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Hyperbolic Temperature Profiles In An Elastic Half-Space By Laser Pulses

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Abstract

The discovery of Lasers in the early sixties has opened up new era in the field of science and technology, in particular in the field of heat transfer. Most Lasers are heat producing devices converting Electro-magnetic energy into thermal energy. Hyperbolic heat conduction is gaining importance not only in engineering sciences but also in geological and biological sciences. It is necessary to consider hyperbolic heat conduction in problems involving short time intervals, and for very high heat fluxes. In this paper we determined the temperature distribution in an elastic half space produced by a flux of radiation energy incident on its boundary by using the modified hyperbolic heat conduction equation which includes the effect of the finiteness of heat propagation velocity. The results are obtained using the General Inverse method which is also known as Fourier's series technique, evaluated the results numerically and represented graphically.

Key words: Heat transfer, Hyperbolic heat conduction, Temperature profile, Laser radiation, Heat flux.

AMS Classification: 34B07, 34B40

1. Introduction

When Laser radiation is incident on an absorbing material, dramatic effects associated with the laser induced temperature change will occur. The analysis consists of deriving solutions to the heat conduction equation for several different models of the spatial and temporal distribution of Laser power.

Ready, J.F.[17], Bechtel, J.H.[2] were the first to use a laser to generate elastic waves in a solid body. In many cases, surface heating with no change of phase is of interest. Using conventional thermo dynamics, Ready, J.F. calculated the temperature rise caused by short high power pulses of radiation. He described the methods by which one may calculate the effects of high power pulses of Laser radiation of specified shapes absorbed at opaque surfaces. Bechtel J.H., obtained analytical and numerical solutions to the heat conduction equation appropriate to the heating of absorbing media with pulsed lasers. The spatial and temporal form of the temperature is determined using several different models of the laser irradiance. Zweing, A.D.[24] developed a steady state model for ablation materials by strongly absorbed long pulse lasers. Yilbas, B.K.[22] examined the unsteady analysis of the conduction limited heat transfer for exponential pulse input with time dependent intensity.

Bekir Sami, Y. and Ahmet, Z.S.[3] have obtained a quasi - steady analytical solution for laser heating process. Burgener, M.L. and Ready, J.F.[4] studied a two layer structure with a continuous wave seanned laser beam and indicated how to apply their method to N layer. Aslam Chaudhry, M and Syed, M.Zubuin [1] have obtained an analytical solution for temperature profile in an infinitely extended isotropic body with a steady and non steady periodical type point heat sources. Galka, A and Wojnar, R[7] have studied a problem of the light solid body interaction in which a thermo mechanical disturbance in an half space is produced by a flux of radiation energy incident on its boundary.

Further analytical solutions in the field concerning to more realistic situation of conversion of light energy into the mechanical energy in which an axially symmetric beam of the light is incident on a half –space were obtained by Danilowskaya and Shefter [6], Germanovich [8], Hetnarski and De Bolt [10] ,Timan and Fosenko [19] and by others.

For transient heat transfer problems with uniform initial temperature, Laplace transform method is the most powerful method. Using Laplace transform, the governing partial differential equation should be transformed into an algebraic equation. If this algebraic equation can be transformed inversely, the required solution of the original problem is obtained. However, in many cases, it is very difficult and complicated to find the inverse Laplace transform. Ichikawa and Kishima [12] have introduced the application of Fourier series techniques to find inverse Laplace transform. In this technique, they obtained an approximate series solution by taking N-terms. Hsin-Sen Chu, Chao-Kuang Chen and Cheng-Iweng [11], Zedan M.F. and Mujahid, A.M.[23] and suryanarayana, N.V.[18] have testified the merit of this technique. They verified that the results obtained by this technique are very close to the analytical solutions. In this paper we used this technique and compared the results with that of Galka, A. and Wojnar, R. [7], The results obtained are very close to those of Galka, A. and Wojnar, R.

2 Hyperbolic Heat Conduction

Transient heat transfer problems usually involve the solution of the classical Fourier heat conduction equation, which is of parabolic character, as a consequence, a perturbed heat signal propagates with an infinite velocity through the medium. That is, if an isotropic homogeneous elastic continuum is subjected to a mechanical or thermal disturbance, the effect of the disturbance will be felt instantaneously at distances infinitely far from its source.

Such a behavior is physically inadmissible and contradicts the existing theories of heat transport mechanisms.

It seems, therefore reasonable to modify the existing theory of heat conduction. To remove the above two deficiencies many investigators such as Maxwell, J.C [15], Morse, P.M and Feshbach, H [16], Chester, M [5], Gurtin, M.E. and Pipkin, A.C. [9], Lebon, G. and Lamberamont, J. [13], Lord H.W and Shulman, Y. [14] have suggested some modifications.

Tisza, L. [20] predicted the possibility of extremely small heat propagation rates (second sound) in liquid helium - II. Chester, M. [5] discussed the possibility of existence of second sound in solids. The second sound effect indicates that wave type mechanism rather than usual diffusion process can transport heat. All these researches lead to the reformulation of existing Fourier heat conduction equation into a damped wave type equation, which is hyperbolic.

Morse, P.M. and Feshbach, H. [16], postulated that the governing transient heat conduction must depend upon the velocity of the propagation of heat 'C'. They assumed that the following equation

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\alpha}\frac{\partial T}{\partial t}=\nabla^2 T \qquad (1)$$

which is hyperbolic, must be the correct governing differential equation for heat conduction problems.

In some cases, the effect of the finite speed of propagation is negligible, however this effect is considerable and important even at the ordinary temperature when the elapsed time during a transient is small. If 'C' is very large, equation (1.1) reduces to classical Fourier heat conduction equation. The estimation for ‘C’ is 1/√3 times the velocity of dilatational wave 'C_1' which is given by $C_1{}^2=(\lambda+2\mu)/\rho$, where λ and μ are Lame's constants. J N Sharma, D S Chandrashekarayya and others have done considerable work in the field.

3 Hyperbolic Formulation - Temperature Distribution

In this paper we considered the problem in which the temperature is produced by a particular heat supply generated by absorption of laser pulse incident on the half space. The Laser beam action is converted instantaneously into heat supply whose time profile and space distribution is modeled by the step and exponential functions respectively. The Laser induced heat makes the half space expand proportionally to the temperature.

We assume that the radiation energy incident on the boundary is homogeneous, and in conversion of the light energy into the thermal energy, the coherence and polarisation of the laser emission can be ignored. To express the validity of the Fourier series technique Venkateswara Reddy K [21] had taken up the parabolic formulation of the problem and compared the results with that of Galka, A. and Wojnar, R. [7].

The Hyperbolic heat conduction equation concerning to our problem is,

$$\frac{K}{C^2}\frac{\partial^2 T}{\partial \bar{t}^2}+C_E\frac{\partial T}{\partial \bar{t}}-K\frac{\partial^2 T}{\partial \bar{x}^2}=\bar{r} \qquad (2)$$

Where $\bar{r}$ denotes the heat supply induced by laser pulse. Here $\bar{x}, \bar{t}$ and T denote the position, time and temperature respectively if the quantities are measured in metric system of units.

The initial and boundary conditions are

$$T(\bar{x},0) = 0, \qquad \left.\frac{\partial T}{\partial \bar{t}}\right|_{t=0} = 0 \quad \text{for} \quad \bar{x} > 0 \tag{3}$$

$$\bar{h}_1 T - \bar{h}_2 \frac{\partial T}{\partial \bar{x}} = 0 \quad \text{for} \quad \bar{x} = 0, \quad \bar{t} > 0 \tag{4}$$

$$\mathrm{T}(\bar{x}, \bar{t}) \to 0 \quad \text{as} \quad \bar{x} \to \infty, \quad \bar{t} > 0 \tag{5}$$

The coefficients $\bar{h}_1 and \bar{h}_2$ in equation (8) are chosen in such a way that

$$\bar{h} = \frac{\bar{h}_1}{\bar{h}_2} \tag{6}$$

Here, K = Thermal conductivity

C_E = Specific heat per unit volume at zero strain.
= density x specific heat

and $\mathrm{C} = \frac{1}{\sqrt{3}} \mathrm{C}_1$

To express the above equations in dimensionless form, we introduce the Chadwick - Sneddon units as follows. C.S. units of length and time are defined by

$$\bar{x}_{cs} = \frac{a}{C_1}$$

$$\bar{t}_{cs} = \frac{\bar{x}_{cs}}{C_1} = \frac{a}{C_1^{\ 2}} \qquad \ldots (7)$$

Where $a = \frac{K}{\rho C_E}$ is the diffusivity and $\mathrm{C}_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ is the longitudinal wave velocity in an unbounded isotropic and homogeneous elastic medium. Here λ and μ are Lame's constants.

The temperature unit is $\bar{T}_{cs} = T_0$

and the heat supply unit is $\bar{r}_{cs} = \frac{(C_1 C_E)^2 T_0}{K} = C_E T_0 \frac{1}{\bar{t}_{cs}}$ …(8)

Using the following non-dimensional variables

$$x = \frac{\bar{x}}{\bar{x}_{cs}}, \qquad t = \frac{\bar{t}}{\bar{t}_{cs}}$$

$$\theta = \frac{T}{\bar{T}_{cs}} = \frac{T}{T_0}, \qquad r = \frac{\bar{r}}{\bar{r}_{cs}}$$

$$h_1 = \bar{h}_1 \quad \text{and} \quad h_2 = \frac{\bar{h}_2}{\bar{x}_{cs}} \tag{9}$$

We get the non-dimensional equation as

$$3\frac{\partial^2\theta}{\partial t^2} + \frac{\partial\theta}{\partial t} - \frac{\partial^2\theta}{\partial x^2} = r \tag{10}$$

and the non-dimensional initial and boundary conditions are

$$\theta(x,0) = 0, \quad \left.\frac{\partial\theta}{\partial t}\right|_{t=0} = 0 \quad \text{for } x > 0 \tag{11}$$

$$h_1\theta - h_2\frac{\partial\theta}{\partial x} = 0 \qquad \text{for } x = 0, \ t > 0 \tag{12}$$

where $h = \frac{h_1}{h_2} > 0$

$$\theta(x, t) \to 0 \quad \text{as} \quad x \to \infty, \quad t > 0 \tag{13}$$

The Laser half-space interaction is modeled by the heat supply

$$\bar{r} = \bar{I}(\bar{t})\, e^{-\bar{k}\bar{x}} \tag{14}$$

Where
$$\bar{I}(\bar{t}) = \begin{cases} \bar{I}_0 \ for\, \bar{t}_* > \bar{t} > 0 \\ 0 \ for\, \bar{t} > \bar{t}_* \end{cases} \tag{15}$$

Using the notations $I_0 = \frac{1}{\bar{r}_{cs}}\bar{I}_0$ & $k = \bar{k}\bar{x}_{cs}$

and Heaviside's step function $H(t) = \begin{cases} 0 \ for \ t \le 0 \\ 1 \ for \ t > 0 \end{cases}$

Equation (18) can be written as

$$r = I(t)\, e^{-kx} \tag{16}$$

where $I(t) = I_0 \left[H(t) - H(t - t_*) \right]$

Therefore the equation (14) can be written as

$$3\frac{\partial^2 \theta}{\partial t^2} + \frac{\partial \theta}{\partial t} - \frac{\partial^2 \theta}{\partial x^2} = I(t)\, e^{-kx} \tag{17}$$

Applying the Laplace transform to equation (21), we get

$$(3s^2 + s)\bar{\theta} - \frac{\partial^2 \bar{\theta}}{\partial x^2} = I_0 \frac{1}{s}\left(1 - e^{-st_*}\right) e^{-kx} \tag{18}$$

Boundary conditions take the form

$$h_1 \bar{\theta} - h_2 \frac{\partial \bar{\theta}}{\partial x} = 0 \qquad \text{for} \quad x = 0$$

$$\bar{\theta}(x, s) \to 0 \quad \text{for} \quad x \to \infty \tag{19}$$

Therefore, a solution of the equation (22), subject to (23) is

$$\bar{\theta}(x, s) = I_0 \frac{1 - e^{-st_*}}{s(3s^2 + s - k^2)} \left[e^{-kx} - \frac{h + k}{h + \sqrt{3s^2 + s}}\, e^{-\sqrt{s}x} \right] \tag{20}$$

Where $h = \frac{h_1}{h_2}$, $h_{2 \neq 0}$

The general inverse transform method known as the Fourier series technique, is used here to obtain θ(x,t) from equation(24). According to this method the inverse transform θ(t) of a function $\bar{\theta}(s)$ in the s-domain is approximated by

$$\theta(t) \cong \frac{e^{ct}}{t}\left[\frac{1}{2}\bar{\theta}(c) + \mathrm{Re}\left\{ \sum_{n=1}^{N} \bar{\theta}(c + in\pi / t) \right\} (-1)^n \right]$$

Therefore from (24), we get

$$\theta(x,t) \cong \frac{e^{ct}}{t}\left[\frac{I_0}{2} \frac{1 - e^{-ct_*}}{c(3c^2 + c - k^2)} \left(e^{-kx} - \frac{h + k}{h + \sqrt{3c^2 + c}}\, e^{-(\sqrt{3c^2 + c})x} \right) \right.$$

$$+\mathrm{Re}\left\{\sum_{n=1}^{N}\left(\frac{I_0(1-e^{-(c+in\pi/t)t_*})}{(c+in\pi/t)(3(c+in\pi/t)^2+(c+in\pi/t)-k^2)}\right)\right.$$

$$\left.\left.\left(e^{-kx}-\frac{h+k}{h+\sqrt{3(c+in\pi/t)^2+(c+in\pi/t)}}e^{-(\sqrt{3(c+in\pi/t)^2+(c+in\pi/t)})x}\right)\right\}(-1)^n\right] \tag{21}$$

After simplification, we get

$$\theta(x,t)\cong\frac{e^{ct}}{t}\left[\frac{1}{2}I_0ab+\sum_{n=1}^{N}\left\{I_0\left(\frac{a_1b_1+a_2b_2}{d_1d_2}\right)e^{-kx}\right.\right.$$
$$-I_0(h+k)\big[(a_1b_1+a_2b_2)(c_1c_3-c_2c_4)$$
$$\left.\left.+(a_1b_2-a_2b_1)(c_1c_4+c_2c_3)\big]\frac{e_1}{d_1d_2d_3}\right\}(-1)^n\right] \tag{22}$$

Where $a=\dfrac{1-e^{-ct_*}}{c(3c^2+c-k^2)}$

$$b=e^{-kx}-\frac{h+k}{h+\sqrt{3c^2+c}}e^{-(\sqrt{3c^2+c})x}$$
$$a_1=3c^3+c^2-ck^2-n^2\pi^2(9c+1)/t^2$$
$$a_2=(9c^2+2c-k^2-3n^2\pi^2/t^2)n\pi/t$$

$$b_1=1-e^{-ct_*}\cos(n\pi/t)t_*$$
$$b_2=e^{-ct_*}\sin(n\pi/t)t_*$$
$$p=3c^2+c-3n^2\pi^2/t^2,\ q=(6c+1)n\pi/t$$
$$\alpha=\tan^{-1}(q/p)$$

$$c_1=h+\sqrt{r}\cos\alpha/2$$

$$c_2 = \sqrt{r}\sin\alpha/2$$

$$c_3 = \cos(x\sqrt{r}\sin\alpha/2)$$

$$c_4 = \sin(x\sqrt{r}\sin\alpha/2)$$

$$d_1 = c^2 + n^2\pi^2/t^2$$

$$d_2 = (3c^2 + c - k^2 - 3n^2\pi^2/t^2)^2 + ((6c+1)n\pi/t)^2$$

$$d_3 = c_1{}^2 + c_2{}^2$$

and

$$e_1 = e^{-(\sqrt{r}\cos\alpha/2)x}$$

4 Numerical Evaluation and Discussion of the Results

In this paper we discussed the problem of determination of the temperature distribution in an elastic half space produced by a flux of radiation energy incident on its boundary. The laser beam action is converted instantaneously into a heat supply whose time profile and space distribution are modeled by the step function and exponential function respectively. The results obtained here by using hyperbolic heat conduction are more accurate and more general which include the effect of finite speed of heat propagation.

The solution for temperature distributions given by (22) is evaluated numerically, for a material whose material constants are given below.

Coefficient of the heat exchange h=11/10, coefficient of light absorption k=10, light intensity $I_0 = k^2 = 100$, ct = 4 and N =150.

The hyperbolic temperature distribution (22) is evaluated numerically by taking duration of laser pulse $t_* = 1$. Figure 1 depicts the variation of temperature with time on the boundary x = 0 and at various cross sections x = 0.25, 0.5, 0.75 and 1.0 of the half space. At the boundary, the temperature increases rapidly after application of the laser pulse until the pulse is switched off, afterwards it decreases. On the boundary x=0 the maximum temperature occurs at t = 2.4. On the cross sections x = 0.25, 0.5, 0.75 and 1.0, the maximum temperature occur respectively at t = 2.65, 3.0, 3.4 and 4.0. As x increases the maximum temperature on various cross sections goes on decreasing. This phenomenon is in agreement with hyperbolic heat conduction since in hyperbolic heat conduction there will be a finite built-up time for the onset of heat flow after a temperature gradient is imposed. In Figure 2 we depicted the variation of temperature corresponding to hyperbolic heat conduction with distance at times t = 1.0, 2.0, 2.5 and 3.0 respectively.

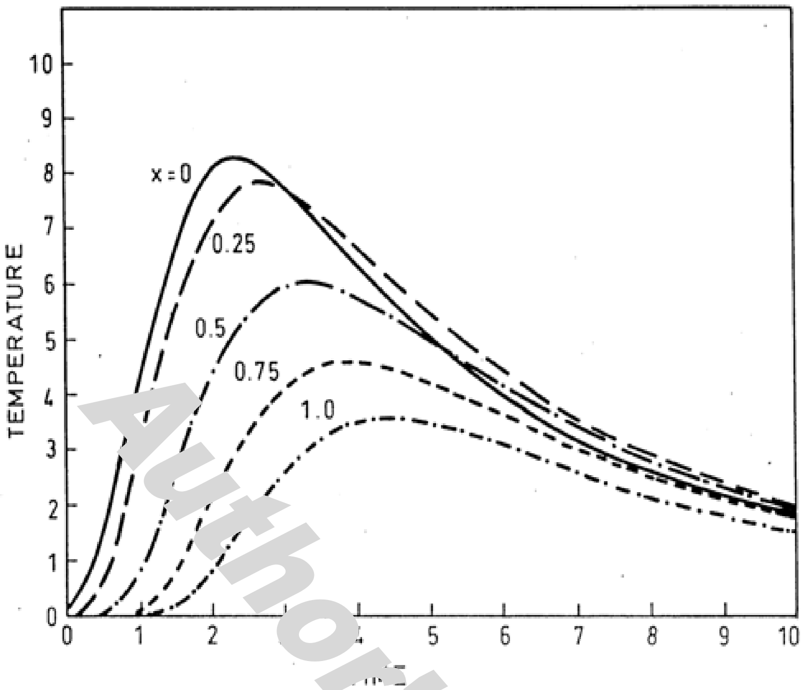

FIGURE 1 THE HYPERBOLIC TEMPERATURE DISTRIBUTION WITH TIME FOR DIFFERENT DISTANCES.

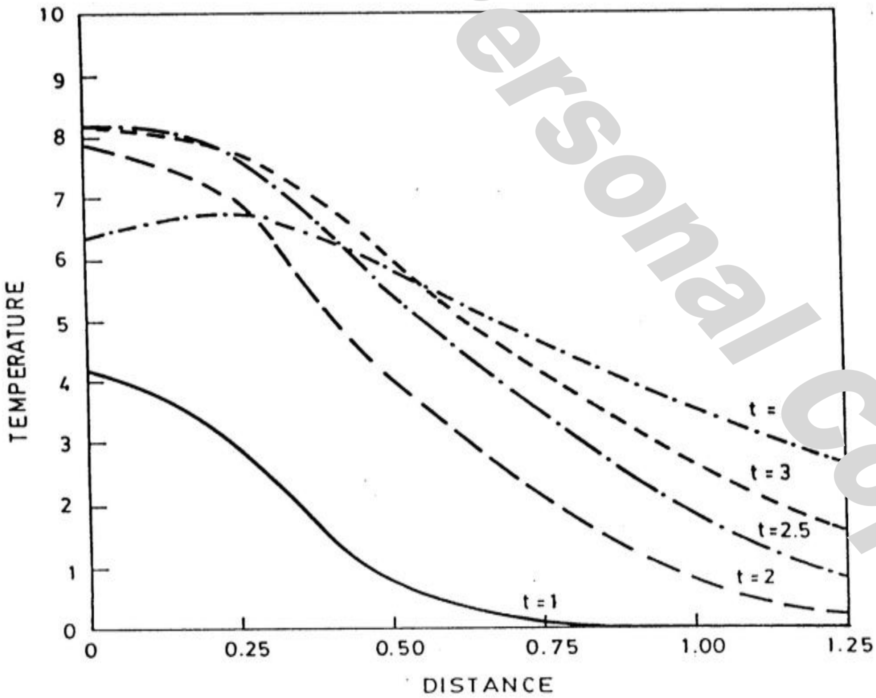

FIGURE 2 THE HYPERBOLIC TEMPERATURE DISTRIBUTION WITH DISTANCE FOR DIFFERENT TIMES

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