ISSN 0976-1411

Available online at www.internationaleJournals.com

International eJournals

International eJournal of Mathematics and Engineering
186 (2012) 1775 – 1789

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

www.InternationaleJournals.com

Run-Upflow of a Rivlin-Ericksen Conducting Fluid Through a Planar Channel with Porous Lining Under a Transverse Magnetic Field - Brinkman Model

D.Malleswari[1], D.Raju[2] and A.LeelaRatnam[3]

[1] Department of Mathematics, Government Degree College for Women, Begumpet, Hyderabad. A.P. E-mail: malleswaridammalapati@yahoo.com

[2] Vidya Jyothi Institute of Technology, AzizNagar Gate, Hyderabad-500 075, India. E-mail: 20122102india@gmail.com

[3]Department of Applied Mathematics, Sri Padmavathi MahilaVisvavidyalayam, Tirupathi, India.E-mail: leeleratnamappikatla@yahoo.com.

ABSTRACT

In this paper, we discussed the Run – up flow of an incompressible viscoelastic Rivlin-Ericksen fluid through a parallel plate channel bounded by a sparsely packed porous bed subjected to a traverse magnetic field. The velocity in both clean fluid as well as porous zones, the stresses and the mass flux are have been evaluated and their behavior has been computationally discussed for variations in the governing parameters.

Keywords: Run – up flow, Visco – elasticity, Brinkman equation, Newtonian fluid.

Mathematical Subject classification: 58 D 30.

1. INTRODUCTION:

Fluids are classified into two categories based on the constitutive equation of the fluid i.e., Newtonian fluids and Non-Newtonian fluids. Fluids like polymer solutions and paints in which stress and rate of strain relationship is nonlinear such are called non-Newtonian fluids. Majority of transportation of such non-Newtonian fluids through uniform or non uniform channels are related to Industrial, Technological or Biomedical problems.

Rivlin-Ericksen fluid is a class of elastico viscous fluids which cannot be characterized by Maxewell's constitutive relation or oldroyd's constitutive relation. Many research workers have paid attention towards the study of Rivlin-Ericksen fluids **(**Sharma and kumar[2], Ozer and Suhubi and Sharma et al [3] **)**. Pattabhi Ramacharyulu and AppalaRaju [1] have studied run-up flow in a generalized porous medium. Ramakrishna [2] discussed a similar problem related to the flow of a dusty viscous fluid in a conduit choosing parallel plate geometry and cylindrical geometry. Raji Reddy and Sambasiva Rao [3] analyzed run-up flow of viscous incompressible fluid through a rectangular pipe, a pipe of equilateral triangular cross-section, parallel plate channel and a cylinder. They solved the problem using ADI numerical technique. Basha [4] extended the analysis of Raji Reddy and Sambasiva Rao [3] by considering visco-elastic Rivlin-Ericksen fluid between parallel plates. He extended this study by taking a second order Rivlin – Ericksen fluid between parallel porous plates subjected to a constant suction.

The Darcy law is noticed to be inadequate to describe real flows of Newtonian and non-Newtonian fluids, which are of great practical interest. Hence a consideration for non-Darcian description for such flows through porous media is warranted. Brinkman [1] suggested a modification to the Darcy's law, which involves viscous stresses to account for the distortion of the velocity profiles near the boundaries. It is established that when the porosity is large this Brinkman equation obtained by adding the Laplacian term in velocity to Darcy's law given satisfactory results.

In this paper, we discuss the Run-up flow of an incompressible Viscoelastic Rivlin – Ericksen fluid in a parallel plate channel bounded below by a sparsely packed porous bed. The flow in the non-porous region (zone 1) is governed by Navier-stokes equations while the Brinkman equation has been used for the momentum equation in the porous bed (zone 2). Initially the flow is due to a pressure gradient with boundaries at rest, and at time t > 0 the pressure gradient is withdrawn the upper plate suddenly moves with a uniform velocity while the lower plate continues to be at rest. The unsteady governing equations are solved as initial value problem. The velocity in both the clean fluid and porous zones has been evaluated and their behaviour is discussed computationally by variations in the governing parameters. Making use of the transform techniques the unsteady governing equations are solved imposing the boundary as well as interfacial continuity conditions. The velocity in both the zones the stresses on the boundaries and the mass flux have been evaluated and their behavior have been computationally discussed.

2. FORMULATION AND SOLUTION OF THE PROBLEM:

We choose the Cartesian system O ((x', y') and in accordance with the run-up flow mechanism in the absence of any extraneous force the flow is unidirectional along the direction of the imposed pressure gradient parallel to the boundary planes.

The equations governing the initial flow in non-dimensional form in zone – 1 and zone – 2 are

$$\frac{d^2u}{dy^2} - M^2Ru = PR \tag{1}$$

$$\frac{d^2u_p}{dy^2} - \lambda D^{-2}u_p - M^2Ru_p = PR \tag{2}$$

Where

$$M^2 = \frac{\sigma\mu_e^2 H_0^2 h}{\rho U} \qquad \text{(The Hartmann number)}$$

$$R = \frac{\rho U h}{\mu} \qquad \text{(The Reynolds number)}$$

$$D^{-2} = \frac{h^2}{k} \qquad \text{(The inverse Darcy parameter)}$$

$$\lambda = \frac{\mu_{eff}}{\mu} \qquad \text{(The ratio of viscosities)}$$

The respective boundary conditions in non-dimensional form are

$$u = 0 \text{ at } y = 1 \tag{3}$$

$$u_p = 0 \; at \; y = 0 \tag{4}$$

The interfacial conditions in non-dimensional form are

$$u = u_p, \frac{du}{dy} = \lambda \frac{du_P}{dy} \;\; at \;\; y = s_1 \tag{5}$$

At t > 0 the momentum equations governing the flow in non-dimensional form are

in Zone – 1

$$\frac{\partial u}{\partial t} = \frac{1}{R}\frac{\partial^2 u}{\partial y^2} + S\frac{\partial^3 u}{\partial t \partial y^2} - M^2 u \tag{6}$$

in Zone – 2

$$\frac{\partial u_p}{\partial t} = \frac{1}{R}\frac{\partial^2 u_p}{\partial y^2} + S\frac{\partial^2 u_p}{\partial t \partial y^2} - (D^2 R)^{-1}\lambda u_p - M^2 u_p \tag{7}$$

Where $S = \frac{\alpha_1}{\rho h^2}$ is the viscoelastic parameter

The corresponding non-dimensional boundary and interfacial conditions are

$$u = 1 \; at \; y = 1 \;\; (t > 0) \tag{8}$$

$$u_p = 0 \; at \; y = 0 \;\; (t > 0)$$

$$\left.\begin{array}{l} u = up \; at \; y = s_1 \\ \dfrac{du}{dy} = \lambda \dfrac{du_p}{dy} \; at \; y = s_1 \end{array}\right\} \tag{9}$$

Solving (1) and (2) subjected the condition (3) to (5)

The expressions for the velocities u and up corresponding to the initial flow are

$$(u)_{t=0} = \frac{Sinh(M\sqrt{R}(1-y))}{d_{25}}\left\{\frac{-PR}{M\sqrt{R}}\left(Sinh(M\sqrt{R}s_1) - Sih(M\sqrt{R})\right) + \right.$$

$$\frac{PRSinh(M\sqrt{R})}{M_1}d_{16} -$$

$$\left[\frac{Sinh(M_1 s_1)\ Sinh(M\sqrt{R})}{\lambda M_1 \dfrac{Sinh(M\sqrt{R}}{M\sqrt{R}}\left(Cosh(M_1 s_1) - \dfrac{PR}{M_1}Sinh(M_1 s_1)\right)Sinh(M\sqrt{R}(1-s_1)) - Sinh(M_1 s_1)}\right]$$

$$\left.\left[\frac{-PR}{M\sqrt{R}}\left(Sinh(M\sqrt{R}(1-s_1)) + d_{30} - \frac{M\sqrt{R}}{M_1}d_{16}\right)\right]\right\}$$

$$+\frac{PR}{M\sqrt{R}}\left(\frac{Sinh(M\sqrt{R}y)}{Sinh(M\sqrt{R})} - 1\right) \tag{10}$$

$$(u_p)_{t=0} = Sinh(M_1 y)\left(\frac{\dfrac{PR}{M\sqrt{R}}\left(Sinh(M\sqrt{R}(1-s_1)) + d_{30} - \dfrac{M\sqrt{R}}{M_1}d_{16}\right)}{\lambda M_1 \dfrac{Sinh(M\sqrt{R}}{M\sqrt{R}}\left(Cosh(M_1 s_1) - \dfrac{PR}{M_1}Sinh(M_1 s_1)\right)Sinh(M\sqrt{R}(1-s_1)) - Sinh(M_1 s_1)}\right)$$

$$+ \frac{PR}{M_1}\left(1 - Cosh(M_1 y)\right)$$

(11)

Where $M_1 = \lambda D^{-2} + M^2 R$

The details of the constant coefficients d_{16} etc. are given in the appendix.

Now solve the governing equation (6) and (7) subjected to the conditions (8) to (9)

Let $\bar{u}, \bar{u}_p$ be the Laplace transforms of u and u_p respectively.

Taking transforms on both sides of (6) and (7) and making use of the expression for the initial velocity (10) and (11) the equations reduce to

$$\frac{d^2\bar{u}}{dy^2} - a^2\bar{u} = \left(\frac{-sR}{1+SRs}\right)\left\{\frac{Sinh(M\sqrt{R}(1-y))}{d_{25}}.\left\{\left(\begin{array}{l}\frac{-PR}{M\sqrt{R}}\left(Sinh(M\sqrt{R}s_1) - Sinh(M\sqrt{R})\right) + \\ \frac{PRSinh(M\sqrt{R})}{M_1}d_{16}\end{array}\right) - \right.\right.$$

$$\left(\frac{Sinh(M_1s_1)\,Sinh(M\sqrt{R})}{\lambda M_1\frac{Sinh(M\sqrt{R})}{M\sqrt{R}}\left(Cosh(M_1s_1) - \frac{PR}{M_1}Sinh(M_1s_1)\right)Sinh(M\sqrt{R}(1-s_1)) - Sinh(M_1s_1))}\right)$$

$$\left.\left.\left(\frac{-PR}{M\sqrt{R}}\left(Sinh(M\sqrt{R}(1-s_1)) + d_{30} - \frac{M\sqrt{R}}{M_1}d_{16}\right)\right)\right\} + \frac{PR}{M\sqrt{R}}\left(\frac{Sinh(M\sqrt{R}y)}{Sinh(M\sqrt{R})} - 1\right)\right\} +$$

$$\frac{MPR^{5/2}}{(1+SRs)}\left(\frac{Sinh(M\sqrt{R}y)}{Sinh(M\sqrt{R})}\right) + \frac{R}{(1+SRs)}\left(M^2R\,Sinh(M\sqrt{R}(1-y))\right) \qquad (12)$$

Where $a^2 = \frac{R(s+M^2)}{(1+SRs)}$

$$\frac{d^2\bar{u}_p}{dy^2} - \beta^2\bar{u}_p = \frac{R}{1+SRs}\left\{\left(\begin{array}{c}\frac{(S+1)Sinh(M_1s_1)}{\lambda M_1\frac{Sinh(M\sqrt{R})}{M\sqrt{R}}\left(Cosh(M_1s_1) - \frac{PR}{M_1}Sinh(M_1s_1)\right).} \\ .Sinh(M\sqrt{R}(1-s_1)) - Sinh(M_1s_1))\end{array}\right)\right.$$

$$\left.\left(\frac{-PR}{M\sqrt{R}}\left(Sinh(M\sqrt{R}(1-s_1)) + d_{30} - \frac{M\sqrt{R}}{M_1}d_{16}\right)\right) + \frac{PR}{M\sqrt{R}}\left(S\left(-Cosh(M_1y)\right) - d_{16}\right)\right\} \qquad (13)$$

Where $\beta^2 = \frac{R}{1+SRs}\left[+(DR)^{-1}\lambda + M^2\right]$

The boundary and the interfacial conditions in the transformed form are

$$\bar{u} = \frac{1}{s} \text{ at } y = 1 \qquad (14)$$

$$\bar{u}_p = 0 \text{ at } y = 0 \qquad (15)$$

$$\bar{u} = \bar{u}_p \;\; and \; \frac{d\bar{u}}{dy} = \lambda\frac{d\bar{u}_p}{dy} \text{ at } y = s_1 \qquad (16)$$

Solving (12) and (13) we obtain

$$\bar{u}=\left(\frac{Cosh(ay)Sinh(a)-Cosh(a)Sinh(ay)}{d_{31}}\right).\ .\left(\frac{P\lambda^{2}\beta Cosh(\beta s_1)\left[Cosh(M_1 s_1)-Cosh(\beta s_1)\right]}{M_1(M_1^2+\beta^2)}\right)-$$

$$-\left\{\frac{1}{sSinh(a)}+\frac{PR^2}{(1+SRs)M\sqrt{R}(M^2R+a^2)Sinh(a)}(d_{27})\right\}.$$

$$.\left[\left\{\frac{\left(Cosh(ay)Sinh(a)-Cosh(a)Sinh(ay)\right)}{d_{31}}\right\}\right.$$

$$\left(Sinh(as_1)\lambda\beta Cosh(\beta s_1)+aCosh(as_1)\right)-Sinh(ay)\Big]-$$

$$-\left\{\left(\frac{\left(Cosh(ay)Sinh(a)-Cosh(a)Sinh(ay)\right)}{d_{31}}\right)\right.$$

$$\left(Sinh(M\sqrt{R}(1-s_1))\lambda\beta Cosh(\beta s_1)+M\sqrt{R}Cosh(M\sqrt{R}(1-s_1))\right)-Sinh(M\sqrt{R}(1-y)).$$

$$.\left\{\frac{sR}{(1+SRs)}\left[\frac{1}{(M^2R+a^2)d_{25}}\left(\frac{-PRSinh(M\sqrt{R}(1-s_1))}{M\sqrt{R}}+\frac{PRSinh(M\sqrt{R})d_{16}}{M_1}+\right.\right.\right.\quad +$$

$$\left(\frac{M\sqrt{R}\,M_1Sinh(M_1s_1)Sinh(M\sqrt{R})}{\lambda M_1Sinh\left(M\sqrt{R}\right)\left[M_1Cosh\left(M_1s_1\right)-PRSinh\left(M_1s_1\right)\right]d_{25}-M\sqrt{R}M_1Sinh(M_1s_1)}\right)$$

$$\left.\left.\left.\left(\frac{PR}{M\sqrt{R}}Sinh(M\sqrt{R}(1-s_1))+d_{30}-\frac{M\sqrt{R}}{M_1}d_{16}\right)\right)\right]+$$

$$+\left.\frac{R^2M^2}{(1+SRs)(M^2R+a^2)}\right\}+\left\{\frac{PR^2}{M\sqrt{R}(1+SRs)(M^2R+a^2)Sinh(M\sqrt{R})}\left(d_{27}\right)\right\}.$$

$$.\left\{\frac{\left(Cosh(ay)Sinh(a)-Cosh(a)Sinh(ay)\right)}{d_{31}}\right.$$

$$\left(Sinh(M\sqrt{R})\lambda Cosh(\beta s_1)+M\sqrt{R}Cosh(M\sqrt{R}s_1)\right)-Sinh(M\sqrt{R}y) \tag{17}$$

$$\bar{u}_p=\frac{PR}{M_1(M_1+\beta^2)}\left(Cosh(M_1y)-Cosh(\beta y)\right)+Sinh(\beta y)\left\{d_{33}d_{34}\left[\frac{PR}{M_1(M_1+\beta^2)}\left(d_7\right)\right]\right\}-$$

$$-Sinh(\beta y)\left\{\frac{1}{sSinh(a)}+\frac{SPR^2}{(1+SRs)Sinh(a)}\left(\frac{1}{M\sqrt{R}(M^2R+a^2)}\right)+\frac{MPR^2}{(1+SRs)(M^2R+a^2)Sinh[a]}\right\}.$$

$$. d_{34}\left[d_{33}Sinh(as_1)+aCosh(as_1)Sinh(\beta S_1)\right]\} -$$

$$-Sinh[\beta y]\left\{\frac{-SR}{(1+SRs)}\left[\frac{1}{(M^2R+a^2)d_{25}}\left(\frac{-PRSinh[M\sqrt{R}(1-s_1)]}{M\sqrt{R}}+\frac{PRSinh(M\sqrt{R})d_{16}}{M_1}+\right.\right.\right.$$

$$+\left(\frac{M\sqrt{R}\,M_1Sinh(M_1s_1)Sinh(M\sqrt{R})}{\lambda M_1SinhM\sqrt{R}[M_1Cosh(M_1s_1)-PRSinh(M_1s_1)]\,SinhM\sqrt{R}(1-s_1)-M\sqrt{R}M_1Sinh(M_1s_1)}\right).$$

$$.\left.\left.\left.\left(\frac{PR}{M\sqrt{R}}SinhM\sqrt{R}(1-s_1)+d_{30}-\frac{M\sqrt{R}}{M_1}d_{16}\right)\right)\right]\right\}.$$

$$. d_{34}\left[Sinh(M\sqrt{R}(1-s_1))d_{33}+M\sqrt{R}Cosh(M\sqrt{R}(1-s_1))]Sinh(\beta)_1)\right] -$$

$$-\mathrm{Sinh}[\beta y]\left\{\frac{PR^2}{M\sqrt{R}(1+SRs)(M^2R+a^2)Sinh[M\sqrt{R}]}\left[-M\sqrt{R}\right]\right\}.$$

$$. d_{34}\left[\mathrm{Sinh}(M\sqrt{R}(1-s_1))d_{33}+M\sqrt{R}\mathrm{Cosh}(M\sqrt{R}s_1)\mathrm{Sinh}(\beta s_1)\right].$$

$$. \left[Sinh(M\sqrt{R}s_1)d_{33}+M\sqrt{R}Sinh(\beta i_1)Cosh(M\sqrt{R}s_1)\right] \tag{18}$$

Taking inverse Laplace Tran form of equations (17) and (18) we obtain

$$u=\frac{Cosh(d_1y)Sinh(d_1)-Cosh(d_1)Sinh(d_1y)}{d_9}d_{18} -$$

$$-\frac{1}{Sinh(M\sqrt{R})}\left\{\begin{matrix}\left[\dfrac{Cosh(M\sqrt{R}y)Sinh(M\sqrt{R})-Cosh(M\sqrt{R})Sinh(M\sqrt{R}y)}{d_6}\right]d_2\\ -Sinh(M\sqrt{R}y)\end{matrix}\right\}-$$

$$-\sum_{n=1}^{\infty}\left[(-1)^n exp(-s_nt)\frac{Cosh(a_ny)Sinh(a_n)-Cosh(a_n)Sinh(a_ny)}{d_{14}}\right]d_{35}-$$

$$-d_{36}\left\{\frac{d_5\left(Cosh(d_4y)Sinh(d_4)-Cosh(d_4)\right)Sinh(d_4y)}{d_8}\left(d_5-d_{20}\right)\right\}-$$

$$-\left\{\frac{d_4R}{(1+SRd_4)}\left[\frac{1}{d_{25}}\left[\frac{-P\sqrt{R}}{M\sqrt{R}}Sinh(M\sqrt{R}(1-s_1)+\frac{PR\,Sinh(M\sqrt{R})}{M_1}d_{16}+\right.\right.\right.$$

$$\left(\frac{M\sqrt{R}\,M_1Sinh(M_1s_1)\,Sinh(M\sqrt{R})}{\lambda M_1Sinh(M\sqrt{R})\left(M_1Cosh(M_1s_1)-PRSinh(M_1s_1)\right)Sinh(M\sqrt{R}(1-s_1))-M\sqrt{R}M_1Sinh(M_1s_1)}\right)$$

$$.\left(\frac{PR}{M\sqrt{R}}Sinh(M\sqrt{R}(1-s_1)+d_{30}-\frac{M\sqrt{R}}{M_1}d_{16}\right)\Bigg]\Bigg]+\left.\frac{R^2M^2}{(1+SRd_4)}\right\}$$

$$\left\{\left[\frac{Cosh(d_4y)Sinh(d_4)-Cosh(d_4)Sinh(d_4y)}{d_8}\right]d_{29}-Sinh(M\sqrt{R}(1-y))\right\}$$

$$-\frac{d_{19}d_{27}}{M\sqrt{R}}\left(\frac{Sinh(d_4y)}{Sinh(d_4)}-\frac{Sinh(M\sqrt{R}y)}{Sinh(M\sqrt{R})}\right)$$

$$u_p=\frac{PR}{M_1}\left\{\left(Cosh(M_1y)-Cosh(\beta_1y)\right)-Sinh(\beta_1y)\left[\frac{d_{11}d_{21}d_7}{d_9}\right]\right\}-$$

$$Sinh(\beta_0y)\left\{\frac{d_{12}d_{22}Sinh(M\sqrt{R})}{d_6}+M\sqrt{R}Cosh(M\sqrt{R}s_1)Sinh(\beta_0s_1)\right\}-$$

$$\sum_{n=1}^{\infty}(-1)^n exp(-s_nt)\left[\left\{PR^2d_{23}(\frac{s_n}{M\sqrt{R}}+M)\right\}\left(d_{15}d_{24}Sinh(a_ns_1)+a_nCosh(a_ns_1)Sinh(\beta_ns_1)\right)\right]$$

$$Sinh(\beta_ny-Sinh(\beta_2y)\left\{\frac{-d_4d_{19}}{PR}\left[\frac{1}{Sinh(M\sqrt{R})d_{25}}\left[\frac{-PRd_{25}}{M\sqrt{R}}+\frac{PR\,d_{16}\,Sinh(M\sqrt{R})}{M_1}-\right.\right.\right.$$

$$-\left(\frac{Sinh(M_1s_1)Sinh(M\sqrt{R})}{M\sqrt{R}\lambda Sinh(M\sqrt{R})(M_1Cosh(M_1s_1)]-PRSinh(M_1s_1))d_{25}}\right)$$

$$\left.\left.\left.\left(\frac{-PR}{M\sqrt{R}}\left(d_{25}+d_{30}-\frac{M\sqrt{R}d_{16}}{M_1}\right)\right)\right]\right]\right\}.$$

$$.\left[d_{26}\left(d_{25}d_{17}+M\sqrt{R}Cosh(M\sqrt{R}(1-s_1))Sinh(\beta_2s_1)\right)\right]$$

$$Sinh[\beta_2 y]\left[d_{19}\left(\frac{d_4}{M\sqrt{R}} - M\sqrt{R}\right)\right].\ d_{26}\left\{d_{17}Sinh(M\sqrt{R}s_1) - M\sqrt{R}Cosh(M\sqrt{R}s_1)Sinh(\beta_2 s_1)\right\}$$

The expressions for d_1, d_2, etc. are given in the appendix.

The shear stresses are calculated using the formula $\tau = \frac{du}{dy}$

$$\tau_{y=0} = \frac{d_{18}d_7d_1Cosh(d_1)}{d_9} + M\sqrt{R}Coth(M\sqrt{R})(\frac{d_2}{d_6}) + \frac{M\sqrt{R}}{Sinh(M\sqrt{R})} +$$

$$+\sum_{n=1}^{\infty}(-1)^n \exp(-s_n t)\left(\frac{Cosh(a_n)}{d_{14}}d_{35}\right) + d_{36}d_5\left(\frac{Cosh(d_4)}{d_8} - 1\right)$$

$$\tau_{y=1} = \frac{d_{18}d_7d_1[-1]}{d_9} + M\sqrt{R}\frac{[-1]}{Sinh(M\sqrt{R})}(\frac{d_2}{d_6}) + \frac{M\sqrt{R}Cosh(M\sqrt{R})}{Sinh(M\sqrt{R})} +$$
$$+\sum_{n=1}^{\infty}(-1)^n exp(-s_n t)\left(\frac{[-1]}{d_{14}}d_{35}\right) - d_{36}d_5\left(\frac{[-1]}{d_8} - \frac{Cosh(d_4)}{d_5}\right) - d_{27}d_{28}Sinh(M\sqrt{R})$$

The mass flux also determine by the formula

Mass flux = $\int_{s_1}^{1} u\, dy$

$$= \frac{d_{18}d_7d_1[-1]}{d_9d_1} - \frac{d_2}{d_6M\sqrt{R}}\left[\frac{[-1+Cosh(M\sqrt{R})]}{Sinh(M\sqrt{R})}\right] + \frac{[Cosh(M\sqrt{R})-1]}{M\sqrt{R}Sinh(M\sqrt{R})} -$$

$$-\sum_{n=1}^{\infty}(-1)^n \exp(-s_n t)\left(\frac{[-1+Cosh(a_n)]}{d_{14}}d_{35}\right) - d_{36}\left(\frac{[-1+Cosh(d_4)]}{d_8} - \frac{Cosh(d_4)}{d_5}\right)(d_5 + d_{20})$$

$$- d_{36}d_4\left[\frac{[Cosh(d_4)-1]}{d_4} - \frac{[Cosh(M\sqrt{R})-1]}{M\sqrt{R}}\right] +$$

$$+\left\{\frac{d_4R}{(1+SRd_4)}\left[\frac{1}{d_{25}Sinh[M\sqrt{R}]}\left(\frac{-PRd_{25}}{M\sqrt{R}} + \frac{PRSinh(M\sqrt{R})(1-Cosh(M_1s_1))}{M_1}\right.\right.\right.$$

$$.\left(\frac{M\sqrt{R}\,M_1Sinh(M_1s_1)Sinh(M\sqrt{R})}{\lambda M_1Sinh(M\sqrt{R})(M_1Cosh(M_1s_1) - PRSinh(M_1s_1))d_{25} - M\sqrt{R}M_1Sinh(M_1s_1)}\right).$$

$$\cdot\Bigg(\frac{PRd_{25}}{M\sqrt{R}}+\Bigg(\frac{Sinh(M\sqrt{R}s_1)}{Sinh(M\sqrt{R})}-1\Bigg)-\frac{M\sqrt{R}}{M_1}d_{16}+\frac{R^2M^2}{1+SRd_4}\Bigg)\Bigg)\Bigg]\Bigg]\Bigg\}.$$

$$\cdot\frac{d_{29}}{d_4}\left(\frac{[-1+Cosh(d_4)]}{d_8}-\left(\frac{1-Cosh(M\sqrt{R})}{M\sqrt{R}}\right)\right)$$

3. DISCUSSIONS:

The behaviour of the axial velocity in both clean fluid [u] and porous regions [u_p] the shear stress on the boundaries and the mass flux have been computationally evaluated for variations in the governing parameters viz. M, R, D^{-1} and S and their profiles are plotted in fig. 1-4. Also the influence of the thickness of the porous bed on the axial velocity has been discussed and the related profiles are plotted in fig. 5.

Figs. 1 and 2 correspond to the axial velocity profiles for variations M and R respectively, fixing the other parameters. We notice that the behaviour of the axial velocity in the porous as well as the clean fluid regions very much depends on the relative magnitudes of these two parameters. Keeping R = 10, when M<4 the axial velocity rises from zero on the lower plate to a maximum near the upper plate before attaining the prescribed velocity on the upper plate. However M>4 we notice that the fluid attains maximum velocity within the porous region near the interface and experiences a sudden retardation at the interface before gradually rising to attain the prescribed velocity on the upper plate [Fig. 1]. For sufficiently large M [~ 10] and $R \geq 20$, we observe that u gradually enhances from rest on the lower plate to a maximum near the upper plate and rapidly retards to attain the prescribed value on the upper plate [Fig. 2]. We may note that u does not experience any retardation at the interface as in the case of lower values of R [Fig. 1]. Fig.3 corresponds to profiles of u for increase in the permeability of the medium k. We may note that as k increases the inverse Darcy parameter D^{-1} reduces. In the porous bed the fluid moves with enhanced velocity as the permeability increases while it moves with reduced velocity in the clean fluid region [Fig. 3]. Fixing the other parameter an increase in the viscoelastic parameter S retards the fluid flow [Fig.4] everywhere in the fluid region. Also for S [<1.5] the axial velocity experiences retardation at the interface before enhancing gradually to attain the prescribed velocity on the upper plate. However for S > 1.5 we notice that such retardation not found near the interface and velocity exhibits a monotonic behaviour in attain its prescribed values on the boundary. The thickness of the porous bed influences the flow both in the porous and clean fluid regions [Fig. 5]. We observe that as the thickness increases the fluid in the porous bed moves with reduced velocity. While in the clean fluid region it experiences relatively higher speeds in [Fig. 5].

The shear stress have been evaluated for lower and upper plates for different governing parameters and tabulated in table -1 and table -2. We find that the shear stresses reducing in their magnitude on either boundary planes for increasing the thickness of the porous bed fixing all the governing parameters. For small thickness of the porous bed the shear stress on the lower plate enhances with M or R the other parameters being fixed.

However the shear stress reduced with increasing S or D^{-1} fixing other parameters. A similar behaviors a notice even for large thickness of the porous bed. On the upper plate the shear stress is once again reduces with increase in the thickness of the porous bed. For variations in the governing parameters also its magnitude reduces with increase in M or S or D^{-1}, keeping the other parameters fixed. However an increase in R enhances the shear stress on the upper plate for all sets of the remaining parameters. The mass flux has been evaluated and tabulated. The mass flux enhances with enhancement in the thickness of the porous bed for all the variations in the governing parameters. It also enhances increase in R while it reduce with increase in S. We also observe that lesser the permeability of the bed higher mass flux in the entire region fixing the other parameters it is interesting note that the mass flux enhances M for M < 8. But later reduces for higher values of M.

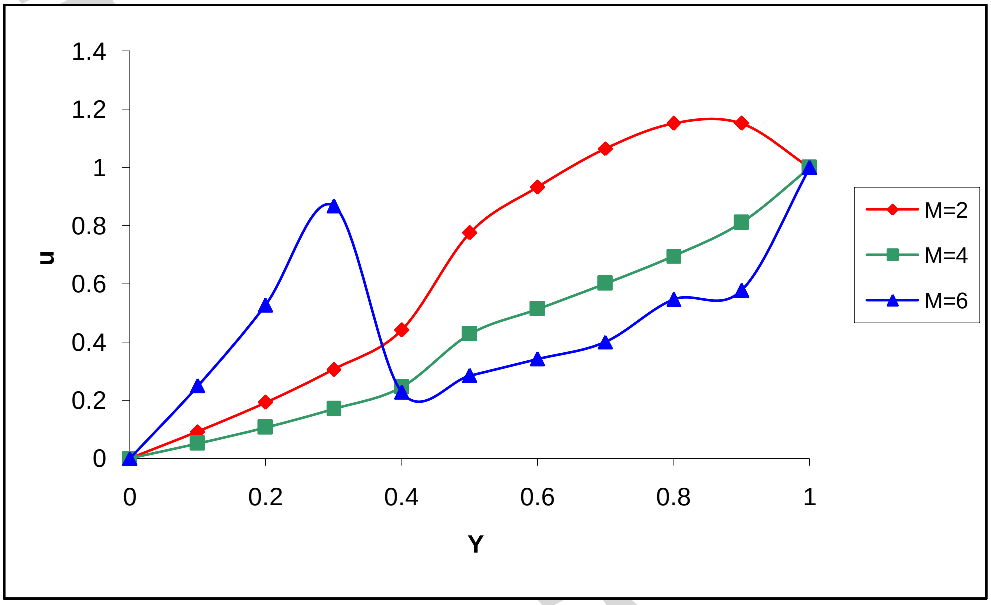

Fig. 1. Variation of axial velocity with M

P = 1, t = 1, $D^{-1} = 10^4$, R = 10, S = 2.5, $\lambda = 1.2$, $s_1 = 0.4$

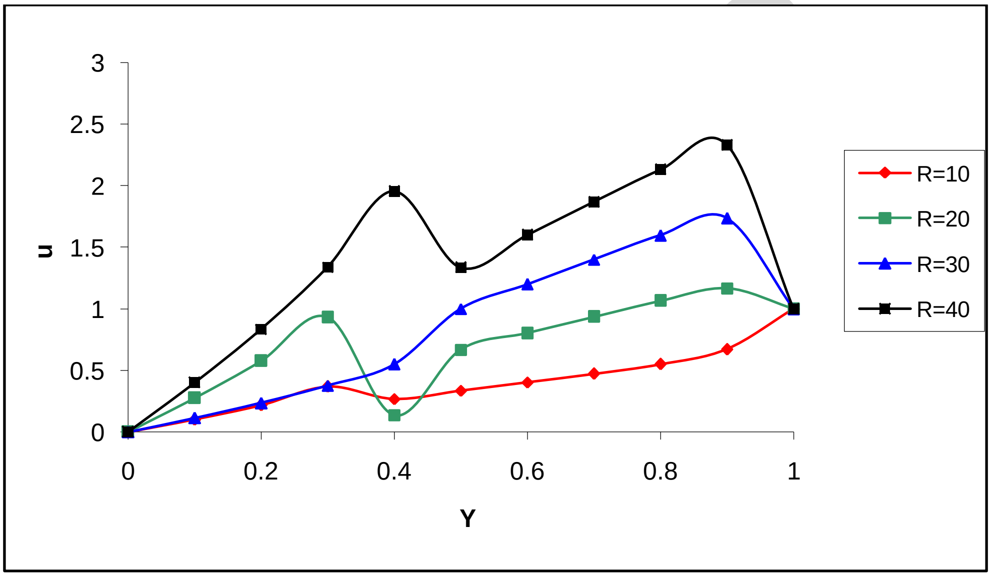

Fig. 2. Variation of axial velocity with R

P = 1, t = 1, $D^{-1} = 10^4$, M = 10, S = 2.5, λ = 1.2, s_1 = 0.4

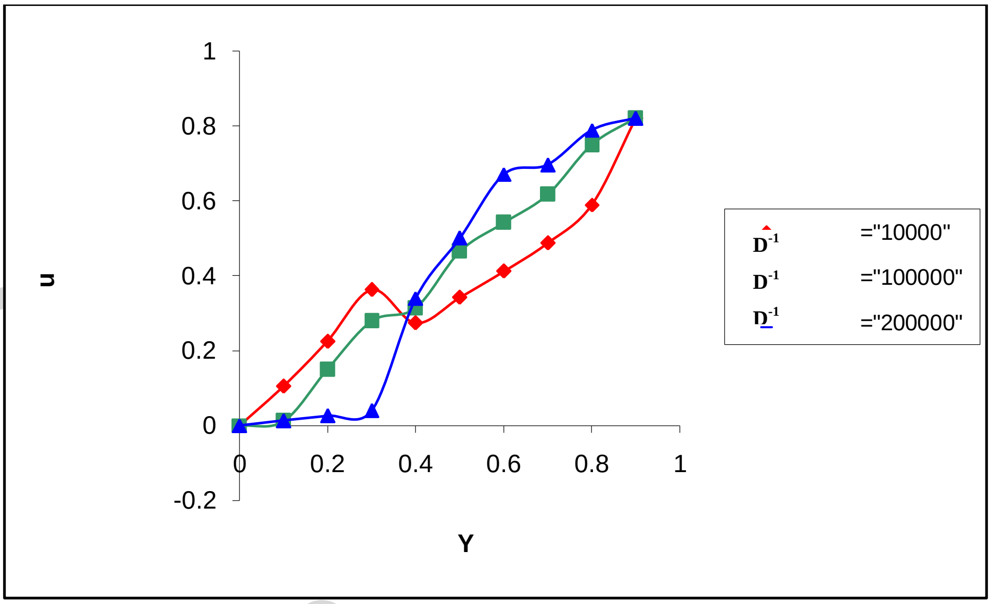

Fig. 3. Variation of axial velocity with D^{-1}

P = 1, t = 1, R = 10, M = 10, S = 2.5, λ = 1.2, s_1= 0.4

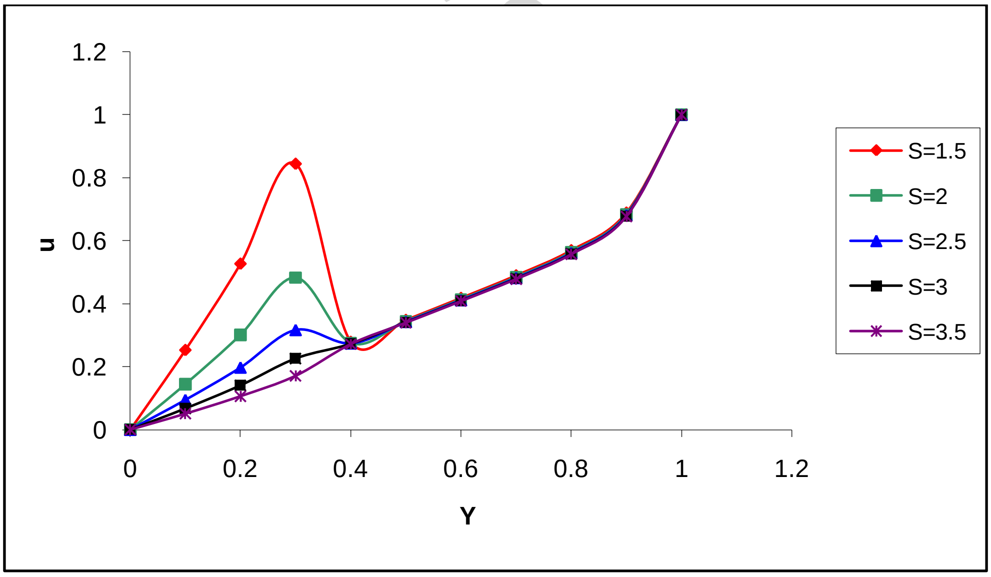

Fig. 4. Variation of axial velocity with S

P = 1, t = 1, R = 10, M = 10, $D^{-1} = 10^4$, λ = 1.2, s_1= 0.4

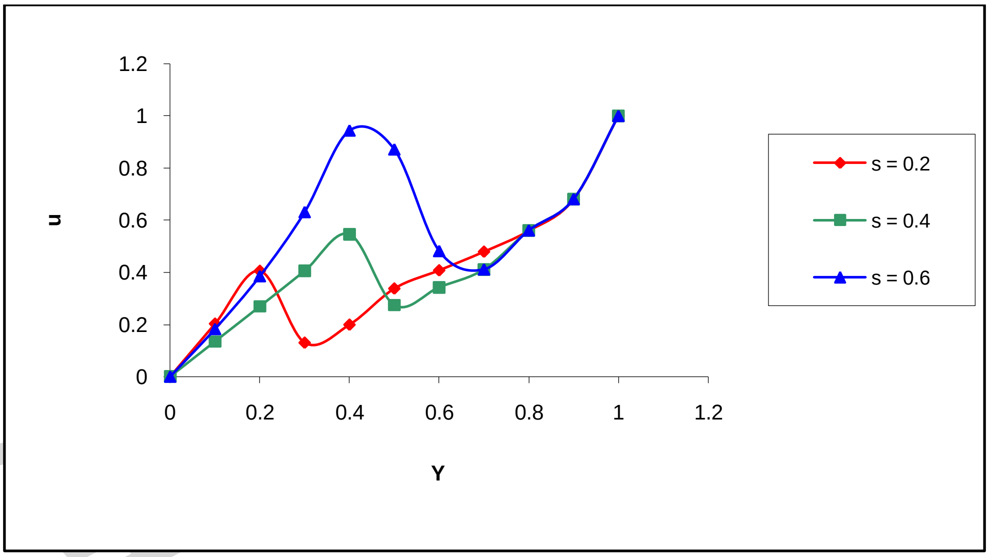

Fig. 5. Variation of axial velocity with s_1

P = 1, t = 1, R = 10, M = 10, S = 2.5, λ = 1.2, $D^{-1} = 10^4$

Table-1

SHEAR STRESS τ AT y= 0

s_1	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
0.2	-11.2568	-12.8673	-14.902	-18.193	-24.5519	-30.6506	-15.9821	-12.8893	-11.9552	-11.5128	-11.2556	-11.2568
0.4	-6.58855	-7.28827	-8.15632	-11.6336	-16.3004	-20.7513	-16.4158	-7.04258	-6.79953	-6.58846	-6.58856	-6.58856
0.6	-5.36491	-5.83101	-6.40156	-9.98149	-14.4209	-18.7113	-7.02967	-5.42504	-5.43003	-5.36485	-5.36491	-6.58856
0.8	-4.77935	-5.13706	-5.57087	-9.05139	-13.3096	-17.4944	-5.65555	-4.41886	-4.75986	-4.77931	-4.77935	-4.77938

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
M	5	8	10	5	5	5	5	5	5	5	5	5
R	10	10	10	20	30	40	10	10	10	10	10	10
S	2.5	2.5	2.5	2.3	2.5	2.5	0.5	1	1.5	2.5	2.5	2.5
D^{-1}	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4

Table-2

SHEAR STRESS τ AT y= 1

s_1	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
0.2	3.36284	3.31979	3.3119	4.49678	5.49036	6.33381	2.9356	3.2949	3.35035	3.36125	3.36284	3.36291

0.4	3.29153	3.29257	3.26685	4.53476	5.50873	6.27725	5.34896	3.26415	3.30162	3.29153	3.2852	3.27701
0.6	3.31317	3.11943	1.66351	4.44693	4.48695	0.611142	3.62499	3.29673	3.34383	3.31317	3.2363	3.2215
0.8	3.29954	-1.03902	-66.7735	2.51592	-3.8185	-2.706	3.57202	3.13089	3.32505	3.29955	3.29954	3.29960

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
M	5	8	10	5	5	5	5	5	5	5	5	5
R	10	10	10	20	30	40	10	10	10	10	10	10
S	2.5	2.5	2.5	2.3	2.5	2.5	0.5	1	1.5	2.5	2.5	2.5
D^{-1}	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4

Table-3

Mass flux

S_1	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
0.2	-39.4015	-55.0495	-35.1759	-75.5317	-97.0398	-140.586	-38.0582	-27.6337	-28.1315	-35.1759	-33.1757	-35.552
0.4	-36.1869	-55.8984	-42.1799	-77.4136	-99.5413	-143.924	-43.6886	-29.608	-29.467	-36.1865	-36.187	-36.1868
0.6	-37.2785	-56.761	-45.1595	-79.3435	-102.107	-147.347	-50.3423	-31.7401	-30.8711	-37.2280	37.2287	-37.3561
0.8	-38.3017	-57.6374	-48.373	-81.3248	-104.739	-150.858	-58.4448	-34.0492	-32.3485	-38.3012	-38.3135	-38.3369
1	-39.4075	-58.5276	-51.8693	-83.3592	-107.441	-154.461	-68.8872	-36.5614	-33.9044	-39.4016	-39.4077	-39.4522

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
M	5	8	10	5	5	5	5	5	5	5	5	5
R	10	10	10	20	30	40	10	10	10	10	10	10
S	2.5	2.5	2.5	2.3	2.5	2.5	0.5	1	1.5	2.5	2.5	2.5
D^{-1}	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4	10^4

REFERENCES:

1. Brinkman, A calculation of the viscous force extended by a flowing fluid on a dense swarm of particles. Appl. Sci. Res., A1, 27 – 34, (1947).
2. Mahaboob Basha, “Viscoelastic fluid flow and the Heat Transfer through porous medium”, Ph.D., thesis, S.K. University, anantapur, pp. 1-19 and pp 37 – 64, (1994).
3. Ozer S., Suhubi E.S. stability of poiseuille flow of an incompressible second grade Rivlin – Ericksen fluid. ARI, 51: 221 – 7 (1999).
4. Pattabhi Ramacharyulu N.Ch. and Appala Raju K., “Run-up flow in a generalized porous medium”, Indian J.Pure and Applied Math., 15(6), pp. 665 – 670; (1984).