

**BI-AXIAL COMPRESSIVE ANALYSIS AND OPTIMIZATION OF
COMPOSITE LAMINATES BY SINGLE POINT
CROSS OVER GENETIC ALGORITHM**

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Abstract:

Critical stage in the structural design process is optimization. In the composite structure design process one must deal with many design variables such as plate thickness, number of layers, and orientation of laminae. Mathematical optimization is practical way for us to design a large structure. Genetic Algorithms are the best suitable for multi variable optimization.

In this paper genetic algorithms are used to optimize composite laminates stacking sequence subjected to biaxial compressive loads.

Key words:

Binary search selection; Crossover; Lamina; Laminate; Mutation; Stacking sequence; Tsai-Wu failure criterion.

1. Introduction:

Laminated plates are one of the simplest and most wide spread practical applications of composite laminates. A plate's buckle when the in-plane compressive load gets so large that the originally flat equilibrium state is no longer stable, and the plate deflects into a non flat (wavy) configuration. The flat equilibrium state has only in-plane forces and under goes only extension, compression, and shear. Thus, the flat equilibrium state is often called the membrane prebuckled state and consists of only in-plane deformations. More comprehensively, the load at which the plate deformed configuration suddenly changes into a different configuration is called the buckling load. Note that with bending extension coupling, an original flat plate under axial in-plane compression bends at all loads prior to bifurcation buckling.

Thus a membrane (flat and uniformly stressed) pre buckled state is not actually possible. However, a first order approximation to the bifurcation buckling load is made by ignoring the pre-

buckling out of plane deflections. A computer program in 'C' –language is used to evaluate stress and strain in any layer of a laminate subjected to buckling loads.

2. Optimization:

Optimization is the process of maximizing or minimizing a desired objective function while satisfying the prevailing constraints. It is the act of obtaining the best result under given circumstances.

In design, construction and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables. Optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function.

3. Genetic algorithm

A **genetic algorithm (GA)** is a search technique used in computing to find exact or approximate solutions to optimization and search problems. Genetic algorithms are categorized as global search heuristics. Genetic algorithms are a particular class of evolutionary algorithms (also known as evolutionary computation) that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover (also called recombination).

3.1 Methodology:

Genetic algorithms are implemented as a computer simulation in which a population of abstract representations (called chromosomes or the genotype or the genome) of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem evolves toward better solutions. Traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible. The evolution usually starts from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. If the algorithm has terminated due to a maximum number of generations, a satisfactory solution may or may not have been reached.

In a practical use the fiber angles in laminate are limited to small sets comprising of $0^\circ, \pm 15^\circ, \pm 30^\circ, \pm 45^\circ, \pm 60^\circ, \pm 75^\circ, \pm 90^\circ$. Hence ply angle, design variables are restricted to take any of the above ply angle. It starts with initial population of design variables, ply angle generated randomly permitted angle. Failure strength of each design is used to generate fitness value indicating its level of performance with respect to other design in the population. Design that performs the best i.e. have the highest fitness value are given the greatest probability of breeding with the other good designs. So that their characteristics can be passed to future generations.

4. Mechanics of Composite Materials:

4.1. Force and Moment Resultants Related to Midplane Strains and curvatures:

The midplane strains and plate curvatures are the unknowns for finding the laminate strains and stress. The stress in each lamina in terms of these unknowns. The stresses in each

laminae be integrated through the laminate thickness to give resultant force and moments (or applied force and moments). Since the force and moments applied to a laminate will be known, the midplane strains and plate curvatures can then be found. This relationship between applied loads and the midplane strains and curvatures is developed in this section.

N_x, N_y = normal force per unit length.

N_{xy} = shear force per unit length.

M_x, M_y = bending moments per unit length.

M_{xy} = twisting moments per unit length.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$\epsilon_x^0, \epsilon_y^0, \gamma_{xy}^0$ = mid plane strains;

$\kappa_x, \kappa_y, \kappa_{xy}$ = mid plane curvatures;

5. Failure in FRP Composites:

5.1. Tsai-Wu Failure Theory:

Tsai-Wu criterion is used for finding whether composite fails or not under combined loading the following function is

$$\left(\frac{1}{X_t} - \frac{1}{Y_t}\right)\sigma_1 + \left(\frac{1}{Y_t} + \frac{1}{Y_c}\right)\sigma_2 + \frac{\sigma_1^2}{X_t X_c} + \frac{\sigma_2^2}{Y_t Y_c} + \frac{\sigma_{12}^2}{S^2} + 2F_{12}\sigma_1\sigma_2 = H$$

The summation conversion for repeated subscripts applies in the above expression. According to Tsai-Wu criterion, a composite fails when the following condition is violated

$$FI \leq 1$$

Six independent strength properties are required in order to apply this criterion. Property F_{12} as involved in the interaction term should be determined from a biaxial test. It should be determined from a biaxial test, e.g. with $\sigma_1 = \sigma_2 = \sigma$. suppose the material fails at $\sigma = \sigma^*$. F_{12} can then be expressed as

$$F_{12} = \frac{1}{2(\sigma^*)^2 \left[1 - \sigma^* (F_1 + F_2) - (\sigma^*)^2 (F_{11} + F_{22}) \right]}$$

To help with the appreciation of this coupling strength properties, it is plotted as function of σ below where $x = \sigma / \sigma_{2t}$. It is clear that, over a wide range where σ is like to locate, the variation of F_{12} is insignificant. It has also been shown in Jones (1999) that the coupling term in the failure function F only contributes negligibly to the value of F . Therefore, an accurate determination of F_{12} is not always crucial.

6. Case study:

Material properties of Graphite/Epoxy Composite Laminates are
 $E_1=181\text{Gpa}$; $E_2=10.3\text{Gpa}$; $G_{12}=7.17\text{Gpa}$; $P_{12}=0.28$;

Case 1:

Load $N_x=N_y = -10000\text{ N}$;

Minimum strain energy:

No of Layers 3

Number of Generations had for solution 16
with crossover probability=0.850000
and Mutation Probability=0.150000
Minimum Strain Energy= 1.477948e+01
Optimum Stacking Sequence-60 30-60

No of Layers 4

Number of Generations had for solution 27
with crossover probability=0.850000
and Mutation Probability=0.150000
Minimum Strain Energy= 8.201891e+00
Optimum Stacking Sequence 60-30 45 90

No of Layers 5

Number of Generations had for solution 14
with crossover probability=0.850000
and Mutation Probability=0.150000
Minimum Strain Energy= 6.800419e+00
Optimum Stacking Sequence 15-60 75 30 0

No of Layers 6

Number of Generations had for solution 20
with crossover probability=0.850000
and Mutation Probability=0.150000
Minimum Strain Energy= 6.081007e+00
Optimum Stacking Sequence-60 15 45 60-60-30

No of Layers 7

Number of Generations had for solution 36
with crossover probability=0.850000
and Mutation Probability=0.150000
Minimum Strain Energy= 5.454776e+00
Optimum Stacking Sequence 75-30 0 0 75 75 15

No of Layers 8
Number of Generations had for solution 1
with crossover probability=0.850000
and Mutation Probability=0.150000
Minimum Strain Energy= 4.789090e+00
Optimum Stacking Sequence-45 0 45 75 75-30-45 0

No of Layers 9
Number of Generations had for solution 1
with crossover probability=0.850000
and Mutation Probability=0.150000
Minimum Strain Energy= 4.387587e+00
Optimum Stacking Sequence-30 75 75 15-15 30-15 90-75

No of Layers 10
Number of Generations had for solution 35
with crossover probability=0.850000
and Mutation Probability=0.150000
Minimum Strain Energy= 1.135211e+00
Optimum Stacking Sequence 30 90 90 75 45-45 15 15 75 30

Case 2:

Load $N_x=N_y = - 10000$ N;

Maximum strength factor:

No of Layers 3

Number of Generations 0

with crossover probability=0.850000

and Mutation Probability=0.150000

Max strength level factor= 0.000000e+00

Optimum Stacking Sequence-75-30-30

No of Layers 4

Number of Generations 8

with crossover probability=0.850000

and Mutation Probability=0.150000

Max strength level factor= 1.088041e-03

Optimum Stacking Sequence 45-45 45 60

No of Layers 5

Number of Generations 18

with crossover probability=0.850000

and Mutation Probability=0.150000

Max strength level factor= 9.776744e-04

Optimum Stacking Sequence-30 60 0 0-15

No of Layers 6

Number of Generations 6

with crossover probability=0.850000

and Mutation Probability=0.150000

Max strength level factor= 1.074683e-03

Optimum Stacking Sequence-30 75 30-15-30-15

No of Layers 7
Number of Generations 28
with crossover probability=0.850000
and Mutation Probability=0.150000
Max strength level factor= 6.686375e-04
Optimum Stacking Sequence-75 30 0-90 75 75-75

No of Layers 8
Number of Generations 35
with crossover probability=0.850000
and Mutation Probability=0.150000
Max strength level factor= 8.487243e-04
Optimum Stacking Sequence 75 15-30-30 75 75 75 75

No of Layers 9
Number of Generations 47
with crossover probability=0.850000
and Mutation Probability=0.150000
Max strength level factor= 8.162160e-04
Optimum Stacking Sequence 75-45-15 0 45 60 75 75 60

No of Layers 10
Number of Generations 40
with crossover probability=0.850000
and Mutation Probability=0.150000
Max strength level factor= 7.390886e-04
Optimum Stacking Sequence-30 30 45 75 90-30-30-45-30-30

7. Conclusion:

This paper gives the Optimized Number of layers and Sequence for a given condition.

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