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WAVE PROPAGATION IN MICRO-ISOTROPIC, MICRO – ELASTIC SOLID IN SPECIAL CASE

K. Somaiah, E. Anjan Rao, A. Chandulal and K. Sambaiah
Department of Mathematics, Kakatiya University, Warangal, A.P., India.

Email: somaiahkamidi@yahoo.com
anjanraoe@yahoo.co.in, chandulal2009@gmail.com

ABSTRACT: Wave propagation in micro – isotropic, micro- elastic solid in case of irrotational micro- rotation and macro- displacements are studied and it is observed that there are seven waves propagate in micro-isotropic medium with four distinct velocities, with the same dispersion and the same cut-off frequency.

Key Words: Waves in Micro-isotropic, Micro-elastic solids, irrotational micro-ratations, macro-displacements.

INTRODUCTION

The number of problems were studied in the micro polar theory discussed by Eringen and Suhubi [1,2]. However, very few problems were solved by taking the principle of micromorphic theory. This may be due to the fact that equations of micromaphic elasticity are more complicated. If one has to consider a problem where micro-motion is not restricted to rotation only, the problem can be considered by micro-morphic theory. Addressing himself to this more general problem but searching for a more tractable formulation Koh [3] developed a theory by extending the concept of coincidence of principal directions of stress and strain in classical elasticity to the micro-elastic medium through the postulate of principle of coincidence. Imposing a particular form of micro – isotropy, Koh [3] obtained special constraints on the elastic moduli there by reducing the number 18 to 10 in this special case. With the above restrictions on the micro-morphic medium, Koh [3] called this new medium as micro-isotropic, micro-elastic medium.

The action of sudden disturbance in the elastic solid medium is transmitted at once to other parts of the body. At the beginning the remote parts of the body remain undisturbed and the deformations produced at a point are propagated through the body in the form of waves. If a region is large then the effects of the boundaries can be dis regarded, it is possible to represent the disturbance as the sum of two waves propagated with velocities that depend only the density and elastic constants of the medium. Indeed, the displacement vector $\vec{u}$ can be represented as a sum of two vectors one of which is solenoidal and other is irrotational. This leads to a consideration of two special types of disturbance for one of which div $\vec{u} = 0$ and for the other curl $\vec{u} = \vec{0}$ in the case of classical elasticity [4]. In this paper, we study the wave propagation in micro-isotropic, micro-elastic solid in special case of irrotational micro rotations and macro-displacements, in this additional waves are found.

BASIC EQUATIONS

The basic equations for a micro- isotropic, micro elastic solids are obtained by Koh [3]. The constitutive equations are:

$$t_{(km)} = A_1 e_{pp}\delta_{km} + 2A_2 e_{km} \quad \text{...} \quad (1)$$
$$t_{[km]} = \sigma_{[km]} = 2A_3\varepsilon_{pkm}(r_p + \phi_p) \quad \text{...} \quad (2)$$
$$\sigma_{(km)} = -A_4\phi_{pp}\delta_{km} - 2A_5\phi_{(km)} \quad \text{...} \quad (3)$$
$$t_{k(mn)} = B_1\phi_{pp,k}\delta_{mn} + 2B_2\phi_{(mn),k} \quad \text{...} \quad (4)$$
$$m_{kl} = -2(B_5\phi_{l,k} + B_4\phi_{k,l} + B_5\phi_{p,p}\delta_{kl}) \quad \text{...} \quad (5)$$

where ( ) denotes symmetric part and [ ] denotes anti – symmetric part and

$$\begin{aligned} A_1 &= \lambda + \sigma_1\,; & B_1 &= \tau_3 \\ A_2 &= \mu + \sigma_2\,; & 2B_2 &= \tau_7 + \tau_{10} \\ A_3 &= \sigma_5\,; & B_3 &= 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10} \\ A_4 &= -\sigma_1\,; & B_4 &= -2\tau_4 \\ A_5 &= -\sigma_2\,; & B_5 &= -2\tau_9 \end{aligned} \quad \text{...} \quad (6)$$

The equations of motion for this material are

$$(A_1 + A_2 - A_3)u_{p,pm} + (A_2 + A_3)u_{m,pp} + 2A_3\varepsilon_{pkm}\phi_{p,k} + \rho fm = \rho\frac{\partial^2 u_m}{\partial t^2} \quad \text{..............} \quad (7)$$

$$B_1\phi_{pp,kk}\delta_{ij} + 2B_2\phi_{(ij),kk} - A_4\phi_{pp}\delta_{ij} - 2A_5\phi_{(ij)} + \rho f_{(ij)} = \frac{1}{2}\rho j\frac{\partial^2\phi_{(ij)}}{\partial t^2} \quad \text{...................} \quad (8)$$

$$2B_3\phi_{p,mm} + 2(B_4 + B_5)\phi_{m,mp} - 4A_3(r_p + \phi_p) - \rho l_p = \rho j\frac{\partial^2\phi_p}{\partial t^2} \quad \text{......................} \quad (9)$$

where $\varphi_p = \frac{1}{2}\varepsilon_{pkm}\phi_{km}$; $r_p = \frac{1}{2}\varepsilon_{pkm}u_{m,k}$;

couple stress tensor $m_{kp} = \varepsilon_{pnm}t_{kmn}$;

body couples $l_p = \varepsilon_{pnm}f_{mn}$

and ε_{pnm} is permutation symbol.

S. parameshwaran and S.L. Koh [5] establish the following constrains on micromorphic elastic constants

$$3A_1 + 2A_2 > 0;\; A_2 > 0;\; A_3 > 0$$
$$3A_4 + 2A_5 > 0;\; A_5 > 0$$
$$3B_1 + 2B_2 > 0;\; B_2 > 0;\; B_3 > 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (10)$$
$$-B_3 < B_4 < B_3;\; B_3 + B_4 + B_5 > 0$$

EQUATIONS OF MOTION:

We write the equations of motion (7)and (9) in terms of

$$\Delta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (11)$$

And $$\Delta^1 = \frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_2}{\partial x_2} + \frac{\partial \phi_3}{\partial x_3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (12)$$

where we call Δ is dilation and Δ^1 is microrotation dilation. The equation (7) in components under the absence of body forces are

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_1}\Delta + (A_2 + A_3)\nabla^2 u_1 + 2A_3\left(\frac{\partial \phi_2}{\partial x_3} - \frac{\partial \phi_3}{\partial x_2}\right) = \rho\frac{\partial^2 u_1}{\partial t^2} \ldots\ldots\ldots \qquad (13)$$

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_2}\Delta + (A_2 + A_3)\nabla^2 u_2 + 2A_3\left(\frac{\partial \phi_3}{\partial x_1} - \frac{\partial \phi_1}{\partial x_3}\right) = \rho\frac{\partial^2 u_2}{\partial t^2} \ldots\ldots\ldots \qquad (14)$$

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_3}\Delta + (A_2 + A_3)\nabla^2 u_3 + 2A_3\left(\frac{\partial \phi_1}{\partial x_2} - \frac{\partial \phi_2}{\partial x_1}\right) = \rho\frac{\partial^2 u_3}{\partial t^2} \ldots\ldots\ldots \qquad (15)$$

where $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$

The equation (9) in components form under the absence of body couples are

$$2B_3\nabla^2\phi_1 + 2(B_4 + B_5)\frac{\partial}{\partial x_1}\Delta^1 - 2A_3(u_{3,2} - u_{2,3}) - 4A_3\phi_1 = \rho j\frac{\partial^2 \phi_1}{\partial t^2} \ldots\ldots\ldots \qquad (16)$$

$$2B_3\nabla^2\phi_2 + 2(B_4 + B_5)\frac{\partial}{\partial x_2}\Delta^1 - 2A_3(u_{1,3} - u_{3,1}) - 4A_3\phi_2 = \rho j\frac{\partial^2 \phi_2}{\partial t^2} \ldots\ldots\ldots \qquad (17)$$

$$2B_3\nabla^2\phi_3 + 2(B_4 + B_5)\frac{\partial}{\partial x_3}\Delta^1 - 2A_3(u_{2,1} - u_{1,2}) - 4A_3\phi_3 = \rho j\frac{\partial^2 \phi_3}{\partial t^2} \ldots\ldots \qquad (18)$$

Now the equations (8) are given by

$$B_1\phi_{pp,kk} + 2B_2\phi_{11,kk} - A_4\phi_{pp} - 2A_5\phi_{11} = \frac{1}{2}\rho j\frac{\partial^2 \phi_{11}}{\partial t^2} \qquad \ldots\ldots\ldots\ldots \qquad (19)$$

$$B_1\phi_{pp,kk} + 2B_2\phi_{22,kk} - A_4\phi_{pp} - 2A_5\phi_{22} = \frac{1}{2}\rho j\frac{\partial^2 \phi_{22}}{\partial t^2} \qquad \ldots\ldots\ldots\ldots \qquad (20)$$

$$B_1\phi_{pp,kk} + 2B_2\phi_{33,kk} - A_4\phi_{pp} - 2A_5\phi_{33} = \frac{1}{2}\rho j\frac{\partial^2 \phi_{33}}{\partial t^2} \qquad \ldots\ldots\ldots\ldots \qquad (21)$$

$$2B_2\phi_{(12),kk} - 2A_5\phi_{(12)} = \frac{1}{2}\rho j\frac{\partial^2\phi_{(12)}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

$$2B_2\phi_{(13),kk} - 2A_5\phi_{(13)} = \rho j\frac{\partial^2\phi_{(13)}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

$$2B_2\phi_{(23),kk} - 2A_5\phi_{(23)} = \rho j\frac{\partial^2\phi_{(23)}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

Adding equations (19) to (21) we get

$$(3B_1 + 2B_2)\phi_{pp,kk} - (3A_4 + 2A_5)\phi_{pp} = \frac{1}{2}\rho j\frac{\partial^2\phi_{pp}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

Subtracting equation (20) from (19) we get the following equation.

$$2B_2(\phi_{11} - \phi_{22})_{kk} - 2A_5(\phi_{11} - \phi_{22}) = \frac{\rho j}{2}\frac{\partial^2}{dt^2}(\phi_{11} - \phi_{22}) \quad \ldots\ldots\ldots\ldots\ldots \quad (26)$$

And subtracting equation (21) from (19) we get

$$2B_2(\phi_{11} - \phi_{33})_{,kk} - 2A_5(\phi_{11} - \phi_{33}) = \frac{\rho j}{2}\frac{\partial^2}{dt^2}(\phi_{11} - \phi_{33}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

Wave propagation in the unbounded micro-isotropic medium is represented by twelve equations (13) to (18) and (22) to (27).

Propagation of waves in micro-isotropic medium when irrotational micro- rotations and macro displacements.

We assume that $\nabla \mathrm{x} \vec{U} = \vec{0}$ $\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$

and $\nabla \mathrm{x} \vec{\phi} = \vec{0}$ $\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$

where $\vec{U} = u_1\vec{i}_1 + u_2\vec{i}_2 + u_3\vec{i}_3$ and $\vec{\phi} = \phi_1\vec{i}_1 + \phi_2\vec{i}_2 + \phi_3\vec{i}_3$.

In view of equation (28) the equations (13) to (15) reduce to

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_1}\Delta + (A_2 + A_3)\nabla^2 u_1 = \rho\frac{\partial^2 u_1}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (30)$$

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_2}\Delta + (A_2 + A_3)\nabla^2 u_2 = \rho\frac{\partial^2 u_2}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_3}\Delta + (A_2 + A_3)\nabla^2 u_3 = \rho\frac{\partial^2 u_3}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

In view of equation (29) the equations (16) to (18) reduce to

$$2B_3\nabla^2\phi_1 + 2(B_4 + B_5)\frac{\partial}{\partial x_1}\Delta^1 - 4A_3\phi_1 = \rho j\frac{\partial^2\phi_1}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (33)$$

$$2B_3\nabla^2\phi_2 + 2(B_4 + B_5)\frac{\partial}{\partial x_2}\Delta^1 - 4A_3\phi_2 = \rho j\frac{\partial^2\phi_2}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (34)$$

$$2B_3\nabla^2\phi_3 + 2(B_4 + B_5)\frac{\partial}{\partial x_3}\Delta^1 - 4A_3\phi_3 = \rho j\frac{\partial^2\phi_3}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (35)$$

Differentiating equations (30), (31), (32) with respect to x_1, x_2, x_3 respectively and adding we get

$$(A_1 + 2A_2)\nabla^2\Delta = \rho\frac{\partial^2}{\partial t^2}\Delta \qquad (36)$$

It can be expressed as $\nabla^2\Delta = \frac{1}{C_1^2}\frac{\partial^2\Delta}{\partial t^2}$ ……………………………….. (37)

where $C_1^2 = \frac{A_1 + 2A_2}{\rho}$ …………………………… (38)

Equation (37) shows that the volume dilation or compression is transmitted in the form of waves through the medium with velocity V_1 and is given by

$$V_1^2 = \frac{A_1 + 2A_2}{\rho}$$

Similarly differentiating equations (33), (34) and (35) with respect to x_1, x_2 and x_3 respectively and adding the resultant equations we get

$$2(B_3 + B_4 + B_5)(\nabla^2 - 4A_3)\Delta^1 = \rho j\frac{\partial^2}{\partial t^2}\Delta^1 \qquad (39)$$

Plane waves advancing in the positive direction of unit vector $\hat{n}$ may be expressed as

$$\Delta^1 = A\exp[ik(\hat{n}.\vec{r} - \nu t)] \qquad (40)$$

where A is a constant, $\vec{r}$ is the position vector, k is the wave number and ν is the velocity of the wave.

Substituting equation (40) in (39) we get a wave whose speed is V_2 and is given by

$$V_2^2 = \frac{2(B_3 + B_4 + B_5)\omega^2}{\rho j(\omega^2 - \omega_1^2)} \qquad (41)$$

where angular frequency $\omega = \nu k$. ………………………………………… (42)

and $\omega_1^2 = \frac{4A_3}{\rho j}$ ………………………………………… (43)

The speed of these wave depends on the frequency and it is dispersive. This wave corresponds to micro – rotation

Now we discuss the waves corresponding to the equation (22) to (27) . For these equations the plane waves advancing in the positive direction of unit vector $\hat{n}$ may be expressed as

$$\phi_{(ij)} = D_{ij}\exp[ik(\hat{n}.\vec{r} - \nu t)] \qquad (44)$$

where D_{ij} are constants and $D_{ij} = D_{ji}$ ………………….. (45)

Substituting equation (44) in the equation (25) we get

$$[(3B_1 + 2B_2)k^2 + (3B_4 + 2A_5)] = \frac{\rho j}{2}\omega^2$$

We then obtain an expression for the phase velocity V_3 and it is given by

$$V_3 = \frac{\omega^2}{k^2} = \frac{2(3B_1 + 2B_2)}{\rho j(\omega^2 - \omega_2^2)}\omega^2 \qquad (46)$$

where $\omega_2^2 = \frac{2(3A_4 + 2A_5)}{\rho j}$ ………………………… (47)

It is longitudinal micro-dilation wave V_3 is real not finite or imaginary accuracy as $\omega > \omega_2$, $\omega = \omega_2$ and $\omega < \omega_2$ respectively.

Now substituting equation (44) in equation (26) we obtain the phase velocity V_4 and it is given by

$$V_4^2 = \frac{\omega^2}{k^2} = \frac{4B_2\omega^2}{\rho j(\omega^2 - \omega_3^2)} \quad \text{(48)}$$

where $\omega_3^2 = \frac{4A_5}{\rho j}$ ………………………………… (49)

This wave we call it as transverse micro-extensional wave.

We can observe that the equations (22) to (24) and (27) are identical with the equation (26). Thus we have another four waves whose phase velocities are V_4, V_5, V_6, V_7 such that all these velocities are equal to V_3. The wave corresponds to the equation (27) we call it as transverse micro-extensional wave. The waves given by equations (22) to (24) are called micro-shear waves. All these waves have the same dispersion relation and the same cut – off frequency.

Under the assumed conditions given by equations (28) and (29) it is observed that there are seven waves propagate in micro-isotropic medium, with four distinct velocities V_1, V_2, V_3 and V_4 with the same dispersion relation and the same cut-off frequency.

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