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Intensity Impulse Response Function of Optical Systems with First-Order Parabolic Amplitude Apodization Filters

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Abstract:

The Intensity Impulse Response Function of an optical system apodized with parabolic amplitude filters of the first order has been studied. The results obtained have been discussed with the help of tables and figures. It is observed that the locations of the first minimum and the secondary maxima of the point spread function do not depend on the value of the apodization parameter. These depend only on the values of the bias or D.C value of the Pupil Function.

Key-words: Apodization, Parabolic filters, Impulse Response Function etc………

1. Introduction:

The Point Spread Function (PSF) is the optical analogue of the Impulse Response Function (IRF) used in communication theory. Different researchers have defined the PSF in different ways, though all of these definitions convey the same meaning. An ideal imaging system is one in which effects of diffraction and aberrations are absent. Further, an imaging system is said to be perfect if the aberrations are absent. However, it is impossible to remove the effects of diffraction in the images formed by an optical system. That is why a perfect optical system is known as diffraction- limited system. The real optical imaging systems are, thus, neither ideal nor perfect. Due to this the image of a point source / object formed by any real optical system is never a point.

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In the present paper, we have studied the Point Spread Function of a circularly symmetric optical system apodized by a set of parabolic filters of the first-order. The results obtained have been shown graphically and in the tabular forms. The important conclusions which can be drawn from our present study have been mentioned.

2. Expression for the point spread Function:

For our chosen system of study, the expression for the Point Spread Function or the Impulse Response Function is given by the following mathematical Equation (1):

[1] $$I(y,z)=\left|2\int_0^1\left(\alpha+\beta r^2\right)e^{-iy\frac{r^2}{2}}J_0(zr)rdr\right|^2$$

Where

y,z = Dimensionless diffraction variables along the axial and transverse directions from the focused image;

$J_0(...)$ = Bessel Function of the First kind and order zero;

$\left(\alpha+\beta r^2\right)$ = Pupil Function representing the set of parabolic filters of the First-order;

α and β are the apodization parameters which control the degree of non-uniform transmission through the exit pupil of the aperture. However, the values of α and β must be chosen such that

[2] $$f(r)=\left(\alpha+\beta r^2\right)$$

must be less than or equal to 1 in order that the passivity condition of optical systems is not violated. At the Gaussion focal plane, $y=0$, the equation [1] becomes

[3] $$I(o,z)=\left[2\int_0^1\left(\alpha+\beta r^2\right)J_0(zr)rdr\right]^2$$

3. Results and Discussions:

Figs.1(a) to 1(d) represent the pupil transmission curves for various values of α =0,0.25,0.50 & 0.75 with β =0.25 to 1.0 it is observed from the figures that for all the values of α, as the values of β as increased, the over-all transmissions increase, maintaining, of course, their super-resolving parabolic shape. The most important feature to be observed in these figures is that for a given value of α, the transmission starts from that particular value of α. For example, when α =0, the transmission is zero upto r=0.1. Therefore, for α =0, irrespective of the value of β the transmission is zero in the range $0\le r\le 0.1$. Over this range, therefore, the system acts as an annular aperture with central obscuration of r=0.1.

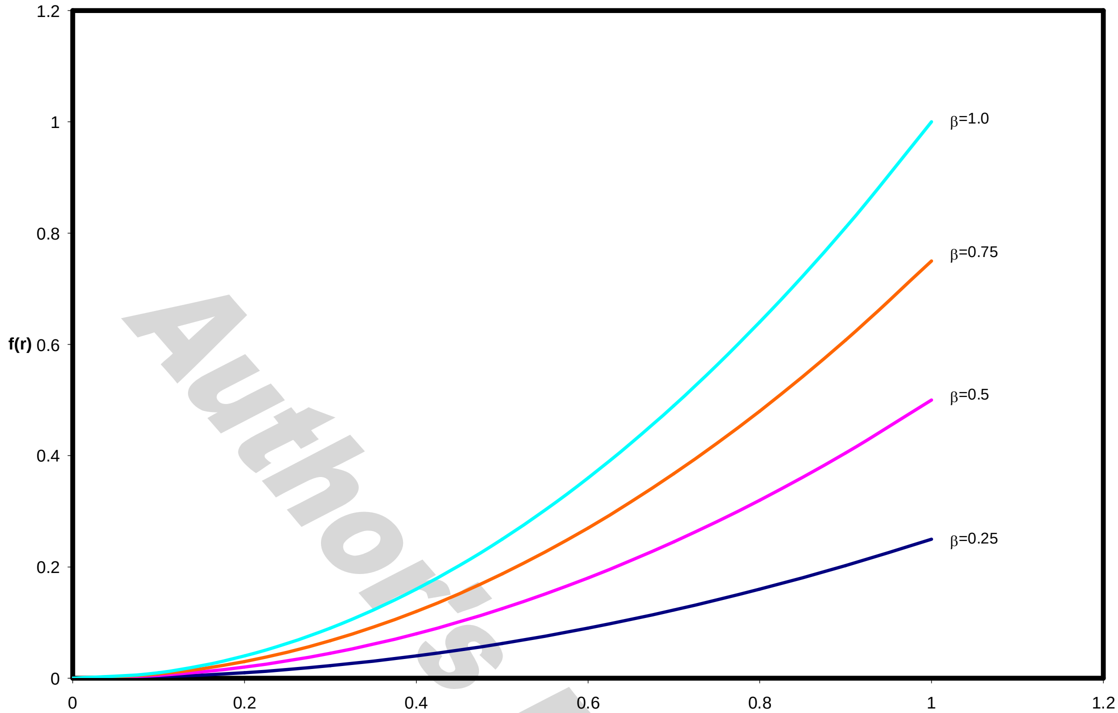

Fig.1 (a): Pupil Transmission curves for various values of β ; α=0

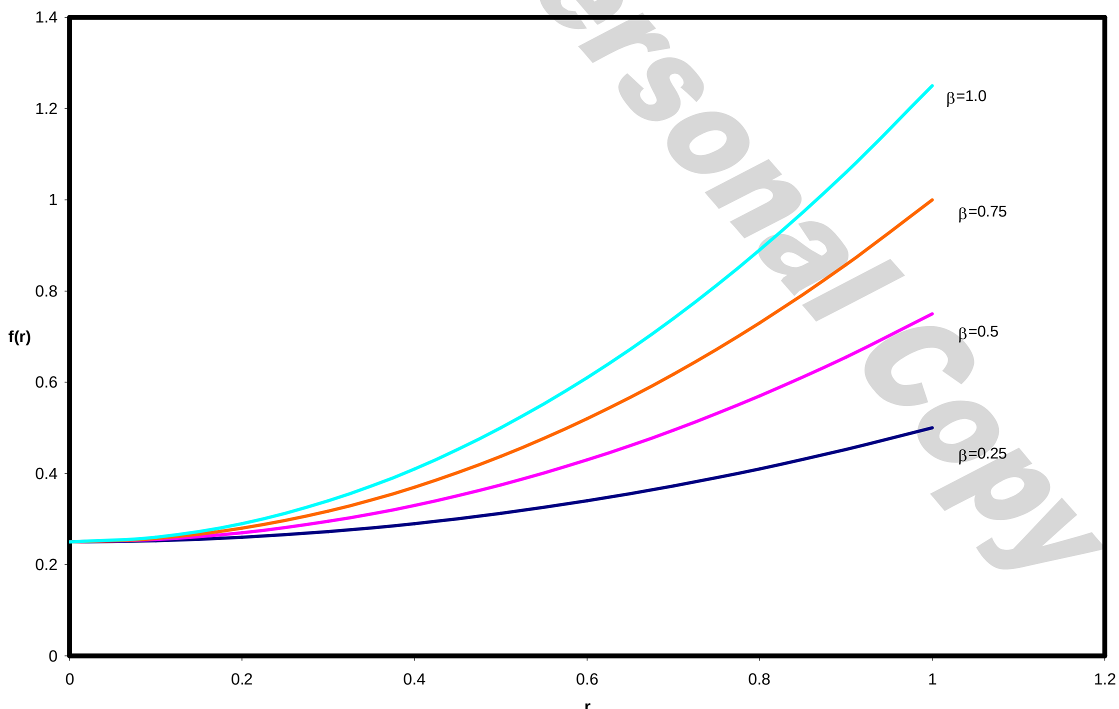

Fig.1 (b): Pupil Transmission curves for various values of β; α=0.25

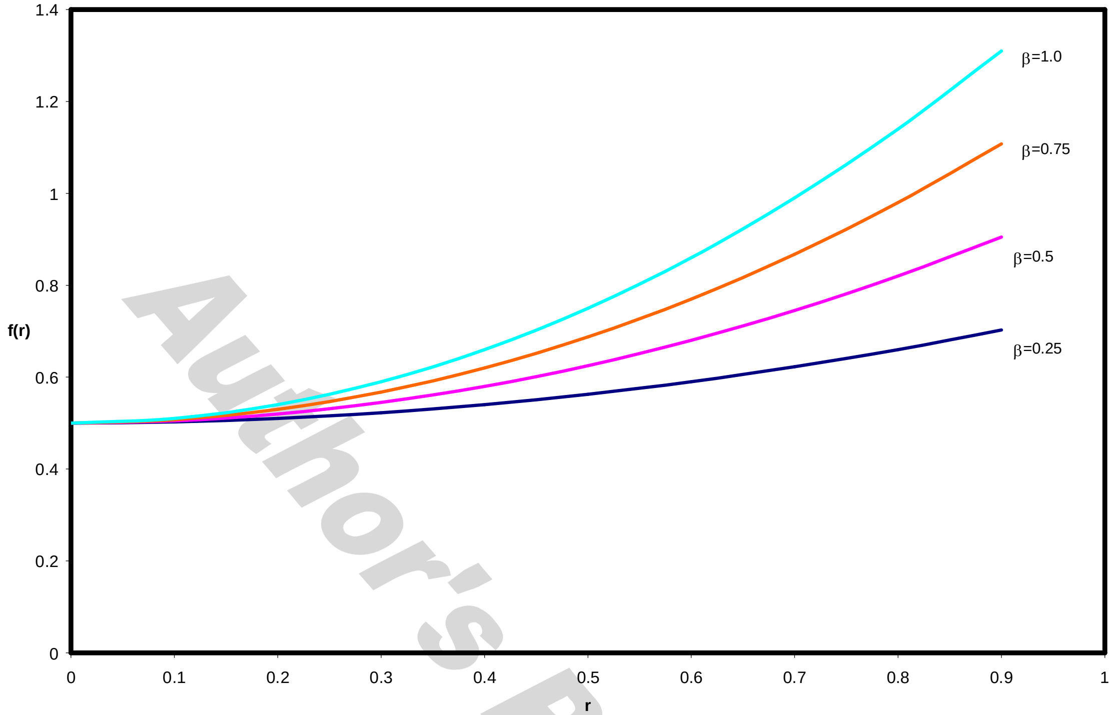

Fig:1 (c): Pupil Transmission curves for various values of β; α =0.50

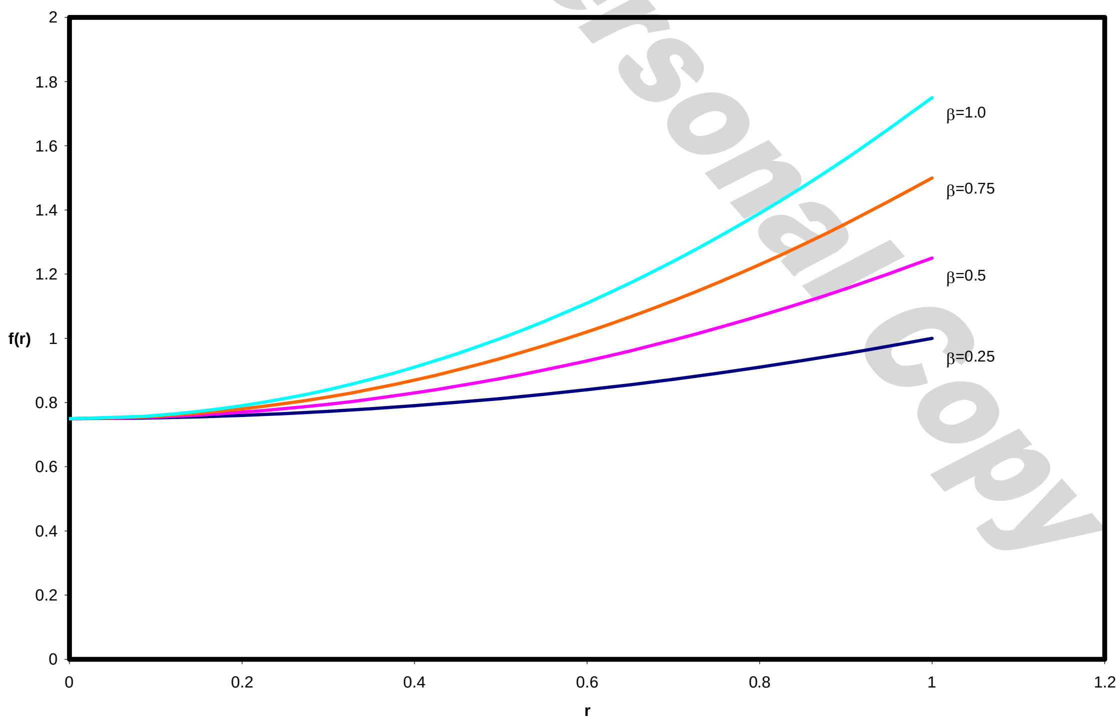

Fig.1 (d): Pupil Transmission curves for various values of β ; α =0.75

We have used the equation $[3]$ to compute the Intensity Point Spread Function for the system under our consideration

In figures 2 (a) to 2 (d) we have plotted the computed values of IPSF for some typical values of α (0, 0.25, 0.50 & 0.75) and β (0, 0.3, 0.6 & 0.9). It is observed from the figures, that for all permissible values of β for a particular value of α, the IPSF values decrease with decreasing values of β; however, the widths of the central maxima remain the same. In order to satisfy the **law of conservation of energy**, the decrease in the peak central energy is compensated by an equal amount of increasing energy in the first secondary maximum. In a previous paper (2), we have discussed about the upward shift of the point of congruence of the APSF curves with increasing values of α. The same phenomenon occurs in the case of IPSF curves also, though the shift of the point of congruence in the IPSF curves is much less compared to that in the APSF curves.

Tables of Locations of various Maxima & Minima with Intensity Values:

Table-1(a)

$\alpha = 0$	β	**Location of First Minima**	**Position of Intensity Values of 1^{st} Second Maxima**
	0	**0**	**0**
	0. 1	$3\left(10^{-6}\right)$	**4. 5(0.000212359)**
	0. 2	$3\left(4\times10^{-6}\right)$	**4. 5(0.00084944)**
	0. 3	$3\left[9\times10^{-6}\right]$	**4. 5(0.00191123)**
	0. 4	$3\left(1.5996\times10^{-5}\right)$	**4. 5(0.00530897)**
	0. 6	$3\left(3.5991\times10^{-5}\right)$	**4. 5(0.00764492)**
	0. 7	$3\left(4.8988\times10^{-5}\right)$	**4. 5(0. 0104056)**
	0. 8	$3\left(6.3984\times10^{-5}\right)$	**4. 5(0.013591)**
	0. 9	$3\left(8.098\times10^{-5}\right)$	**4. 5(0. 0172011)**
	1. 0	$3\left(9.997\times10^{-5}\right)$	**4. 5(0. 0212359)**

Table-1(b)

$\alpha = 0.25$	β	Location of First Minima	Position of Intensity Values of 1^{st} Second Maxima
	0	3.5(0.000385)	5(0.001073)
	0.1	3.5(0.000156)	5(0.002172)
	0.2	$3.5\left(2.89\times10^{-5}\right)$	5(0.003655)

	0.3	$3.5\left(3.07\times10^{-6}\right)$	5(0. 005521)
	0.4	$3.5\left(7.88\times10^{-5}\right)$	5(0. 007771)
	0.5	3.5 (0. 000256)	5(0.010404)
	0.6	3.5(0.000535)	5(0.013421)
	0.7	3.5(0.000915)	5(0.016821)
	0.8	3.5(0.001397)	5(0.020605)
	0.9	3.5(0.001981)	5(0.024372)
	1.0	3.5(0.002666)	5(0.029323)

Table-1(c)

$\alpha = 0.5$	β	Location of First Minima	Position of Intensity Values of 1st Second Maxima
	0	4(0.0002726)	5(0. 004223)
	0.1	4(0.0008361)	5(0.0062987)
	0.2	4(0.0017075)	5(0.0086885)
	0.3	4(0.002866)	5(0.0114619)
	0.4	4(0.0043735)	5(0.0146189)
	0.5	4(0.006182)	5(0.0181594)
	0.6	4(0.0082706)	5(0.0226835)
	0.7	4(0.0106809)	5(0.0263911)
	0.8	4(0.0133989)	5(0.310822)
	0.9	4(0. 0164248)	5(0.0361569)
	1.0	4(0.0197584)	5(0.416151)

Table-1(d)

$\alpha = 0.75$	β	Location of First Minima	Position of Intensity Values of 1^{st} Second Maxima
	0	3.5(0.00346639)	5(0.0096773)
	0.1	3.5(0.00267812)	5(0.0125713)
	0.2	3.5(0.0019914)	5(0.0158685)
	0.3	3.5(0.00140622)	5(0.0195492)
	0.4	3.5(0.00092258)	5(0. 0236134)
	0.5	3.5(0.0005405)	5(0.0280612)
	0.6	3.5 (0. 00025995)	5(0.0389925)
	0.7	$3.5\left(8.0956\times10^{-5}\right)$	5(0.0381074)
	0.8	$3.5\left(3.5\times10^{-6}\right)$	5(0. 0437058)
	0.9	$3.5\left(2.7599\times10^{-5}\right)$	5(0.0496878)
	1.0	3.5(0.000153)	5 (0. 056053)

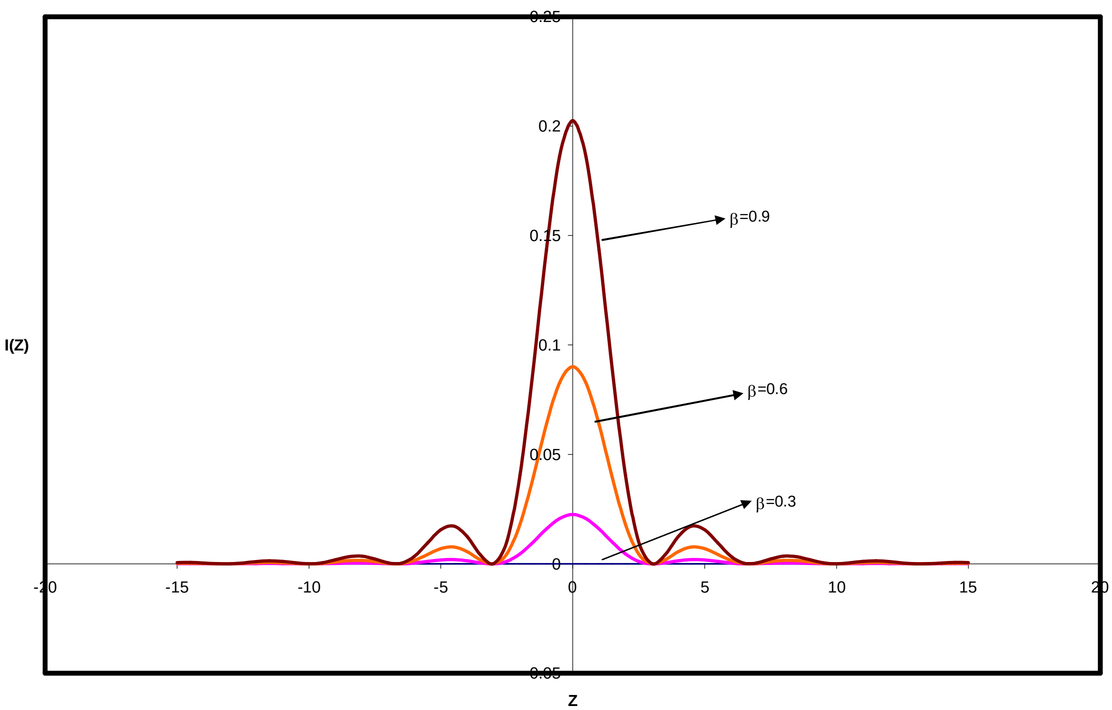

Fig.2(a) IPSF curves for various values of β ; α=0

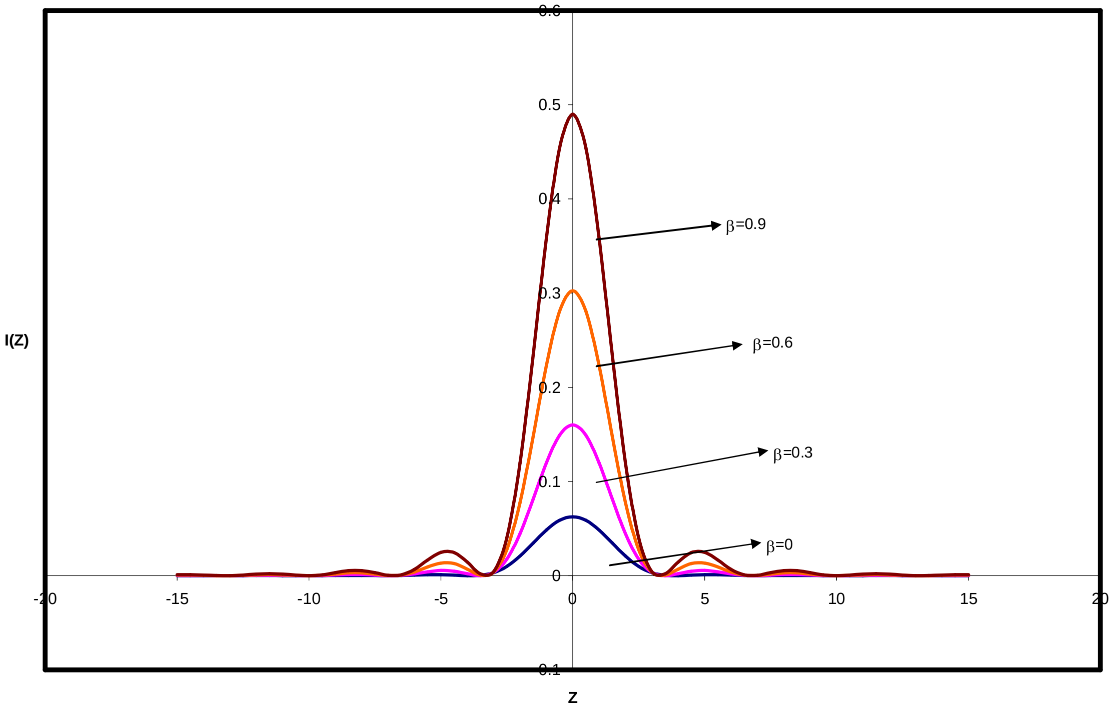

Fig.2(b) IPSF curves for various values of β ;α =0.25

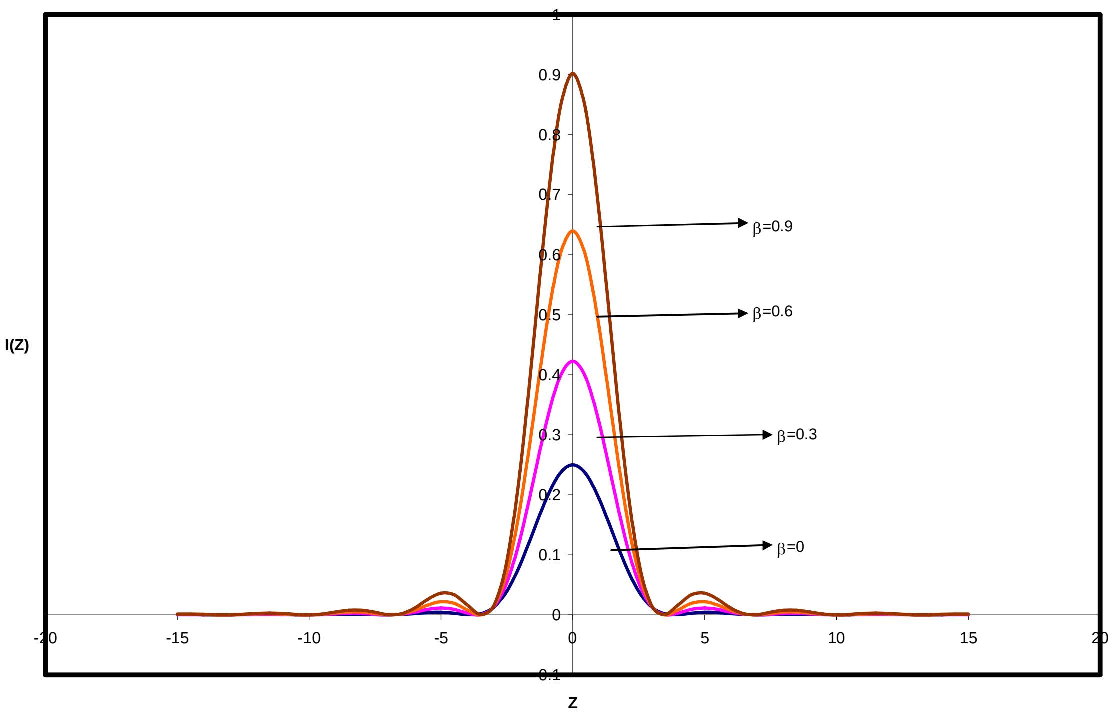

Fig.2 (c) IPSF curves for various values of β ; α=0.50

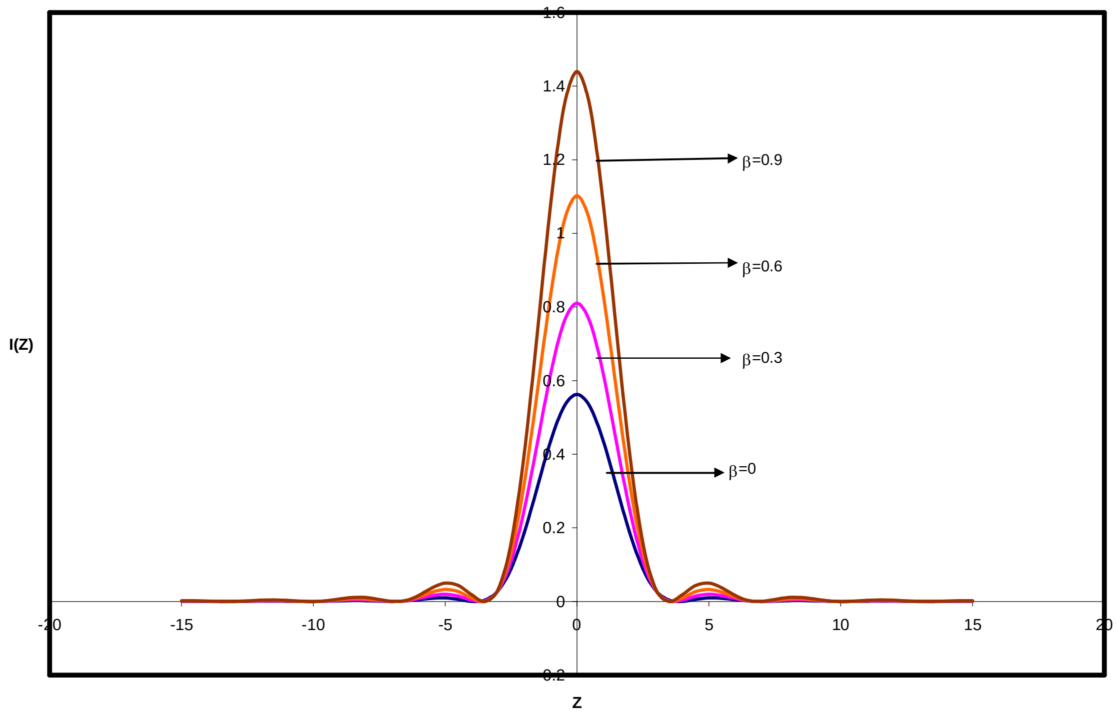

Fig.2(d) IPSF curves for various values of β; α=0.75

In the tables 1(a) to 1(d), we have shown the location of first minimum, positions and the intensity values of the secondary maxima, for various values of β for a particular value of α. It is interesting to observe that the locations of the first minimum and the secondary maxima do not depend on the value of β. It depends only on the values of α; as the value of α is increased, the position of the first minima and the various secondary maxima increase, thereby reducing the resolving power of the system, according to the classical Rayleigh criterion of resolution. However, when the value of α is very high viz., α = 0.75, there is a reversal of the above trend in the positions of the first minima and the secondary maxima.

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