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Diffusion of Chemically Reactive Species and Heat Transfer Effects on Accelerated Vertical Plate with Uniform Heat Flux

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ABSTRACT

An exact analysis of unsteady flow past an uniformly accelerated infinite vertical plate with uniform heat flux in the presence of homogeneous chemical reaction of first order has been studied. The temperature from the plate to the fluid is raised at constant rate and the concentration level near the plate is also raised to C'_w . The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time. It was observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It was also observed that the velocity increases with decreasing values of the chemical reaction parameter.

Keywords: linearly, chemical reaction, accelerated, vertical plate, heat flux.
Mathematics Subject Classification: 76R10

1. INTRODUCTION

Processes involving coupled heat and mass transfer occur frequently in nature. It occurs not only due to temperature difference, but also due to concentration difference or the combination of these two. Quite often, there exist certain industrial processes involving continuous surfaces that move steadily through an otherwise quiescent ambient environment for which a correct assessment of the axial temperature and concentration variation of the material are given relevant importance.

The effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre and Young [2] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das *et al* [3] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das *et al* [4]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Gupta *et al* [5] have studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar [8]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [7]. Basant Kumar Jha and Ravindra Prasad [1] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources . The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [6].

However the chemical reaction effects on linearly accelearted vertical plate with prescribed heat flux and mass diffusion is not studied in the litearture. Hence, it is proposed to study first order chemical reaction effects on unsteady flow past an uniformly accelerated infinite vertical plate in the presence of uniform heat flux and mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. Such a study found useful in hot extrusion of steel, the lamination and meltspinning processes in the extrusion of polymers.

2. Governing Equations

Here the unsteady flow of a viscous incompressible fluid past an uniformly accelerated vertical infinite plate with uniform heat flux in the presence of chemical reaction of first order has been considered. The unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and the concentration C'_∞. At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_0^3}{\nu} t'$ in its own plane against gravitational field. The temperature from the plate to the fluid is raised at an uniform rate and the mass is diffused from the plate to the fluid uniformly. It is also assumed that there exists first order chemical reaction between the fluid and the species concentration. The reaction is assumed to take place entirely in the stream. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \tag{2}$$

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$$\frac{\partial C'}{\partial t'} = D\,\frac{\partial^2 C'}{\partial y^2} - K_l C' \tag{3}$$

with the following initial and boundary conditions:

$$\begin{array}{lllllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all} & y, t' \le 0 \\ t' > 0: & u = \dfrac{u_0^{\,3} t'}{\nu}, & \dfrac{\partial T}{\partial y} = -\dfrac{q}{k}, & C' = C_w & \text{at} & y = 0 \\ & u \to 0 & T \to T_\infty, & C' \to C'_\infty & \text{as} & y \to \infty \end{array} \tag{4}$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0},\; t = \frac{t' u_0^2}{\nu},\; Y = \frac{y u_0}{\nu},$$

$$\theta = \frac{T - T_\infty}{\left(\dfrac{q\nu}{k u_0}\right)},\; Gr = \frac{g\nu\beta(T_w - T_\infty)}{u_0^3},\; C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\; Gc = \frac{g\beta^* q \nu^2}{k u_0^4} \tag{5}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k}, \quad K = \frac{\nu K_1}{u_0^2}, \quad Sc = \frac{\nu}{D}$$

in equations (1) to (4), lead to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{lllllll} & U = 0, & \theta = 0, & C = 0 & \text{for all} & Y, t \le 0 \\ t > 0: & U = t, & \dfrac{\partial \theta}{\partial Y} = -1, & C = 1 & \text{at} & Y = 0 \\ & U \to 0, & \theta \to 0, & C \to 0 & \text{as} & Y \to \infty \end{array} \tag{9}$$

All the physical variables are defind in the nomenclature.The dimensionless governing equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

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$$\theta = 2\sqrt{t}\left[\frac{\exp(-\eta^2 \Pr)}{\sqrt{\pi}\sqrt{\Pr}} - \eta erfc(\eta\sqrt{\Pr})\right] \tag{10}$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc}) erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \tag{11}$$

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$$\begin{aligned} U = {} & t\left[(1+2\eta^2)erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] \\ & - \frac{at\sqrt{t}}{\sqrt{\Pr}}\left[\begin{aligned} & \frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1+\eta^2 \Pr)\exp(-\eta^2 \Pr) - \eta(6+4\eta^2)erfc(\eta) \\ & + \eta\sqrt{\Pr}(6+4\eta^2 \Pr)erfc(\eta\sqrt{\Pr}) \end{aligned}\right] \\ & - b\left[\exp(2\eta\sqrt{ct}).erfc(\eta+\sqrt{ct}) + \exp(-2\eta\sqrt{ct}).erfc(\eta-\sqrt{ct})\right] + 2b.erfc(\eta) \\ & - b\left[\exp(2\eta\sqrt{KtSc}).erfc(\eta\sqrt{Sc}+\sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc}).erfc(\eta\sqrt{Sc}-\sqrt{Kt})\right] \\ & + b\exp(ct)\left[\exp(-2\eta\sqrt{Sc(K+c)t}).erfc(\eta\sqrt{Sc}-\sqrt{(K+c)t})\right. \\ & \qquad \left. + \exp(2\eta\sqrt{Sc(K+c)t}).erfc(\eta\sqrt{Sc}+\sqrt{(K+c)t})\right] \end{aligned} \tag{12}$$

Where, $a = \dfrac{Gr}{3(1-Pr)}, b = \dfrac{Gc}{2c(1-Sc)}, c = \dfrac{K.Sc}{1-Sc}$ and $\eta = \dfrac{Y}{2\sqrt{t}}$.

3. Results and Discussion

For physical understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr and t upon the nature of the flow and transport. The value of Prandtl number Pr are chosen such that they represent water (Pr=7.0). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number, chemical reaction parameter and time.

The temperature profiles are calculated for water from equation (10) and these are shown in Figure 1. for different ($t = 0.2, 0.4, 0.6, 1$). It was observed thet the temperature increases with increasing values of the time t.

Figure 2 represents the effect of concentration profiles for different Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$), K=0.2 and t=0.2. The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It was observed that the wall concentration increases with decreasing values of the Schmidt number. Figure 3 illustrates the effect of the concentration profiles for different values of the chemical reaction parameter ($K = 0.2,2,5,10$) and Sc=0.6 at $t = 0.4$. The effect of chemical reaction parameter is important in concentration field.

Figure 4 depicts the effect of the velocity for different values of the reaction parameter ($K = 0.2,7,15), Sc = 0.6, Gr = Gc = 2$ and $t = 0.3$. The trend shows that the velocity increases with decreasing chemical reaction parameter. The effect of velocity for different ($t = 0.2,0.4,0.6), Gr = Gc = 2$ and K=5 are studied and presented in figure 5. It was observed that the velocity increases with increasing values of t. Figure 6. demonstrates the effects of different thermal Grashof number ($Gr = 2,10$) and mass Grashof number ($Gc = 2,10$), Sc=0.6 and K=15 on the velocity at time $t = 0.5$. It was observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

4. Conclusion

Theoretical syudy of unsteady flow past an uniformly accelerated infinite vertical plate in the presence of heat flux and uniform mass diffudion have been analyzed. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and t are studied . The conclusion of the study are as follows:

(i) The velocity increases with increasing values of Gr, Gc and t.
(ii) The transient velocity increases with decreasing Schmidt number Sc or chemical reaction parameter K.
(iii) The wall concentration increases with decreasing Schmidt number or chemical reaction parameter.

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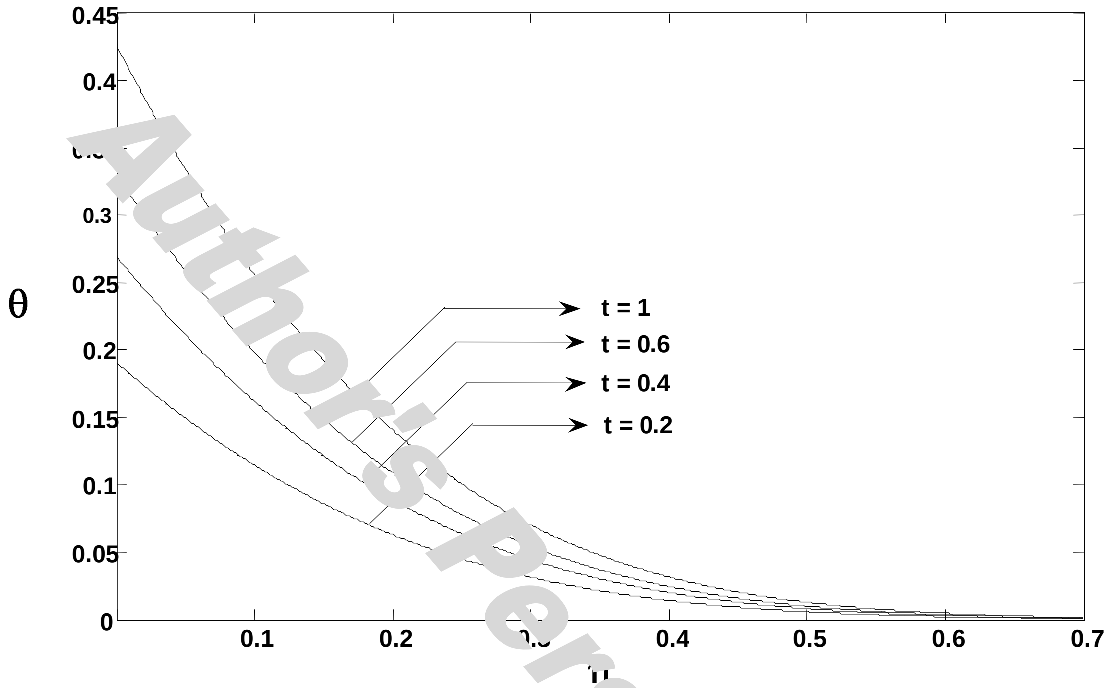

Figure 1. Temperature Profiles for different values of t

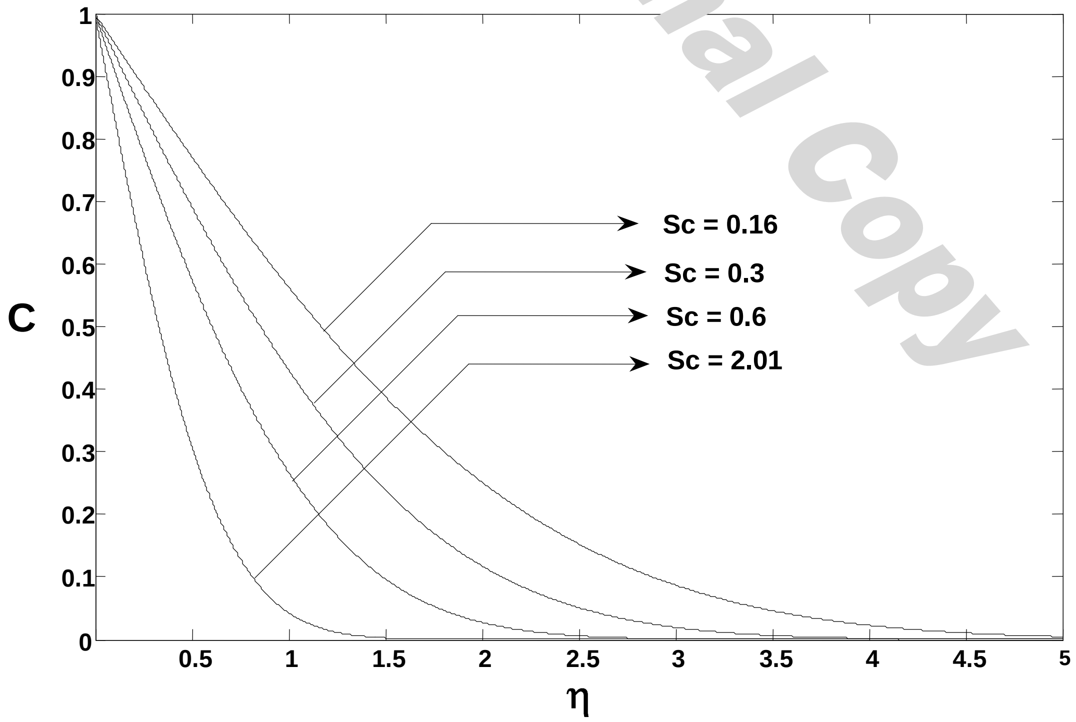

Figure 2. Concentration profiles for different values of Sc

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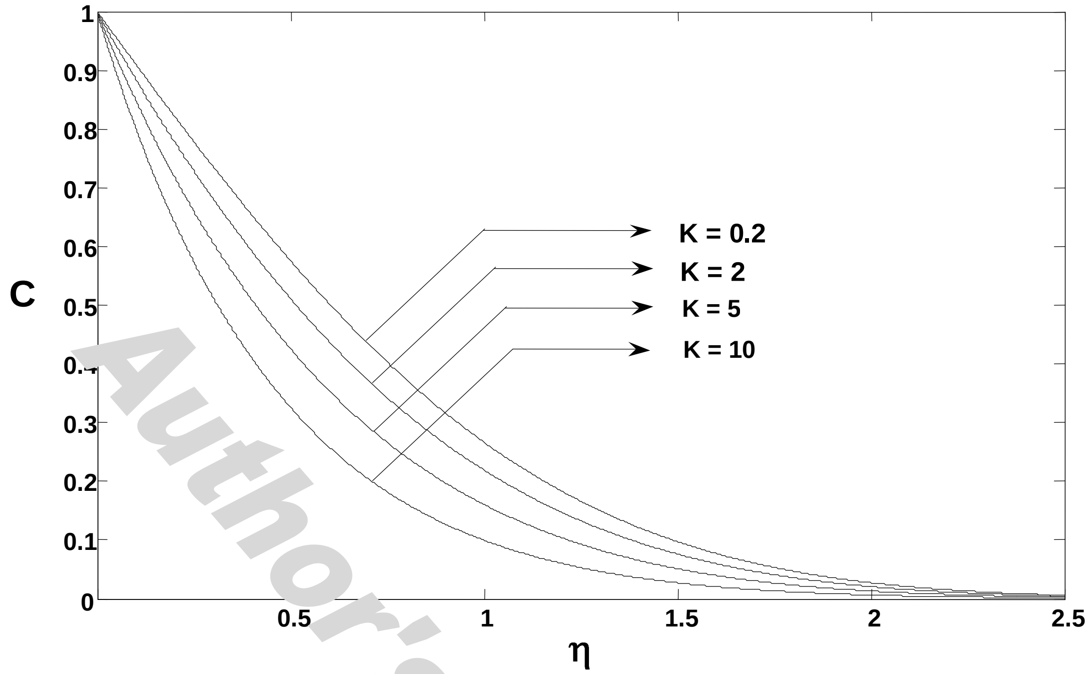

Figure 3. Concentration profiles for different values of K

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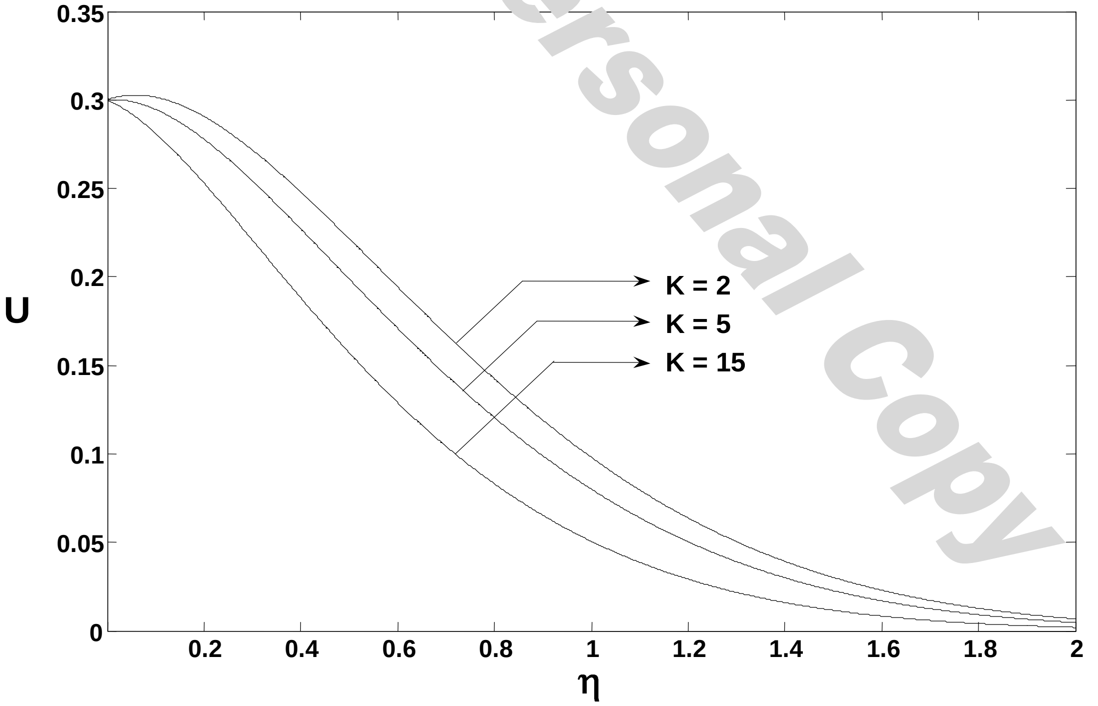

Figure 4. Velocity profiles for different values of K

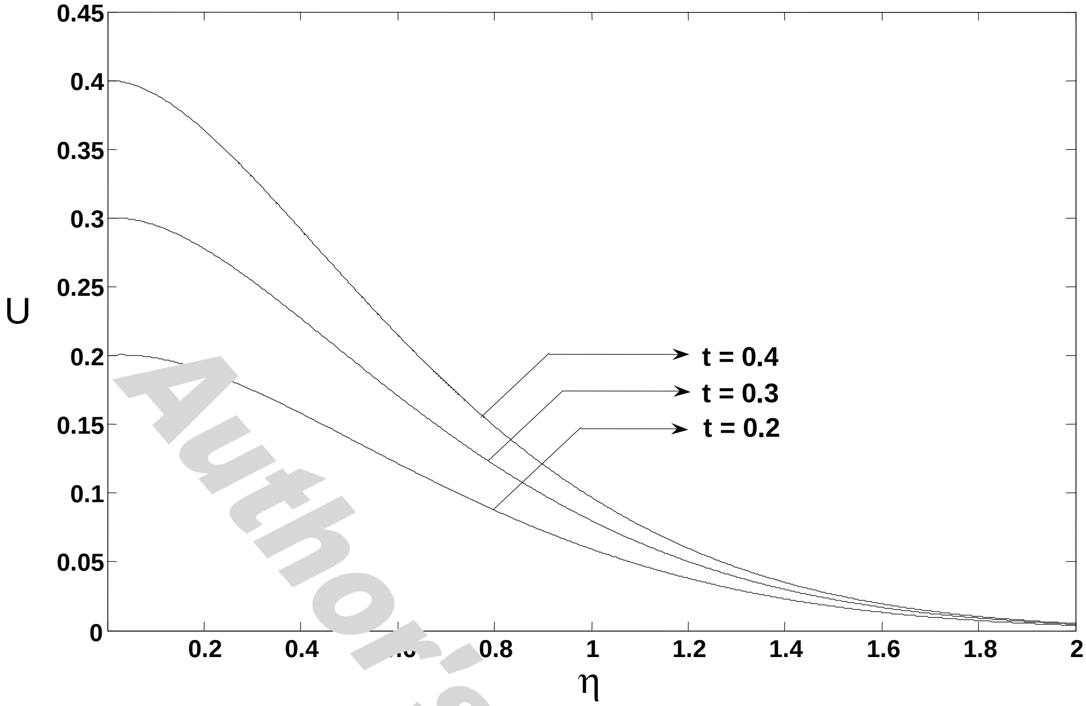

Figure 5. Velocity profiles for different values of t

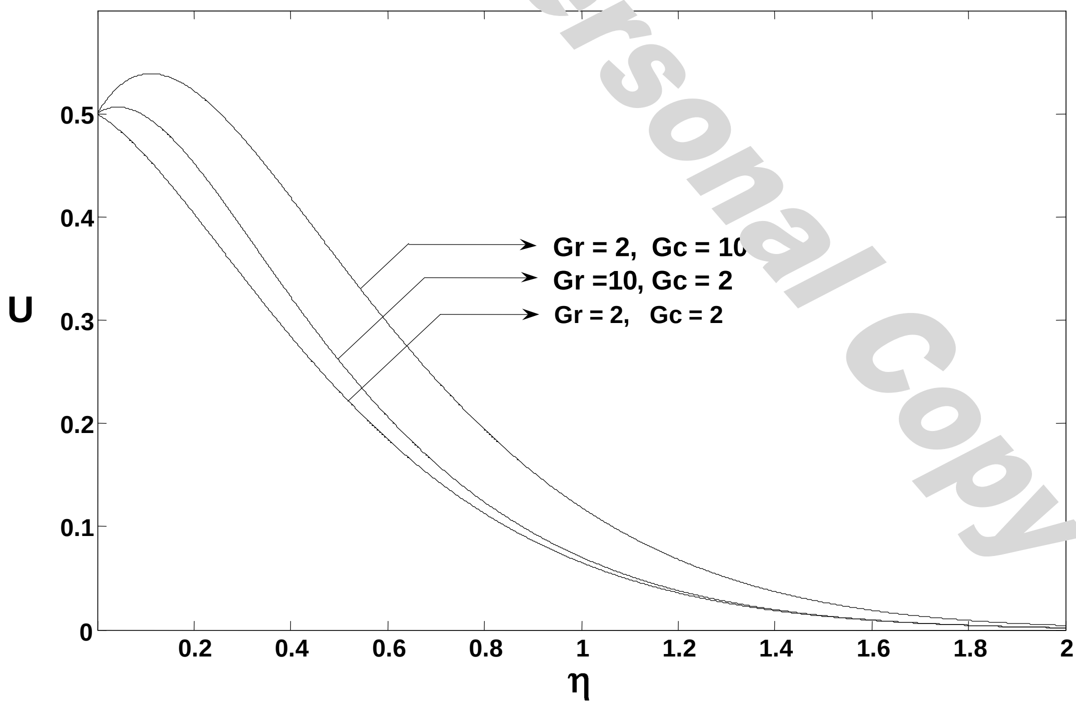

Figure 6. Velocity profiles for different values of Gr, Gc

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