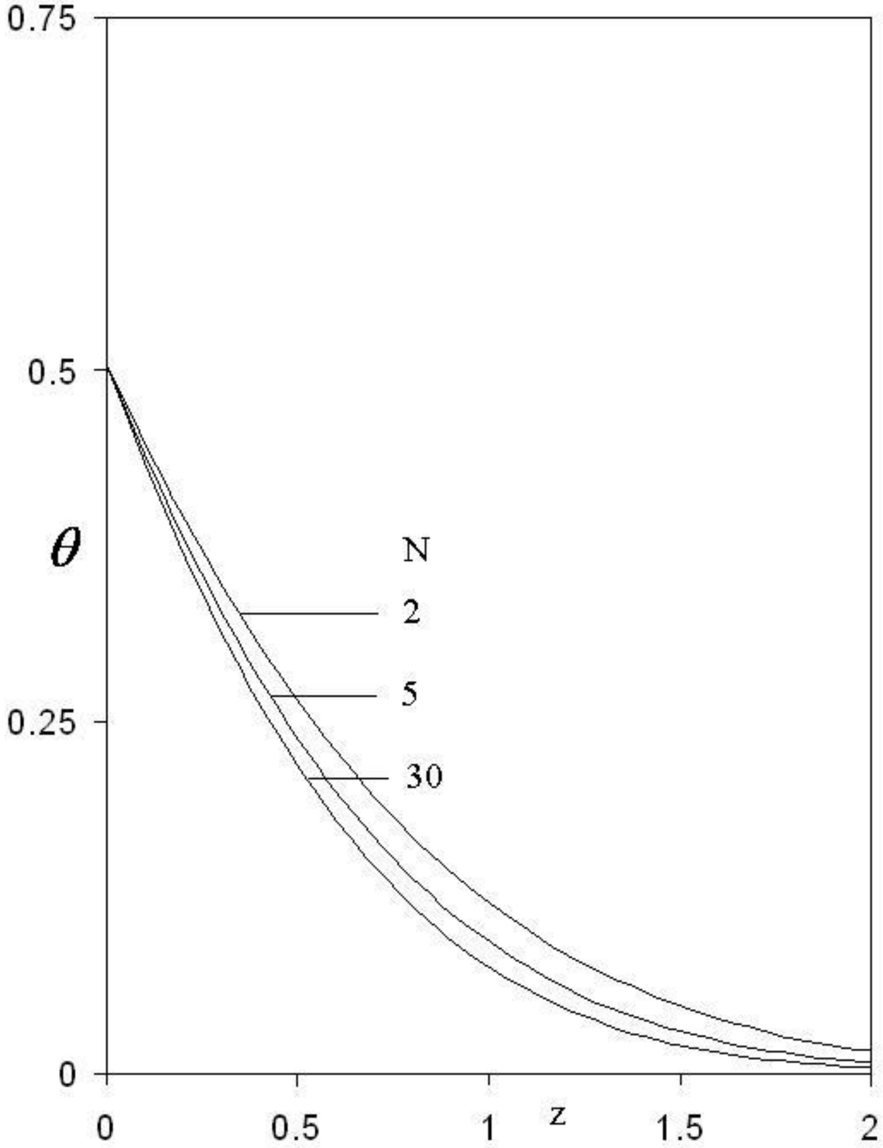

Figure.5: Temperature profiles for different N

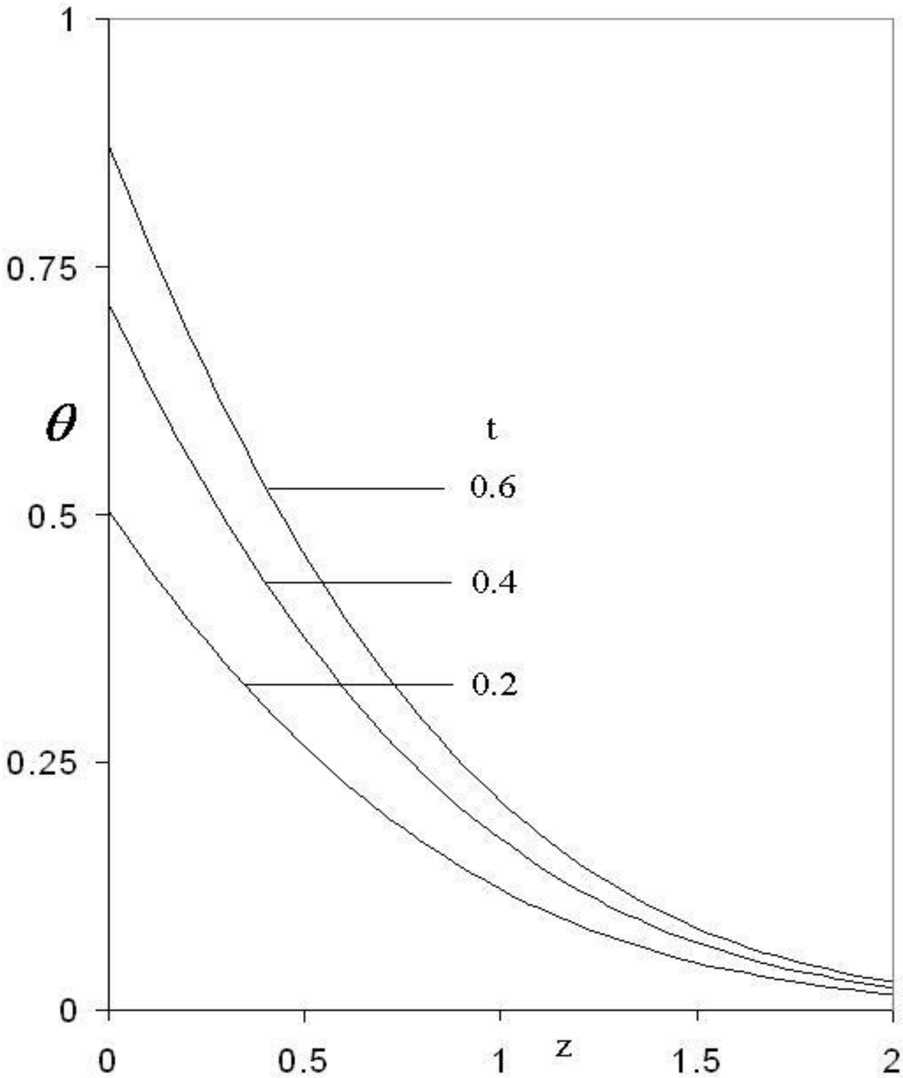

Figure.6: Temperature profiles for different t

z 0.6

2 0.4