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Duo Noetherian Γ-Semigroups

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ABSTRACT

In this paper the terms noetherian Γ-semigroup, Γ-closed Γ-semigroup and center of a Γ-semigroup are introduced. It is proved that if S is a noetherian Γ-semigroup containing proper Γ-ideals, then S has a maximal Γ-ideal. It is proved that if H is the collection of all Γ-ideals in a Γ-closed duo Γ-semigroup S which are not principal and $H \neq \emptyset$, then there exists a prime Γ-ideal of S which is not a principal Γ-ideal. It is proved that if every prime Γ-ideal including S is principal in a Γ-closed duo Γ-semigroup S, then every Γ-ideal in S is principal. It is proved that if S is a Γ-closed duo Γ-semigroup, which is a union of finite number of principal Γ-ideals and every proper prime Γ-ideal is principal, then every Γ-ideal is an intersection of a principal Γ-ideal and an S-Primary Γ-ideal. Also it is proved that if S is a Γ-closed duo Γ-semigroup, which is a union of finite number of principal Γ-ideals and every proper prime Γ-ideal of S is principal and S = S ΓS then every proper Γ-ideal is principal. If S is a duo Γ-semigroup such that $S \neq S\Gamma S$ and every maximal Γ-ideal is principal then it is proved that (1) S has at most two maximal Γ-ideals and (2) if P is a proper prime Γ-ideal of S then either P is a principal Γ-ideal or P = x ΓP for some $x \in S$. If every maximal Γ-ideal in a Γ-closed duo Γ-semigroup S is principal and $S \neq S\Gamma S$, $< x >^{w} = \emptyset$ for every $x \in S$, then it is proved that S is a union of two principal Γ-ideals and every Γ-ideal is an intersection of a prime Γ-ideal and an S-primary Γ-ideal. If S is a noetherian or archimedian duo Γ-semigroup such that S = $\bigcup_{i=1}^{n} < x_i >$ and suppose $a \notin < x_i \Gamma a >$ for all $a \in S$, which is not a product of power of x_i`s, then it is proved that S is finitely generated and in particular if S is noetherian strongly cancellative Γ-semigroup without identity then S is finitely generated. If S is a duo Γ-semigroup which is a union of finite number of principal Γ-ideals and if S =$S\Gamma S$, then it is proved that S contains Γ-idempotent elements. If S is a strongly Γ-cancellable duo Γ-semigroup which is a union of finite number of principal Γ-ideals, then it is proved that S contains identity if and only if S =$S\Gamma S$. In an archimedian duo Γ-semigroup S, if S is a union of finite number of principal Γ-ideals or S contains a maximal Γ-ideal which is finitely generated, then it is proved that every proper Γ-ideal is principal and S is a union of at most two principal Γ-ideals. It is proved that if A is a finitely generated Γ-ideal of a duo Γ-semigroup S, A = A ΓB for some Γ-ideal B and $a \in A$ then $a \in a\Gamma b$ for some $b \in B$. If S be a duo Γ-semigroup containing no Γ-idempotents except perhaps the identity 1 and P is a finitely generated prime Γ-ideal contained properly in $x\Gamma S$ for some $x \in S$ and $x\Gamma S \neq S$, then it is proved that (1) P does not contain any strongly Γ-cancellable element and (2) if A is finitely generated Γ-ideal containing a strongly Γ-cancellable element then A ≠ AΓB for any proper Γ-ideal B. It is proved that if A is a finitely generated Γ-ideal of a duo Γ-semigroup S and A^w = B such that A ΓB = $\bigcap Q_\alpha$ where Q_α`s are primary Γ-ideals, then A ΓB = B. If S is a noetherian duo Γ-semigroup without Γ-idempotents except perhaps identity, then it is proved that for any Γ-ideal A, $A^w \subseteq Z$ where Z is the set of all non-cancellable elements and $A^w = \emptyset$ if S is strongly Γ-cancellative. If S is a noetherian Γ-closed duo Γ-monoid with a unique maximal Γ-ideal M = < m > for some $m \in S$ and if $x \in M$ then it is proved that $x = (m\Gamma)^r u$, u is a unit or $x \in M^w$ with $x = m$ Γx Γs. If S is a noetherian duo Γ-monoid with a unique maximal Γ-ideal M = < m > for some $m \in S$ and if P is a proper prime Γ-ideal of S such that P ≠ M, then it is proved that P ⊆ M^w. If S is a noetherian duo Γ-monoid with a unique maximal Γ-ideal M = < m > for some $m \in S$ and if S has no Γ-idempotents except 1, then it is proved that M^w is a prime Γ-ideal and also if Z ≠ M where Z is the set of all non cancellable elements of S, then Z = M^w. If T is a Γ-closed duo Γ-semigroup and S is a duo Γ-semigroup such that S is a Γ-subsemigroup of T and T = x ΓS^1 for some $x \in T$ and if S is noetherian then it is proved that T is noetherian. Further an analogue of Hilbert basis theorem has obtained for duo Γ-semigroups.

Mathematical subject classification (2010) : 20M07; 20M11; 20M12.

KEY WORDS : chained Γ-semigroup, duo chained Γ-semigroup, noetherian Γ-semigroup and center of a Γ-semigroup.

1.INTRODUCTION :

Γ-semigroup was introduced by SEN and SAHA [12] as a generalization of semigroup. ANJANEYULU [4] make a study on the structure of duo chained semigroups and duo Notherian semigroups. He showed that every Noetherian cancellative duo chained semigroup without idempotents which is not globally idempotent is cyclic and satisfies TAMURA's concentric condition and characterize archimedian duo chained semigroups without idempotents. Further he proved that in a duo semigroup which is a union of finite number of principal ideals, if every proper prime ideal is principal, then every ideal is an intersection of a principal ideal and an S-primary ideal, and if this semigroup is globally idempotent, then every proper ideal is prime. Also he proved that every Noetherian cancellative duo semigroup without identity is finititely generated. Finally he showed that every cancellative Notherian duo semigroup is a direct product of the additive semigroup on nonnegative integers and a group and also he extend ' The analogue of HILBERT basis Theorem' to duo semigroups. In this paper we make a study on the structure of duo Noetherian Γ-semigroups and obtained some characterizations of duo Noetherian Γ-semigroups. Also we extend 'The analogue of HILBERT basis Theorem' to duo Γ-semigroups.

2. PRELIMINARIES:

DEFINITION 2.1 : Let S and Γ be any two non-empty sets. Then S is said to be a **Γ-*semigroup*** if there exist a mapping from $S \times \Gamma \times S$ to S which maps $(a, \gamma, b) \rightarrow a\ \gamma\ b$ satisfying the condition : $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in$ S and $\alpha, \beta \in \Gamma$.

NOTE 2.2 : Let S be a Γ-semigroup. If A and B are two subsets of S, we shall denote the set $\{ a\gamma b : a \in A, b \in B$ and $\gamma \in \Gamma \}$ by AΓB.

DEFINITION 2.3 : A nonempty subset A of a Γ-semigroup S is said to be a ***left* Γ-*ideal*** of S if $s \in S, a \in A, \alpha \in \Gamma$ implies $s\alpha a \in A$.

NOTE 2.4 : A nonempty subset A of a Γ-semigroup S is a left Γ- ideal of S iff SΓA⊆A.

DEFINITION 2.5 : A nonempty subset A of a Γ-semigroup S is said to be a ***right* Γ-*ideal*** of S if $s \in S, a \in A, \alpha \in \Gamma$ implies $a\alpha s \in A$.

NOTE 2.6 : A nonempty subset A of a Γ-semigroup S is a right Γ- ideal of S iff AΓS⊆A.

DEFINITION 2.7 : A nonempty subset A of a Γ-semigroup S is said to be a ***two sided* Γ- *ideal*** or simply a **Γ- *ideal*** of S if $s \in$ S, $a \in$ A, $\alpha \in \Gamma$ imply $s\alpha a \in$ A, $a\alpha s \in$ A.

NOTE 2.8 : A nonempty subset A of a Γ-semigroup S is a two sided Γ-ideal iff it is both a left Γ-ideal and a right Γ- ideal of S.

THEOREM 2.9 : The nonempty intersection of any two (left or right) Γ-ideals of a Γ-semigroup S is a (left or right) Γ-ideal of S.

THEOREM 2.10 : The nonempty intersection of any family of (left or right) Γ-ideals of a Γ-semigroup S is a (left or right) Γ-ideal of S.

THEOREM 2.11 : The union of any two (left or right) Γ-ideals of a Γ-semigroup S is a (left or right) Γ-ideal of S.

THEOREM 2.12 : The union of any family of (left or right) Γ-ideals of a Γ-semigroup S is a(left or right) Γ-ideal of S.

DEFINITION 2.13 : A Γ- semigroup S is said to be a ***left duo Γ- semigroup*** provided every left Γ- ideal of S is a two sided Γ- ideal of S.

DEFINITION 2.14 : A Γ- semigroup S is said to be a ***right duo* Γ- *semigroup*** provided every right Γ- ideal of S is a two sided Γ- ideal of S.

DEFINITION 2.15 : A Γ- semigroup S is said to be a ***duo* Γ- *semigroup*** provided it is both a left duo Γ- Semigroup and a right duo Γ- semigroup.

THEOREM 2.16 : A Γ- semigroup S is a duo Γ- semigroup if and only if $x\Gamma S^1 = S^1\Gamma x$ for all $x \in$ S .

THEOREM 2.17 : Let A be a Γ-ideal in a duo Γ-semigroup S and *a, b* ∈ *S*. Then *a*Γ*b* ⊆ A if and only if < *a* > Γ < *b* > ⊆ A.

COROLLARY 2.18 : Let A be a Γ-ideal in a duo Γ-semigroup S. Then for any natural number n, $(a\,\Gamma)^{n-1}a \subseteq A$ implies $(\langle a \rangle \Gamma)^{n-1} \langle a \rangle \subseteq A$.

DEFINITION 2.19 : A Γ-ideal A of a Γ-semigroup S is said to be a ***maximal* Γ*-ideal*** provided A is a proper Γ-ideal of S and A is not properly contained in any proper Γ-ideal of S.

DEFINITION 2.20 : A Γ- ideal P of a Γ-semigroup S is said to be a ***completely prime* Γ*- ideal*** provided *x, y* ∈ S and *x*Γ*y* ⊆ P implies either *x* ∈ P or *y* ∈ P.

DEFINITION 2.21 : A Γ- ideal P of a Γ-semigroup S is said to be a ***prime* Γ*- ideal*** provided A, B are two Γ-ideals of S and AΓB ⊆ P ⇒ either A ⊆ P or B⊆ P.

COROLLARY 2.22 : A Γ- ideal P of a Γ-semigroup S is a prime Γ- ideal iff *a, b* ∈ S such that $a\Gamma S^1 \Gamma b \subseteq P$, then either *a* ∈ P or *b* ∈ P.

THEOREM 2.23 : Let S be a duo Γ-semigroup. A Γ-ideal P of S is prime Γ-ideal if and only if P is a completely prime Γ-ideal.

DEFINITION 2.24 : If A is a Γ-ideal of a Γ-semigroup S, then the intersection of all prime Γ-ideals of S containing A is called ***prime* Γ*-radical*** or simply **Γ*-radical*** of A and it is denoted by **√A** or ***rad A***.

DEFINITION 2.25 : If A is a Γ-ideal of a Γ-semigroup S, then the intersection of all completely prime Γ-ideals of S containing A is called ***complete prime* Γ*-radical*** or simply ***complete* Γ*-radical*** of A and it is denoted by ***c. rad A***.

NOTE 2.26 : If A is a Γ-ideal of a Γ-semigroup S then ***rad* A = A_3** and ***c.rad* A = A_4.**

THEOREM 2.27 : If A is a Γ-ideal of a duo Γ-semigroup S, then *rad* A = *c.rad* A.

NOTATION 2.28 : If A is a Γ-ideal of a Γ-semigroup S, then we associate the following four types of sets.

A_1 = The intersection of all completely prime Γ-ideals of S containing A.

$A_2 = \{x \in S : (x\Gamma)^{n-1}x \subseteq A$ for some natural number $n\}$

A_3 = The intersection of all prime ideals of S containing A.

$A_4 = \{x \in S : (\langle x \rangle \Gamma)^{n-1} \langle x \rangle \subseteq A$ for some natural number $n\}$

THEOREM 2.29 : If A is a Γ- ideal of a Γ-semigroup S, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.

THEOREM 2.30 : If A is a Γ-ideal in a duo Γ-semigroup S then $A_1 = A_2 = A_3 = A_4$.

DEFINITION 2.31 : A Γ- semigroup S is said to be an ***archimedian* Γ- *semigroup*** provided for any $a,b \in$ S, there exists a natural number n such that $(a\Gamma)^{n-1}a \subseteq < b >$.

DEFINITION 2.32 : A Γ-semigroup S is said to be a ***strongly archimedean* Γ-*semigroup*** provided for any a, b ∈ S, there is a natural number n such that $(< a >\Gamma)^{n-1} < a > \Gamma < b >$.

THEOREM 2.33 : If S is a duo Γ -semigroup, then the conditions (1) S is strongly Archimedean, (2) S is Archimedean and (3) S has no proper prime Γ -ideals are equivalent.

DEFINITION 2.34 : A Γ-ideal A of a Γ-semigroup S is said to be a ***left primary* Γ-*ideal*** provided

i) If X, Y are two Γ-ideals of S such that XΓY ⊆ A and Y ⊈ A then X ⊆ √A.

ii) √A is a prime Γ-ideal of S.

DEFINITION 2.35 : A Γ-ideal A of a Γ-semigroup S is said to be a ***right primary* Γ-*ideal*** provided

i) If X, Y are two Γ-ideals of S such that XΓY ⊆ A and X ⊈ A then Y ⊆ √A.

ii) √A is a prime Γ-ideal of S.

DEFINITION 2.36 : A Γ-ideal A of a Γ-semigroup S is said to be a ***primary* Γ-*ideal*** provided A is both a left primary Γ-ideal and a right primary Γ-ideal.

THEOREM 2.37 : Let A be a Γ-ideal of a Γ-semigroup S. Then X, Y are two Γ-ideals of S such that XΓY ⊆ A and Y ⊈ A ⇒ X ⊆ √A if and only if *x*, *y* ∈ S, < *x* > Γ < *y* > ⊆ A and *y* ∉ A ⇒ *x* ∈ √A.

THEOREM 2.38 : Let A be a Γ-ideal of a Γ-semigroup S. Then X, Y are two Γ-ideals of S such that XΓY ⊆ A and X ⊈ A ⇒ Y ⊆ √A if and only if *x*, *y* ∈ S, < *x* > Γ < *y* > ⊆ A and *x* ∉ A ⇒ *y* ∈ √A.

DEFINITION 2.39 : A Γ-ideal A of a Γ-semigroup S is said to be ***semiprimary*** provided √A is a prime Γ-ideal of S.

DEFINITION 2.40 : A Γ-semigroup S is said to be a ***semiprimary* Γ-*semigroup*** provided every Γ-ideal of S is a semiprimary Γ-ideal.

THEOREM 2.41 : Every left primary or right primary Γ-ideal of a Γ-semigroup is a semiprimary Γ-ideal.

DEFINITION 2.42 : Let P be any prime Γ-ideal in a Γ-semigroup S. A primary Γ-ideal A in S is said to be ***P-primary*** or P is a ***prime Γ-ideal belonging to A*** provided $\sqrt{A}$ = P.

DEFINITION 2.43 : Let S be any prime Γ-ideal in a Γ-semigroup S. A primary Γ-ideal A in S is said to be S-***primary*** or S is a ***prime Γ-ideal belonging to A*** provided $\sqrt{A}$ = S.

THEOREM 2.44 : If A_1, A_2, . . . , A_n are P-primary Γ-ideals in a Γ-semigroup S, then $\bigcap_{i=1}^{n} A_i$ is also a P-primary Γ-ideal.

DEFINITION 2.45 : A Γ-ideal A in a Γ-semigroup S is said to have a (***left***, ***right***) ***primary decomposition*** if $A = A_1 \cap A_2 \cap \ldots \cap A_n$ where each A_i is a (left, right) primary Γ-ideal. If no A_i contains $A_1 \cap A_2 \cap \ldots \cap A_{i-1} \cap A_{i+1} \cap \ldots \cap A_n$ and the Γ-radicals P_i of the Γ-ideals A_i are all distinct, then the primary decomposition is said to be ***reduced***. If P_i is minimal in the set $\{ P_1, P_2, \ldots, P_n \}$ then P_i is said to be ***isolated prime***.

THEOREM 2.46 : Every Γ-ideal in a (left, right) duo noetherian Γ-semigroup S has a reduced (right, left) primary decomposition.

NOTE 2.47 : If S is a Γ-semigroup and $a \in$ S then we denote $< a >^{w} = \bigcap_{n=1}^{\infty} (< a > \Gamma)^{n-1} < a >$.

NOTE 2.48 : If S is a duo Γ-semigroup then $< a >^{w} = \bigcap_{n=1}^{\infty} < (a\Gamma)^{n-1} a > = \bigcap_{n=1}^{\infty} (a\Gamma)^{n-1} a\Gamma S^{1}$ **.**

DEFINITION 2.49 : An element a of Γ- semigroup S is said to be ***semisimple*** provided $a \in < a > \Gamma < a >$, that is, $< a > \Gamma < a > = < a >$.

DEFINITION 2.50 : An element a of Γ- semigroup S is said to be an ***α-idempotent*** if $a\alpha a = a$ for $\alpha \in \Gamma$.

DEFINITION 2.51 : An element a of Γ- semigroup S is said to be an ***idempotent*** or Γ-***idempotent*** if $a\alpha a = a$ for all $\alpha \in \Gamma$.

NOTE 2.52 : ***a*** is an idempotent of S iff a is an α-idempotent for all $\alpha \in \Gamma$.

NOTE 2.53 : If an element ***a*** of a Γ- semigroup S is *an* ***idempotent,*** then $a\Gamma a = a$.

DEFINITION 2.54 : A Γ-semigroup S is said to be an ***idempotent*** **Γ-*semigroup*** provided every element of S is an α –idempotent for some $\alpha \in \Gamma$.

DEFINITION 2.55 : An element a of a Γ-semigroup S is said to be ***regular*** provided $a = a\alpha x\beta a$, for some $x \in$ S, $\alpha, \beta \in \Gamma$. i.e, $a \in a\Gamma S\Gamma a$.

DEFINITION 2.56 : A Γ- semigroup S is said to be *a* ***regular*** **Γ- *semigroup*** provided every element of S is regular.

THEOREM 2.57 : If S is a duo Γ-semigroup, then *a* is regular if and only if *a* is semisimple for any element *a* ∈ S.

THEOREM 2.58 : If a Γ-semigroup S contains regular elements then S contains idempotents.

THEOREM 2.59 : If a Γ-semigroup S regular then S contains Γ - idempotents.

DEFINITION 2.60 : A **Γ**-semigroup S with identity element is called a ***Γ-monoid.***

DEFINITION 2.61 : A Γ-semigroup S is said to be a ***Γ-group*** if

(1) ∃ e ∈ S ∋ $a\Gamma e = e\Gamma a = a$ for all a ∈ S.

(2) every element a ∈S has a α-inverse in S for some $\alpha \in \Gamma$.

3. DUO NOETHERIAN Γ −SEMIGROUPS :

DEFINITION 3.1 : A Γ-semigroup S is said to be a ***noetherian Γ-semigroup*** if ascending chain of Γ-ideals becomes stationary. i.e., if $A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots$is an ascending chain of Γ-ideals of S, then there exists a natural number m such that $A_m = A_n$ for all natural numbers $n \geq m$.

NOTE 3.2 : A Γ-semigroup S is noetherian if and only if every Γ-ideal of S is a union of finite number of principal Γ-ideals of S.

THEOREM 3.3 : If S is a noetherian Γ-semigroup containing proper Γ-ideals then S has a maximal Γ-ideal.

***Proof* :** Let A_1 be a proper Γ-ideal of S. If A_1 is not a maximal Γ-ideal of S, then there exists a proper Γ-ideal A_2 of S such that $A_1 \subset A_2$. If A_2 is not a maximal Γ-ideal of S, then there exists a proper Γ-ideal A_3 of S such that $A_1 \subset A_2 \subset A_3$. By continuing this process we get an ascending chain of proper Γ-ideals of S. Since S is noetherian, the chain $A_1 \subset A_2 \subset A_3 \ldots$ is stationary. It is a contradiction. Therefore there exists a maximal Γ-ideal of S.

DEFINITION 3.4: A Γ-ideal A of a Γ-semigroup S is said to be ***Γ-closed*** if a, b ∈ S, **α** ∈ Γ, $a\alpha b \in A \Rightarrow a\,\Gamma b \subseteq A$.

DEFINITION 3. 5: A Γ-semigroup S is said to be ***Γ-closed*** if every Γ-ideal of S is Γ-closed.

NOTE 3.6 : In a Γ-closed Γ-semigroup S, $< a\alpha b > = < a\Gamma b >$ where a, b ∈ S and **α** ∈ Γ.

THEOREM 3.7 : Let H be the collection of all Γ-ideals in a Γ-closed duo Γ-semigroup S which are not principal. If H ≠ ∅ then there exists a prime Γ-ideal which is not a principal Γ-ideal.

***Proof* :** Let H = { $A_\alpha : \alpha \in \Delta$ } be the collection of all Γ-ideals in a duo Γ-semigroup S, which are not principal. If $\bigcup_{\alpha\in\Delta} A_\alpha = < x >$ for some $x \in S$, then $x \in A_\beta$ for some $\beta \in \Delta$. Therefore

$< x > \subseteq A_\beta \subseteq \bigcup_{\alpha\in\Delta} A_\alpha = < x >$ and hence $A_\beta = < x >$. Then $A_\beta \notin$ H. It is a contradiction. Hence $\cup_{\alpha\in\Delta} A_\alpha$ is not principal. So $\cup_{\alpha\in\Delta} A_\alpha \in$ H. Thus H satisfies all the conditions of Zorn`s lemma. By Zorn`s lemma, H has a maximal element say P. Suppose if possible P is not a prime Γ-ideal. Then there exists $a, b \in S$ such that $a\Gamma b \subseteq P$ and $a \notin P$ and $b \notin P$. Since P is maximal in H, $P \cup < b > \notin H$. Therefore $P \cup < b >$ is a principal Γ-ideal. Then $P \cup < b > = < x >$ for some $x \in S$. If $x \in P$ then we get P = $< x >$ and hence $b \in P$. It is not true. Hence $x \notin P$. Therefore $x \in < b >$and hence $< b > = < x >$. Hence P $\subseteq < b >$. Now $P' = \{s \in S : s\Gamma b \subseteq P\}$ is a Γ-ideal of S. Then clearly $a \in P'$ and $a \notin P$. Therefore $P \subset P'$ and $P \neq P'$. By the maximality of P in H, we get $P' \notin H$. Therefore $P' = < y >$ for some $y \in S$. Now $y \in P' \Rightarrow y\Gamma b \subseteq P \Rightarrow < y\Gamma b > \subseteq P$. Let $t \in P$. Since $P \subseteq < b >$, we have $t = s\gamma b$ for some $s \in S, \gamma \in T$. Now $s\gamma b \in P$. Since S is Γ-closed, $s\Gamma b \subseteq P$. Hence $s \in P' = < y >$. Therefore $s = r\beta y$ for some $r \in S, \beta \in \Gamma$. Now $t = s\gamma b = (r\beta y)\gamma b = r\beta(y\gamma b) \in < y\gamma b > \Rightarrow t \in < y\gamma b > = < y\Gamma b >$. Therefore we have $P \subseteq < y\Gamma b >$. Hence $P = < y\Gamma b >$. Thus P $\notin$ H. It is a contradiction. Therefore P is a prime Γ-ideal.

COROLLARY 3.8 : If H is the collection of all Γ-ideals in a Γ-closed duo Γ-semigroup S, which are not finitely generated and H≠∅, then there exists a prime Γ-ideal which is not finitely generated.

THEOREM 3.9 : If every prime Γ-ideal including S is principal in a Γ-closed duo Γ-semigroup S, then every Γ-ideal in S is principal.

Proof **:** Let H be the collection of all Γ-ideals in S which are not principal. If H≠∅ then by theorem 3.7, H contains a proper prime Γ-ideal which is not principal. It is a contradiction. Hence H = ∅. Therefore every Γ-ideal in S is principal.

COROLLARY 3.10: If every prime Γ-ideal including S is finitely generated in a Γ-closed duo Γ-semigroup S, then every Γ-ideal in S is finitely generated.

THEOREM 3.11 : If S is a Γ-closed duo Γ-semigroup, which is a union of finite number of principal Γ-ideals and every proper prime Γ-ideal is principal, then every Γ-ideal is an intersection of a principal Γ-ideal and an S – Primary Γ-ideal.

Proof **:** First we prove that every primary Γ-ideal Q such that $\sqrt{Q} \neq S$ is a principal Γ-ideal. Now $\sqrt{Q} = P$ is a proper prime Γ-ideal of S. Since every prime Γ-ideal is principal, we have P = $< a >$ for some $a \in S$. Therefore by theorem 2. 29, there exists $n \in N$ such that $(a\Gamma)^{n-1}a \subseteq Q$ and hence $(P\Gamma)^{n-1}P = (< a > \Gamma)^{n-1} < a > = < (a\Gamma)^{n-1}a > \subseteq Q$. Now in the case, when Q is contained in every power of P, we have Q = $(P\Gamma)^{n-1}P = < (a\Gamma)^{n-1}a >$. On the other hand, there exists $m \in N$ such that Q $\subseteq (P\Gamma)^{m-1}P$ and Q$\nsubseteq (P\Gamma)^m P$. Now A = $\{x \in S : x\Gamma(a\Gamma)^{m-1}a \subseteq Q\}$ is a Γ-ideal of S and Q = $A\Gamma(P\Gamma)^{m-1}P$. Since $Q \nsubseteq (P\Gamma)^m P$, we get A $\nsubseteq P$. Since Q is a primary Γ-ideal, $(P\Gamma)^{m-1}P \subseteq Q$ and hence Q = $(P\Gamma)^{m-1}P = < (a\Gamma)^{m-1}a >$. Therefore Q is a principal Γ-ideal. By note 3. 2, S is a noetherian Γ-semigroup. Thus by theorem 2.46, every Γ-ideal A is of the form $Q_1 \cap Q_2 \cap \ldots \cap Q_n$ where each Q_i is a primary Γ-ideal such that $P_i = \sqrt{Q_i} \neq \sqrt{Q_j} = P_j$ for i≠ j. We may assume that $P_i \neq S$ for $i = 1, 2, .., m$ and $P_i = S$ for $m+1 \leq i \leq n$. Clearly $\sqrt{Q_{m+1} \cap Q_{m+2} \cap \ldots \cap Q_n}$ = S. Therefore by theorem 2.44, $Q_{m+1} \cap Q_{m+2} \cap \ldots \cap Q_n$ is a S – primary Γ-ideal. Now we claim that $Q_1 \cap Q_2 \cap \ldots \cap Q_m = Q_1 \Gamma Q_2 \Gamma \ldots \Gamma Q_m$, which proves that $Q_1 \cap Q_2 \cap \ldots \cap Q_m$ is principal. Without loss of generality, we may assume that P_1 is maximal in $\{P_i\}_{i=1}^{m}$. P_2 is maximal in $\{P_i\}_{i=2}^{m}$ and so on. This means that $P_i \subseteq P_j$ for all $i < j$.

Now assume for $r < m$, $Q_1 \cap Q_2 \cap \ldots \cap Q_r = Q_1 \Gamma Q_2 \Gamma \ldots \Gamma Q_r$.

Therefore $Q_1 \cap Q_2 \cap \ldots \cap Q_r \cap Q_{r+1} = (Q_1 \Gamma Q_2 \Gamma \ldots \Gamma Q_r) \cap Q_{r+1} = < a > \cap Q_{r+1}$ for some $a \in S$. Let $x \in < a > \cap Q_{r+1}$. Now $x \in a\Gamma y \subseteq Q_{r+1}$.

If $a \in P_{r+1}$, then $\sqrt{< a >} = \sqrt{Q_1 \cap Q_2 \cap \ldots \cap Q_r} = P_1 \cap P_2 \cap \ldots \cap P_r \subseteq P_{r+1}$ and thus since P_{r+1} is prime, $P_i \subseteq P_{r+1}$ for some $i \leq r$. It is a contradiction. So $a \notin P_{r+1}$. Since Q_{r+1} is a primary Γ-ideal, we have $y \in Q_{r+1}$ and hence $x \in < a > \Gamma Q_{r+1}$. So $< a > \cap Q_{r+1} = < a > \Gamma Q_{r+1}$. Therefore by induction $Q_1 \cap Q_2 \cap \ldots \cap Q_m = Q_1 \Gamma Q_2 \Gamma \ldots \Gamma Q_m$. Thus every Γ-ideal is an intersection of a principal Γ-ideal and an S – primary Γ-ideal.

THEOREM 3.12 : Let S be a Γ-closed duo Γ-semigroup, which is a union of finite number of principal Γ-ideals. If every proper prime Γ-ideal of S is principal and S = S ΓS then every proper Γ-ideal is principal.

Proof **:** Since S is a **Γ**-closed duo Γ-semigroup which is a union of finite number of principal Γ-ideals, S = $\bigcup_{i=1}^{n} < x_i >$ where $x_i \notin < x_j >$ for all $i \neq j$. Since S = S ΓS, $x_i \in < x_i \Gamma x_i >$ for i = 1, 2, . ., n. Thus x_i is semi simple and hence by theorem 2.57, x_i is regular. By theorem 2.59, $< x_i > = < e_i >$ for some Γ – idempotent e_i in S. Let A be any proper Γ-ideal such that $\sqrt{A}$ = S. Therefore $e_i \in (e_i\Gamma)^{n-1}e_i \subseteq A$ for all i = 1, 2, . . ., n. Therefore $x_1, x_2, \ldots, x_n \in A$ and hence S =A. It is a contradiction. Therefore there exists no Γ-ideal of A of S such that $\sqrt{A}$ = S. By theorem 3.9, every proper Γ-ideal is principal.

THEOREM 3.13 : If S is a duo Γ-semigroup such that S ≠ $S\Gamma S$ and every maximal Γ-ideal is principal then S has at most two maximal Γ-ideals.

Proof **:** Let S be a duo Γ-semigroup such that S ≠ $S\Gamma S$. Suppose that every maximal Γ-ideal is principal. Let $a \in S \setminus S$ ΓS. Then S\{ a } is a maximal Γ-ideal. Therefore S\{ a } = < b > for some $b \in S$. Clearly $a \neq b$. Let $b \in S$ ΓS. Then S\{ a } = < b > ⊆ $S\Gamma S$ and hence S\{ a } = $S\Gamma S$. Let M be a maximal Γ-ideal of S. Then M = < c > for some $c \in S$. If $c \in S$ ΓS then M ⊆ $S\Gamma S$. Since M is maximal, M = S ΓS = S\{ a }. If $c \notin S$ ΓS then $c \notin$ S\{ a } and hence $c = a$. Thus M = < a >. So if $b \in S$ ΓS, S can have at most two maximal Γ-ideals, namely S\{ a } and < a >. Let $b \notin S$ ΓS. Then S = < b > ∪{ a } = { a }∪{ b }∪ S ΓS. Let M = < c > be a maximal Γ-ideal. If $c \notin S$ ΓS then $c = a$ or $c = b$. Then M = S\{ a } or M = S\{ b }. If $c \in S$ ΓS then $M = S$ ΓS and hence M is properly contained in a proper Γ-ideal S\{ a }. It is a contradiction. Hence S has at most two maximal Γ-ideals.

THEOREM 3.14 : Let S be a duo Γ-semigroup such that S ≠ $S\Gamma S$ and every maximal Γ-ideal is principal. If P is a proper prime Γ-ideal of S then either P is a principal Γ-ideal or P = x ΓP for some $x \in S$.

Proof **:** Let P be any proper prime Γ-ideal and $a \in S\setminus S$ ΓS. Now S\{ a } is a maximal Γ-ideal. Therefore S\{ a } = < b > for some $b \in S$. If $a \notin$ P then P ⊆ S\{ a } = < b >. If $b \in P$ then P = < b >. If $b \notin P$ then P = b ΓP, since P is a prime Γ-ideal. Let $a \in P$. If $b \in P$ then P = S. If $b \notin$ P then P ⊆ S\{ b }. Since S\{ b } is maximal Γ-ideal, we have P ⊆ S\{ b }= < x > for some $x \in S$. If $x \in P$ then P = < x >. If $x \notin$ P, let $y \in P$. Then $y \in < x >$. So $y \in x\Gamma S \subseteq P$ for some $s \in S$. Since P is prime, $s \in P$. Hence $y \in P \subseteq x\Gamma P$. Clearly $x\Gamma P \subseteq P$. Hence P = < x > or P = x ΓP for some $x \in S$.

THEOREM 3.15 : If every maximal Γ-ideal in a Γ-closed duo Γ-semigroup S is principal and S ≠ $S\Gamma S$, $< x >^{w} = \emptyset$ for every $x \in S$, then S is a union of two principal Γ-ideals and every Γ-ideal is an intersection of a prime Γ-ideal and an S-primary Γ-ideal.

Proof **:** Let P be any proper prime Γ-ideal of S. By theorem 3.14, either P is a principal Γ-ideal or P = x ΓP for some $x \in S$. If P = x ΓP for some $x \in S$, then $(x\Gamma)^n P = P$ for all natural numbers n. Thus $P = \bigcap_{n=1}^{\infty} (x\Gamma)^n P \subseteq \bigcap_{n=1}^{\infty} < (x\Gamma)^{n-1} x > = < x >^{w} = \emptyset$. It is a contradiction. Therefore P = < x > for some $x \in S$. Thus every proper prime Γ-ideal is a principal Γ-ideal. If $a \in S\setminus S\Gamma$S then by hypothesis, the maximal Γ-ideal S\{a} is of the form < b > for some $b \in S$. Therefore S = { a } ∪ < b > = < a > ∪ < b >. By theorem 3.11, every Γ-ideal of S is an intersection of a prime Γ-ideal and an S-primary Γ-ideal of S.

THEOREM 3.16 : Let S be a duo noetherian Γ-semigroup such that S = $\bigcup_{i=1}^{n} < x_i >$. Suppose $a \notin < x_i\Gamma a >$ for all $a \in S$, which is not a product of power of x_i`s. Then S is finitely generated. In particular if S is noetherian strongly Γ-cancellative Γ-semigroup without identity then S is finitely generated.

Proof **:** Suppose that there exists an element a such that a is not a product of x_i`s. If $a = x_i\alpha_1 s_1$ for $\alpha_1 \in \Gamma$, where $a \neq s_1$ is not a product of power of $x_i$`s. Hence $s_1 = x_j\alpha_2 s_2$ for $\alpha_2 \in \Gamma$, where s_2 is not product of powers of x_i`s. If $s_2 \in < s_1 >$ then $s_2 = s_1\alpha_3 r$ for some $r \in S^1$, $\alpha_3 \in \Gamma$ and hence $s_1 = x_j\alpha_2(s_1\alpha_3 r) \in < x_j \Gamma s_1 >$, which is not true. Hence $< s_1 > \subset < s_2 >$. By continuing this process, we get a nonterminating chain of Γ-ideals $< s_1 > \subset < s_2 > \subset < s_3 > \subset \cdots$ Since S is noetherian, it is a contradiction. So S is finitely generated. If S is a strongly Γ-cancellative Γ-semigroup and if $a = a\beta_1(b\beta_2 a)$ for $\beta_1, \beta_2 \in \Gamma$, then $b\beta_2 a$ is an identity in S. It is a contradiction. So $a \notin < x_i \Gamma a >$ for all $a \in S$. As above, we have S is finitely generated.

THEOREM 3.17 : Let S be a duo Γ-semigroup which is a union of finite number of principal Γ-ideals. If S =$S\Gamma S$, then S contains Γ-idempotent elements.

Proof **:** Suppose that S = $\bigcup_{i=1}^{n} < x_i >$ and $x_i \notin < x_j >$ for $i \neq j$ and S =$S\Gamma S$. Since S =$S\Gamma S$, we have $x_i \in < x_i > \Gamma < x_i >$ for each i = 1, 2, 3, ... , n. Therefore each x_i is semi simple in S. By theorem 2.57, x_i is regular in S and hence by theorem 2.58, S contains Γ-idempotents.

THEOREM 3.18 : Let S be a strongly Γ-cancellable duo Γ-semigroup which is a union of finite number of principal Γ-ideals. Then S contains identity if and only if S =$S\Gamma S$.

Proof **:** Suppose that S is a strongly Γ-cancellable duo Γ-semigroup and S =$S\Gamma S$. By theorem 3.17, S contains Γ-idempotent element say e. Let $a \in S$. Then $a\alpha(e\beta e) = a\alpha e$. Since S is strongly Γ-cancellative, $a\alpha e = a$. Similarly $e\alpha a = a$. Then e is the identity in S. Therefore S contains the identity. Conversely suppose that S contains the identity. Then clearly S =$S\Gamma S$.

THEOREM 3.19 : Let S be a duo archemedian Γ-semigroup. If S is a union of finite number of principal Γ-ideals, then every proper Γ-ideal is principal and S is a union of at most two principal Γ-ideals.

Proof **:** Suppose that S =$\bigcup_{i=1}^{n} < x_i >$. Let H be the collection of all proper Γ-ideals which are not principal. If H ≠ ∅ then clearly H is a partially ordered set under set inclusion. Let $\{A_\alpha\}$ be a chain of Γ-ideals in H. If S = $\bigcup A_\alpha$then $x_i \in A_i$ for some natural number i. If we take j = max $\{1, 2, 3, \ldots, n\}$ then $x_i \in A_j$ for i = 1, 2, 3, …, n. So S = $\bigcup_{i=1}^{n} < x_i > \subseteq A_j \subseteq S$. and hence A_j= S. It is a contradiction. Hence S ≠$\bigcup A_\alpha$. If $\bigcup A_\alpha = < a >$ for some $a \in S$, then $a \in A_i$ for some i and hence $A_i = < a >$, which is not true. Thus $\bigcup A_\alpha \in$ H. Therefore H satisfies the hypothesis of Zorn`s lemma. By Zorn`s lemma, there exists a maximal element P in H. By corollary 3.8, P is a prime Γ-ideal of S. Since S is a duo archemedian Γ-semigroup, by theorem 2.33, S has no proper prime Γ-ideals. It is a contradiction. Hence H = ∅. Therefore every proper Γ-ideal of S is a principal Γ-ideal. Let S = $\bigcup_{i=1}^{n} < x_i >$ with $x_i \notin < x_j >$ for $i \neq j$. If $n > 2$, then $S \neq < x_1 > \cup < x_2 >$. Since $< x_1 > \cup < x_2 >$;is a proper Γ-ideal, $< x_1 > \cup < x_2 >$ is a principal Γ-ideal. Thus either $< x_1 > \subseteq < x_2 >$ or $< x_2 > \subseteq < x_1 >$. This contradicts the choice of x_i`s. Thus $n \leq 2$,

THEOREM 3.20 : Let S be an archemedian duo Γ-semigroup. If S contains a maximal Γ-ideal which is finitely generated, then every proper Γ-ideal is principal and S is a union of at most two principal Γ-ideals.

Proof **:** Suppose that S contains a maximal Γ-ideal M which is finitely generated. Let $a \in S/M$. Since M is maximal, $S = M \cup < a >$. So S is a union of finite number of principal Γ-ideals. Therefore by theorem 3.19, every Γ-ideal is principal and S is a union of at most two principal Γ-ideals.

THEOREM 3.21 : Let S be an archemedian duo Γ-semigroup with S =$\bigcup_{i=1}^{n} < x_i >$.

If $a \notin < x_i$ Γa> for all $a \in S$, which is not a product of powers of x_i`s, then S is finitely generated.

Proof **:** Let S be an archemedian duo Γ- semigroup with S $=\bigcup_{i=1}^{n} < x_i >$. By theorem 3.19, S is a union of at most two principal Γ-ideals. By theorem 3.16, S is finitely generated.

THEOREM 3.22 : Let A be a finitely generated Γ-ideals of a duo Γ-semigroup S. If A = A ΓB for some Γ-ideal B and if $a \in A$ then $a \in a\Gamma b$ for some $b \in B$.

Proof **:** Suppose that A is a finitely generated Γ-ideal of a duo Γ- semigroup S. Without loss of generality, assume that A $=\bigcup_{i=1}^{n} < x_i >$ with $x_i \notin < x_j >$ for $i \neq j$. Suppose that A = A ΓB for some Γ-ideal B of S. Then A = A ΓB = $(\bigcup_{i=1}^{n} < x_i >)$ ΓB = $\bigcup_{i=1}^{n}(x_i$ ΓB). Let $a \in A$. If $a = x_i$ for some i where $1 \leq i \leq n$, then $x_i \notin < x_j >$ for $i \neq j$. So $x_i \in x_i$ ΓB. Thus $a \in a\Gamma b$. If $a \neq x_i$ for all $1 \leq i \leq n$, then $a \in < x_i >$ for some $1 \leq i \leq n$. Therefore $a = x_i\alpha s$ for some $s \in S$, $\alpha \in \Gamma$. So $a = x_i\alpha s \in (x_i\alpha s)\ \Gamma B = a\Gamma B$. Therefore $a \in a\Gamma b$ for some $b \in B$.

THEOREM 3.23 : Let S be a duo Γ-semigroup containing no Γ-idempotents except perhaps the identity 1. If P is a finitely generated prime Γ-ideal contained properly in $x\Gamma$S for some $x \in S$ and $x\Gamma S \neq$ S, then P does not contain any strongly Γ-cancellable element.

Proof **:** Suppose that S is a duo Γ- semigroup containing no Γ-idempotents except the identity 1 and P is a finitely generated prime Γ-ideal such that $P \subset x$ ΓS for some $x \in S$ and x ΓS≠ S. Since $P \subset x$ ΓS, $x \notin P$. Clearly $P \subseteq x\Gamma$P. Let $p \in P$. Since $P \subset x$ ΓS, $p \in x$ ΓS. So $p = x\alpha s$ for some $\alpha \in \Gamma$, s $\in S$. Now $x\alpha s \in P$, P is prime $\Rightarrow s \in P$. Therefore $p = x\alpha s \in x$ ΓP and hence $P \subseteq x\Gamma$P. Therefore $P = x\Gamma$P. Assume that a is a strongly cancellable element in P. By theorem 3.22, $a = a\beta b$, $b \in x\Gamma s$. Therefore $a\beta b = (a\beta b)\beta b = a\beta(b\beta b)$. Since a is strongly Γ-cancellative, we have $b = b\beta b$. It is a contradiction. Hence P does not contain strongly cancellable elements.

THEOREM 3.24 : Let S be a duo Γ-semigroup containing no Γ-idempotents except perhaps the identity 1 and P be a finitely generated prime Γ-ideal contained properly in $x\Gamma S$ for some $x \in S$ and $x\Gamma S \neq$ S. If A is finitely generated Γ-ideal containing a strongly Γ-cancellable element then A ≠ AΓB for any proper Γ-ideal B.

Proof **:** Suppose that A is a finitely generated Γ-ideal containing a strongly Γ-cancellable element say a. Suppose if possible A = AΓB for some proper Γ-ideal B. Now $a \in A = A$ΓB implies that $a = a\alpha b$ for some $b \in B$, $\alpha \in \Gamma$. Therefore $a\alpha b = (a\alpha b)\alpha b = a\alpha(b\alpha b)$. Since a is strongly Γ-cancellative, $b\alpha b = b$. Therefore B contains Γ-idempotent elements. It is a contradiction. Hence A ≠ AΓB.

THEOREM 3.25 : Let A be a finitely generated Γ-ideal of a duo Γ-semigroup S and A^w = B such that A ΓB = $\cap Q_\alpha$ where Q_α`s are primary Γ-ideals. Then A ΓB = B.

Proof **:** Since A, B are two Γ-ideals of a duo Γ- semigroup S, clearly we have AΓB ⊆ B. Let $\sqrt{Q_\alpha} = P_\alpha$ for each α. Since each $Q_\alpha$is a primary Γ-ideal of S, $\sqrt{Q_\alpha} = P_\alpha$ is a prime Γ-ideal of S for each α. Now AΓB = $\cap Q_\alpha \subseteq Q_\alpha$ for each α. Let $A \nsubseteq P_\alpha$. Since AΓB ⊆ Q_α for each α. $A \nsubseteq P_\alpha = \sqrt{Q_\alpha} \Rightarrow$ B ⊆ Q_α. Let $A \subseteq P_\alpha = \sqrt{Q_\alpha}$. Since A = $\bigcup_{i=1}^{n} < x_i >$, $x_i \in P_\alpha = \sqrt{Q_\alpha}$ for

i = 1, 2, 3, . . ., n. Then $(x_i\Gamma)^{r_i-1}x_i \subseteq Q_\alpha$ for i = 1, 2, 3, . . ., n. Let m = Max $\{r_1, r_2, \ldots, r_n\}$. Then $(A\Gamma)^{m-1}A \subseteq Q_\alpha$. Since B = $A^w = \bigcap_{m=1}^{\infty}(A\Gamma)^{m-1}A \subseteq (A\Gamma)^{m-1}A \subseteq Q_\alpha$.

Thus B ⊆ $\bigcap Q_\alpha$ = AΓB. Therefore AΓB = B.

THEOREM 3.26 : Let S be a noetherian duo Γ-semigroup without Γ-idempotents except perhaps identity. Then for any Γ-ideal A, $A^w \subseteq Z$ where Z is the set of all non-strongly Γ-cancellable elements and $A^w = \emptyset$ if S is strongly cancellative.

***Proof* :** If $A^w = \emptyset$, then clearly $A^w \subseteq Z$. If $A^w \neq \emptyset$, then by theorem 3.25, $A\Gamma A^w = A^w$. Therefore by theorem 3.24, we have A^w does not contain strongly Γ-cancellable elements. Therefore $A^w \subseteq Z$. If S is strongly Γ-cancellative then Z = ∅ and hence $A^w \subseteq Z \Rightarrow A^w = \emptyset$.

THEOREM 3.27 : Let S be a noetherian Γ-closed duo monoid with a unique maximal Γ-ideal M = < *m* > for some *m* ∈ *S*. If $x \in M$ then $x = (m\Gamma)^r u$, *u* is a unit or $x \in M^w$ with $x = m\,\Gamma x\,\Gamma s$.

***Proof* :** Let $x \in M$. Now $x \in M = < m >$ implies that $x = m\Gamma t_1$ for some $t_1 \in S$. If t_1 is not a unit, then $t_1 = m\Gamma t_2$ for some $t_2 \in S$ and hence $x = m\Gamma(m\Gamma t_2) = (m\Gamma)^2\, t_2$. By proceding the same process, we get $x = (m\Gamma)^n\, t_n$ for some natural number *n*, for some $t_n \in S$. If t_n is a unit for some natural number *n*, then $x = (m\Gamma)^n u$ where $u = t_n$, is a unit. If t_n is not a unit, then $x = (m\Gamma)^n\, t_n$ for *n* = 1, 2, 3, . . .and hence $x = (m\Gamma)^n t_n \subseteq \bigcap_{n=1}^{\infty} < (m\Gamma)^{n-1} m > \bigcap_{n=1}^{\infty} (< m > \Gamma)^{n-1} < m >$ $= < m >^w = M^w$. Therefore $x \in M^w$. Since S is noetherian, the chain $< t_1 > \subseteq < t_2 > \subseteq < t_3 >$ . . . is stationary. Therefore there exists a natural number *n* such that $< t_n > = < t_{n+1} >$ for all natural numbers *n*. Therefore $t_{n+1} = t_n \Gamma s$ for some $s \in S$. Now $x \in (m\Gamma)^n t_n = (m\Gamma)^n t_{n+1} = (m\Gamma)^n t_n \Gamma s = x\,\Gamma s$. $\therefore x = (m\Gamma)^n t_n = m\Gamma(m\Gamma)^{n-1} t_n = m\Gamma x \Gamma s$.

THEOREM 3.28 : Let S be a noetherian duo monoid with a unique maximal Γ-ideal M = < *m* > for some *m* ∈ *S*. If P is a proper prime Γ-ideal of S such that P ≠ M, then P ⊆ M^w.

***Proof* :** Let P be any proper prime Γ-ideal of S such that P ≠ M. Then there exists $x \in$ M such that $x \notin P$. By theorem 3.14, P = $x\,\Gamma$P. Thus $P = (x\Gamma)^n P$ for all natural numbers *n*. Therefore $P = \bigcap_{n=1}^{\infty}(x\Gamma)^n P \subseteq \bigcap_{n=1}^{\infty} < (x\Gamma)^{n-1} x > \subseteq M^w$ and hence P ⊆ M^w.

THEOREM 3.29 : Let S be a noetherian duo monoid with a unique maximal Γ-ideal M = < *m* > for some *m* ∈ *S*. If S has no Γ-idempotents except 1, then M^w is a prime Γ-ideal and also if Z ≠ M where Z is the set of all non strongly cancellable elements of S, then Z = M^w.

***Proof* :** Suppose that S has no Γ-idempotent elements except 1 and Z is the set of all non strongly cancellable elements of S. Therefore Z is a proper prime Γ-ideal of S. If Z ≠ M, then by theorem 3.28, Z ⊆ M^w. Since Z is a prime Γ-ideal of S and M is Γ-ideal of S, by theorem 3.26, $M^w \subseteq Z$ and hence Z = M^w.

THEOREM 3.30 : Let T be a Γ-closed duo Γ-semigroup and S be a duo Γ-semigroup such that S is a Γ-subsemigroup of T and T = $x\,\Gamma S^1$ for some $x \in T$. If S is noetherian then T is noetherian.

***Proof* :** Suppose that S is noetherian duo Γ - semigroup. Let A^1 be a proper Γ -ideal of T. Let A = $\{a \in S : x\Gamma a \subseteq A^1\}$. Let $a \in A$ and $s \in S$.
Now $a \in A \Rightarrow x\Gamma a \subseteq A^1$. Now $s \in S \subseteq T \Rightarrow s \in T$.

Since A^1 is a Γ -ideal of T, $(x\Gamma a)\Gamma s \subseteq A^1$. So $x\Gamma(a\Gamma s) \subseteq A^1$. Thus $a\Gamma s \subseteq A$. Therefore A is a right Γ -ideal of S. Since S is a duo Γ – semigroup, A is a Γ -ideal of S. Since S is noetherian, A $= \bigcup_{i=1}^{n} < a_i > = \bigcup_{i=1}^{n} a_i\Gamma S^1$. Write B $= \bigcup_{i=1}^{n} x\Gamma a_i\Gamma S^1$. Let $y \in A^1$. Clearly $y \neq x$ and $y \in T = x\Gamma S^1$. So $y = x\alpha s$ for some $s \in S$, $\alpha \in \Gamma$. Since $x\alpha s = y \in A^1$ and A^1 is Γ-closed we get $x\Gamma s \subseteq A^1$ and hence $s \in A$ and so $s \in < a_i > = a_i\Gamma S^1$ for some i. Therefore $y = x\alpha s \in x\Gamma a_i\Gamma S^1 \subseteq B$. Therefore $A^1 \subseteq B$. Let $z \in B$. Then $z \in x\Gamma a_i\Gamma S^1$. Since $a_i\Gamma S^1 \subseteq A$, we have $x\Gamma a_i\Gamma S^1 \subseteq A^1$ and hence $z \in A^1$. Therefore $B \subseteq A^1$ and hence $A^1 = B$. Now $\bigcup_{i=1}^{n} x\Gamma a_i\Gamma S^1 \subseteq A^1 = B = \bigcup_{i=1}^{n} x\Gamma a_i\Gamma S^1$. Hence $A^1 = \bigcup_{i=1}^{n} x\Gamma a_i\Gamma S^1$. ∴ T is noetherian.

DEFINITION 3. 31 : Let S be a Γ-semigroup. The set $\mathfrak{C}(S) = \{a \in S : a\Gamma s = s\Gamma a \text{ for all } s \in S\}$ is called the ***center*** of S.

THEOREM 3.32 : (ANALOGUE OF HILBERT BASIS THEOREM) :

Let T be a Γ-closed duo Γ-semigroup and S be a duo Γ-semigroup such that S is a Γ-subsemigroup of T. Suppose T $= \bigcup_{i=1}^{n} x_i\Gamma S$, x_i\`s are in the centre of T, $x_i\alpha x_j \in x_i\Gamma S$ or $x_j\Gamma S$ for some $\alpha \in \Gamma$, $i \neq j$ and $S \subseteq x_i\Gamma S$ for every i. Then T is noetherian if S is noetherian.

***Proof* :** We prove the theorem by using induction on the number n of generators of T. *i.e.* the number of x_i\`s. By theorem 3.30, the result is evident for all T with one generator. Suppose that the theorem is true for all T with the number of generators $\leq n-1$. Let A^1 be a Γ -ideal of T. Let $A = \{a \in S : x_1\Gamma a \subseteq A'\}$. Clearly A is a Γ -ideal of S and hence A $= \bigcup_{i=1}^{n} a_i\Gamma S^1$, $a_i \in S$. If $T_1 = \bigcup_{i=2}^{n} x_i\Gamma S$ and $L_1 = A^1 \cap T_1$, then T_1 is a Γ −subsemigroup of T containing S and L_1 is a

Γ-ideal in T_1. Then by induction, $L_1 = \bigcup_{i=1}^{r} b_i \Gamma T^1$. We first claim that $A^1 = B \cup L_1$ where $B = \bigcup_{i=1}^{m} x_1 \Gamma a_i \Gamma S^1$. We claim that $A' = B \bigcup L_1$. If $x \in B$ then $x \in x_1 \Gamma a_i \subseteq A'$ or $x \in x_1 \Gamma a_i \Gamma S \subseteq A'$ and hence $x \in A^1$. Thus $B \subseteq A'$. Clearly $L_1 \subseteq A'$. Therefore $B \bigcup L_1 \subseteq A'$. Let $x \in A'$. If $x \in T_1$ then $x \in A' \bigcap T_1 = L_1$ and hence $x \in B \bigcup L_1$. If $x \notin T_1$ then $x \in x_1 \Gamma S$ and hence $x = x_1 \gamma s$ for some $\gamma \in \Gamma$, $s \in S$. Since T is Γ-closed, A' is a Γ-closed Γ-ideal of T. So $x_1 \gamma s = x \in A' \Rightarrow x_1 \Gamma S \subseteq A' \Rightarrow s \in A$. $x \in x_1 \Gamma s \subseteq x_1 \Gamma a_i \Gamma S'$ for some $i \Rightarrow x \in B$. Therefore $A' \subseteq B \bigcup L_1$. Hence $A' = B \bigcup L_1$. Now $A' = B \bigcup L_1 = \bigcup_{i=1}^{m} x_i \Gamma a_i \Gamma S^1 \bigcup_{i=1}^{r} b_i \Gamma T^1$. Therefore A' is finitely generated Γ-ideal. Hence T is noetherian.

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