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Chemical Reaction Effects on Flow Past an Accelerated Vertical Plate with Variable Temperature in the Presence of Rotating Fluid

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ABSTRACT

An exact analysis of rotation effects on unsteady flow of an incompressible fluid past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of chemical reaction of first order. The plate temperature is raised linearly with time and the concentration level near the plate is also raised to C'_w. The dimensionless governing equations are solved using Laplace-transform technique. The velocity profiles, temperature and concentration are studied for different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It is also observed that the velocity increases with decreasing magnetic field parameter or rotation parameter Ω.

Keywords: rotation, accelerated, vertical plate, heat and mass transfer, chemical reaction.

1. INTRODUCTION

In many chemical engineering processes, there is a chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glass ware and food processing.

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself.

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Gupta et al [5] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [6] extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis al [9].

MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh [7]. Mass transfer effects on flow past a uniformly accelerated vertical plate was studied by Soundalgekar [11]. Basant Kumar Jha and Ravindra Prasad [1] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. Singh [10] studied MHD flow past an impulsively started vertical plate in a rotating fluid. Rotation effects on hydromagnetic free convective flow past an accelerated isothermal vertical plate was studied by Raptis and Singh [8]. Recently, the rotation effects on flow past a vertical plate in the presence of thermal radiation was analyzed by Vijayalakshmi [12].

Hence, it is proposed to study the effects of rotation on free-convection flow of an incompressible viscous fluid past a linearly accelerated infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a study is found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors, magnetic suppression of molten semi-conducting materials and meteorology.

2. GOVERNING EQUATIONS

Consider the unsteady flow of an incompressible fluid past a uniformly accelerated infinite vertical plate when the fluid and the plate rotate as a rigid body with a uniform angular velocity Ω' about z′-axis in the presence of chemical reaction of first order. Initially, the temperature of the plate and concentration near the plate are assumed to be T_∞ and C'_∞. At time $t' > 0$, the plate starts moving with a velocity $u = u_0 t'$ in its own plane and the temperature from the plate is raised linearly with time and the concentration level near the plate are also raised to C'_w . Since the plate occupying the plane $z' = 0$ is of infinite extent, all the physical quantities depend only on z′ and t′. Assume that there is a homogeneous chemical reaction between the plate and the fluid in a rotating system under the usual Boussinesq's approximation in dimensionless form is as follows:

$$\frac{\partial U}{\partial t} - 2\Omega V = Gr\theta + Gc\,C + \frac{\partial^2 U}{\partial Z^2} \tag{1}$$

$$\frac{\partial V}{\partial t} + 2\Omega U = \frac{\partial^2 V}{\partial Z^2} \tag{2}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Z^2} \tag{3}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Z^2} - KC \tag{4}$$

With the following initial and boundary conditions:

$$\begin{array}{llll} u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all } y,\ t' \le 0 \\ t' > 0: \quad u = u_0 t', & T = T'_\infty + (T'_w - T'_\infty)A\,t', & C' = C'_w & \text{at } y = 0 \\ u \to 0, & T \to T_\infty, & C' \to C'_\infty & \text{as } y \to \infty \end{array} \tag{5}$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{(\nu u_0)^{1/3}},\ V = \frac{v}{(\nu u_0)^{1/3}},\ t = t'\left(\frac{u_0^{\ 2}}{\nu}\right)^{1/3},\ Z = z\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ Gr = \frac{g\beta(T_w - T_\infty)}{u_0},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gc = \frac{g\beta^*(C'_w - C'_\infty)}{u_0} \tag{6}$$

$$\Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D}, K = K_l\left(\frac{\nu}{u_0^{\ 2}}\right)^{\frac{1}{3}}$$

Where, $A = \left(\frac{u_0^{\ 2}}{\upsilon}\right)^{1/3}$.

The rotating free-convection flow past an accelerated vertical plate is described by coupled partial differential equations (1) to (4) with the prescribed boundary conditions (5). To solve the equations (1) and (2), we introduce a complex velocity q = U + iV, equations (1) and (2) can be combined into a single equation:

$$\frac{\partial q}{\partial t} = Gr\theta + GcC + \frac{\partial^2 q}{\partial Z^2} - mq \tag{7}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} & q = 0, & \theta = 0, & C = 0 & \text{for all} & Z, t \le 0 \\ t > 0: & q = t, & \theta = t, & C = 1 & \text{at} & Z = 0 \\ & q \to 0, & \theta \to 0, & C \to 0 & \text{as} & Z \to \infty \end{array} \tag{8}$$

where, $m = 2i\Omega$.

3. METHOD OF SOLUTION

The dimensionless governing equations (3), (4) and (7), subject to the initial and boundary conditions (8), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$
\begin{aligned}
q = &\left(\frac{t}{2}+c+cat+d\right)\left[\exp(2\eta\sqrt{2i\Omega t})\,erfc(\eta+\sqrt{2i\Omega t})+\exp(-2\eta\sqrt{2i\Omega t})\,erfc(\eta-\sqrt{2i\Omega t})\right]\\
&-\eta\sqrt{\frac{t}{2i\Omega}}\left[\frac{1}{2}+ac\right]\left[\exp(-2\eta\sqrt{2i\Omega t})\,erfc(\eta-\sqrt{2i\Omega t})-\exp(2\eta\sqrt{2i\Omega t})\,erfc(\eta+\sqrt{2i\Omega t})\right]\\
&-2c\,erfc(\eta\sqrt{\Pr})-c\exp(at)[\exp(2\eta\sqrt{(2i\Omega+a)t})\,erfc(\eta+\sqrt{(2i\Omega+a)t})\\
&\qquad+\exp(-2\eta\sqrt{(2i\Omega+a)t})\,erfc(\eta-\sqrt{(2i\Omega+a)t})]\\
&+c\exp(at)[\exp(2\eta\sqrt{at\Pr})\,erfc(\eta\sqrt{\Pr}+\sqrt{at})\\
&\qquad+\exp(-2\eta\sqrt{at\Pr})\,erfc(\eta\sqrt{\Pr}-\sqrt{at})]\\
&-d\exp(bt)[\exp(2\eta\sqrt{(2i\Omega+b)t})\,erfc(\eta+\sqrt{(2i\Omega+b)t})\\
&\qquad+\exp(-2\eta\sqrt{(2i\Omega+b)t})\,erfc(\eta-\sqrt{(2i\Omega+b)t})]\\
&+d\exp(bt)[\exp(-2\eta\sqrt{Sc(k+b)t})\,erfc(\eta\sqrt{Sc}-\sqrt{(k+b)t})\\
&\qquad+\exp(2\eta\sqrt{Sc(k+b)t})\,erfc(\eta\sqrt{Sc}+\sqrt{(k+b)t})]\\
&-2act\left[\left(1+2\eta^2\Pr\right)erfc(\eta\sqrt{\Pr})-\frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\sqrt{\Pr})\right]\\
&-d[\exp(2\eta\sqrt{Sckt})\,erfc(\eta\sqrt{Sc}+\sqrt{kt})+\exp(-2\eta\sqrt{Sckt})\,erfc(\eta\sqrt{Sc}-\sqrt{kt})] \qquad (9)
\end{aligned}
$$

$$\theta = t\left\{\left(1+2\eta^2\Pr\right)erfc(\eta\sqrt{\Pr})-\frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\Pr)\right\} \qquad (10)$$

$$C = \frac{1}{2}[\exp(2\eta\sqrt{Sckt})\,erfc(\eta\sqrt{Sc}+\sqrt{kt})+\exp(-2\eta\sqrt{Sckt})\,erfc(\eta\sqrt{Sc}-\sqrt{kt})] \qquad (11)$$

where $a=\frac{m}{\Pr-1}$, $b=\frac{m-KSc}{Sc-1}$, $c=\frac{Gr}{2a^2(1-\Pr)}$, $d=\frac{Gc}{2b(1-Sc)}$ and $\eta=\frac{Z}{2\sqrt{t}}$

In order to get the physical insight into the problem, the numerical values of q have been computed from (9). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$
\begin{aligned}
erf(a+ib) &= erf(a)+\frac{\exp(-a^2)}{2a\pi}[1-\cos(2ab)+i\sin(2ab)]\\
&+\frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}[f_n(a,b)+ig_n(a,b)]+\in(a,b)
\end{aligned}
$$

where, $f_n = 2a-2a\cosh(nb)\cos(2ab)+n\sinh(nb)\sin(2ab)$

$$g_n = 2a\cosh(nb)\sin(2ab)+n\sinh(nb)\cos(2ab)$$

$$|\in(a,b)| \approx 10^{-16}|erf(a+ib)|$$

4. Discussion of Results

For physical understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr, m and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which correspond to water-vapor. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr = 0.71) and water (Pr = 7.0). The numerical values of the velocity, temperature and concentration fields are computed for different physical parameters like Prandtl number, rotation parameter, magnetic field parameter, thermal Grashof number, mass Grashof number, Schmidt number and time.

The temperature profiles are calculated for water and air at t=0.2 and these are shown in Figure 1. The effect of the Prandtl number plays an important role in temperature field. It is observed that the temperature increases with decreasing Prandtl number. This shows that the heat transfer is more in air than in water.

Figure 2. represents the effect of concentration profiles for different Schmidt number (Sc = 0.16, 0.3, 0.6, 2.01), K=0.2 and t=0.2. The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number. The concentration profiles for different chemical reaction parameter (K=0.2,2,5) are presented in figure 3. The trend shows that the wall concentration increases with decreasing values of the chemical reaction parameter K.

The primary velocity profiles for different thermal Grashof number (Gr = 2, 5,10), mass Grashof number (Gc = 2, 5,10), K= 0.5, Ω =0.5, Pr = 7 and t = 0.5 are presented in figure 4. The trend shows that the velocity decreases with increasing values of thermal Grashof number or mass Grashof number.

Figure 5. demonstrates the effects secondary velocity profiles for different thermal Grashof number (Gr =2, 5, 10), mass Grashof number (Gc = 5, 5,10), K=8, and Ω = 0.1 at time t = 0.1. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The secondary velocity profiles for different rotation parameter (Ω = 0.5, 1),Gr = Gc = 10, Pr = 7, and t = 0.05 are shown in figure 6. It is observed that the velocity decreases with increasing values of the rotation parameter Ω.

5. CONCLUSION

Theoretical solution of flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of chemical reaction of first order had been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the rotation parameter or chemical reaction parameter K.

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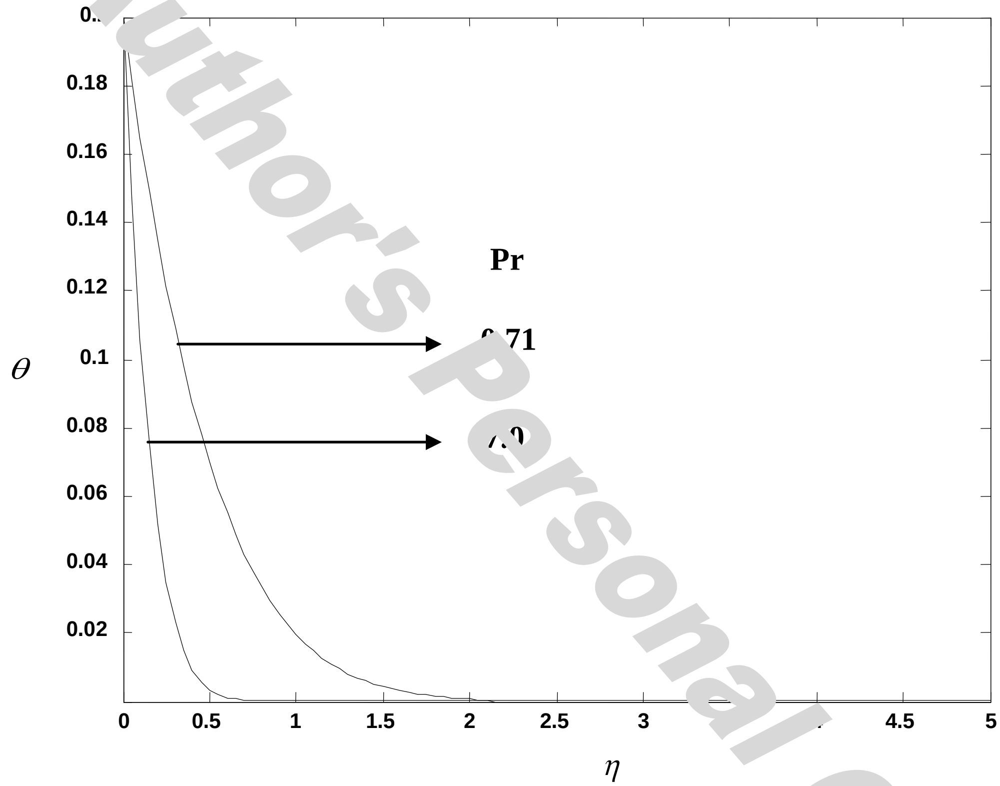

Figure 1. Temperature Profiles for different Pr

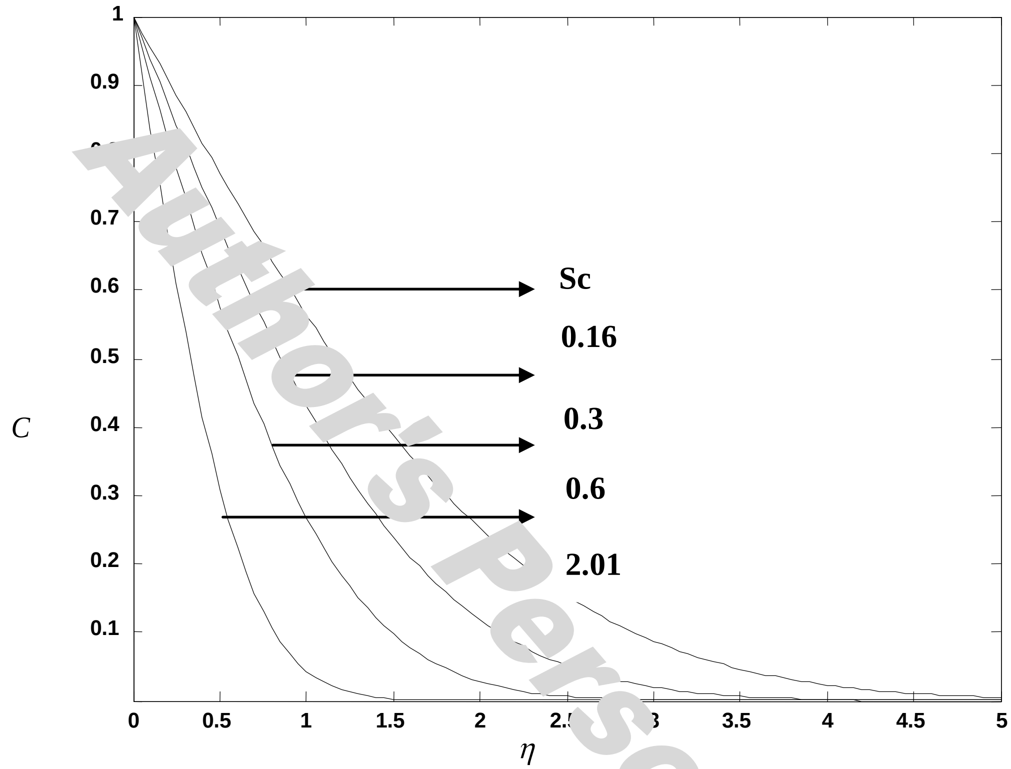

Figure 2. Concentration Profiles for different Sc

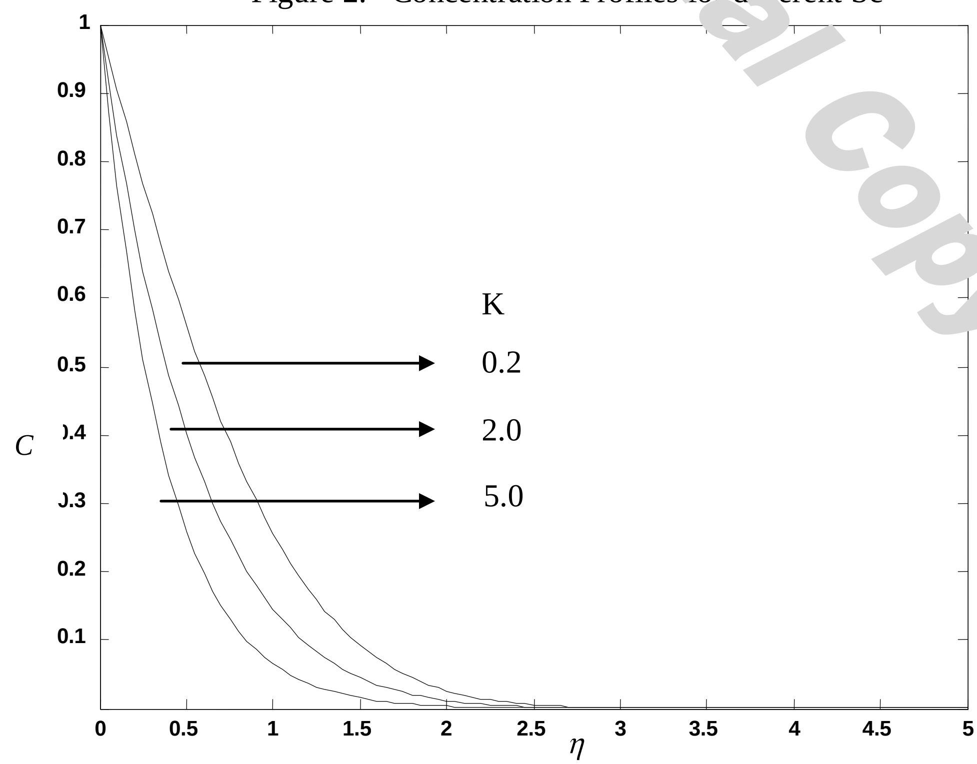

Figure 3. Concentration Profiles for different K

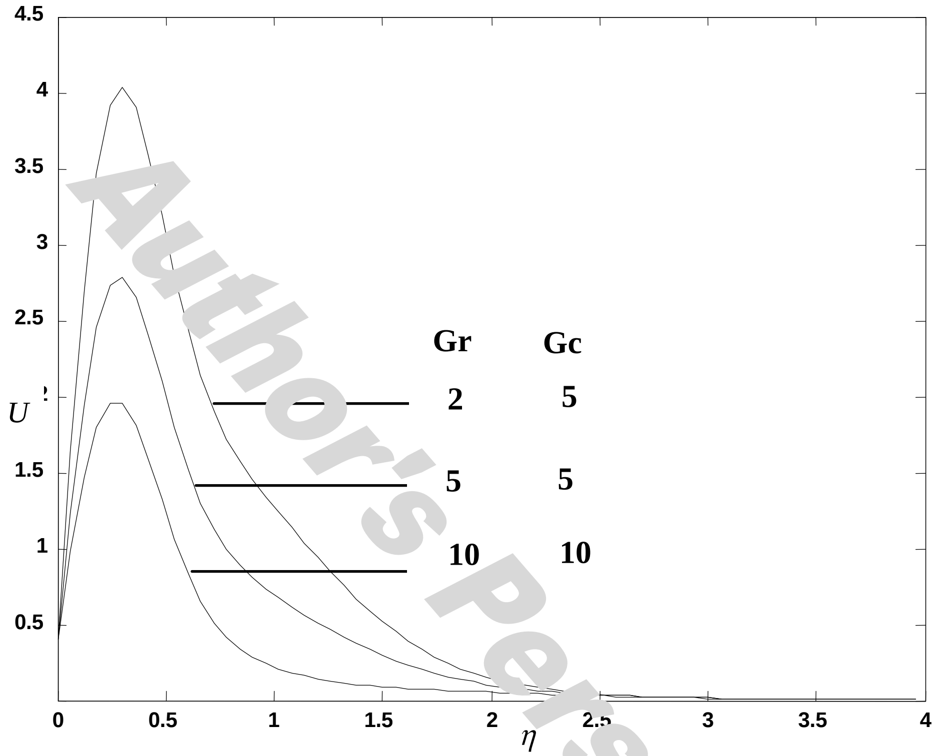

Figure 4. Primary velocity for different values for Gr and Gc

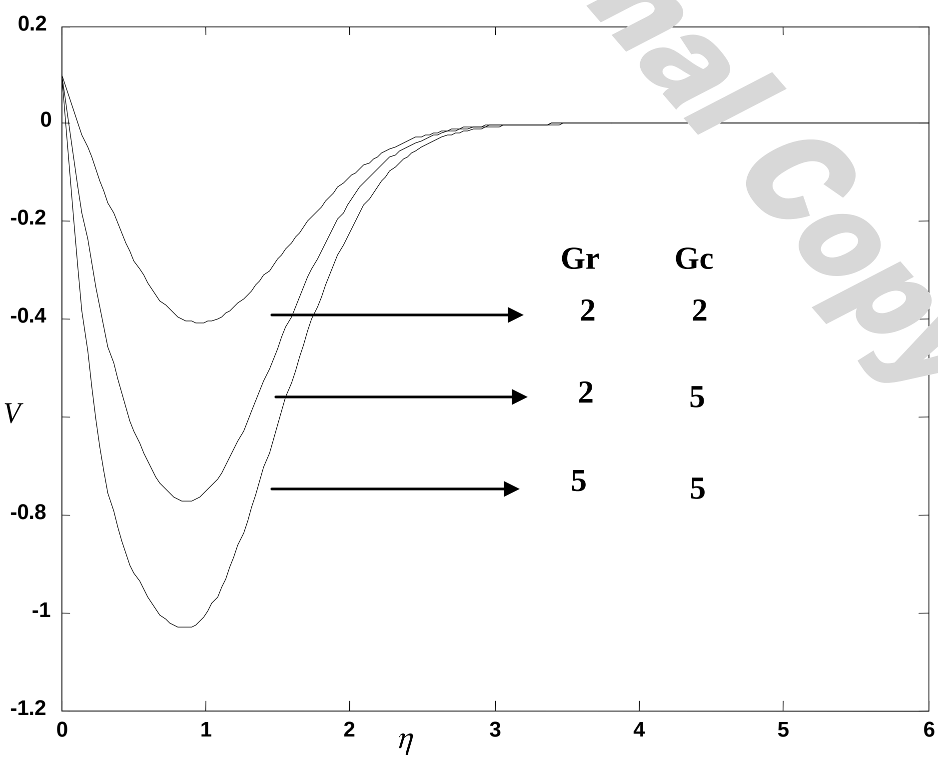

Figure 5. Secondary velocity for different values for Gr and Gc

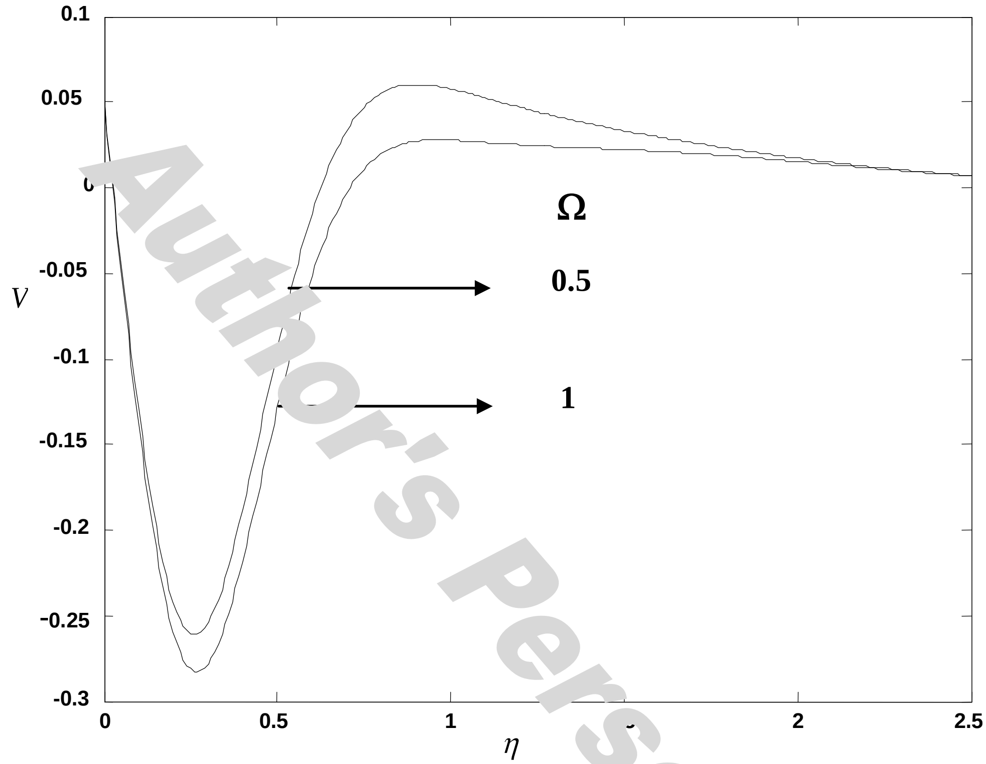

Figure 6. Secondary velocity for different values of Ω