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SOLUTION OF FLOW PROBLEMS BY STABILITY

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ABSTRACT:

We have physical situation of which an equivalent mathematical model is constructed. We solve this mathematical model and obtained; a solution there two main techniques of solving a stability problem namely normal mode technique and the energy method. The normal mode technique is the most important technique that was widely being used so far and is also being successfully used in now a days in the leaner theory of stability. The normal mode technique essentially consists in expressing an arbitrary disturbances as a super position of certain basic mode called the normal mode. Stability of the system is the examined with respect to these modes. Since the principle of super position hold in case of linear theory of stability, the stability analysis for each mode can be made separately. In this process the perturbation are resolved in to dynamically independent wave like components satisfying the linearised equations and the boundary conditions of problems. Further the suitable set of normal modes must be complete for such an expansion to be possible.

Key words: 1.Stability 2.Normal mode 3.Perturbation 4.Flow 5.Fluid
Subarea : Fluid mechanics
Broadarea: Mathematics

INTRODUCTION

The vast application of fluid mechanics brings its.importance and significance has receiving a great value during last few decades .many of the physical theory and phenomena are evaluated successfully with it help and now we have a better understanding of that particular aspects .in practical aspects we have physical situation which we want to understand and explain .in this process of understanding it ,a mathematical modal is constructed. whenever the modal brings into the knowledge the modal so constructed is highly complicated .if at all which our modal to be realistic and accurate. for the specific concerned ,the compressibility of air ,its thermally conducting nature and other factors introduce much complications in the study of numerical and mathematical modal of various situations .needless to say that one is bound to assume certain approximations and assumptions regarding the nature of fluid and the flow boundaries, as in trend, these are not well explained and further, because to take the fast, consider the realistic situation, more and more problem becomes complicated create in original mode. although worth the help of these approximation can be able to solve more and more complicated and acute problems. this may be possible that certain error may come into contact with our knowledge. so a carefully study required in the respect of practical as result concerned. We have a physical situation of which as equivalent mathematical modal is constructed .this can be solve it is mathematical modal and get a solution of that particular modal. We consider a solution in the neighborhood of this theoretical and observe its time behavior .the neighborhood solution brings theoretical solution is frequently known as perturbed solution. Either more modal can be made for such type of practice, if brings the approximate result as perturbed solution result .this is solution for theoretical practice, and known as perturbed solution . we observed this perturbation solution approaches nearer to the basic solution as time passes ,this system is called stable .in other hand if the result will not nearer to basic solution ,the system will be called unstable. although our best effort goes to the discard such a solution ,while not a single chance to survive in any physical situation. therefore ,perturbations(an alteration in the normal state or path of a system or moving object)are generally correlate through assumptions and approximations, while providing a successful mathematical model corresponding to a given physical and practical model phenomena. so the stability theory may be rises and seems as a confrontational of the theoreticians with disturbances, irregularities and imperfections which are always present in experiments. So this can be calculated that the success of any mathematical model depends upon approximations and assumptions. Our best effort must-right approach for practical solution.

Mathematical Formulation : We shall, explain the idea of stability, instability and marginal state of the system with the help of a example whose differential equation together with initial conditions is given by

$$\frac{dy}{dt} = ay$$

$$Y(0) = Y_0$$

On solving this initial value problem, we get

$$Y = Y_0 e^{at}$$

(i) Now $a < 0$ than y decays exponentially as $t \to \infty$ and therefore the system is stable.
(ii) If $a > 0$ then y grows exponentially as $t \to \infty$ and the system becomes unstable.
(iii) If $a = 0$ then system is in marginal state.

In most of the experiments and physical observations, a perturbation, instead of growing without limit, tends to a finite value as the time approaches infinity. If this value be small though not infinitesimal, then the basic solution, through not stable in the sense of linear theory, represents an approximate functioning of the physical system. There are two main techniques of solving a stability problem ; namely,
(i) Normal mode technique and
(ii) The energy method.

The normal mode technique is the most important technique that has widely been used so far and is also being successfully used now a days in the linear theory of stability. The normal mode technique essentially consists in expressing an arbitrary disturbances as a superposition of certain basic mode called the normal mode. Stability of the system is then examined with respect to these modes. Since the principle of superposition holds in case of linear theory of stability, the stability analysis for each mode can be made separately. In this process, the perturbations are resolved into dynamically independent wave like components satisfying the linearised equations and the boundary conditions of the problem. Further, the suitable set of normal modes must be complete for such an expansion to be possible. The normal modes of a system are defined as the possible forms of time function constituting the time behaviour of the quantities in linear system which is originally disturbed and is deprived of the external point. In the usual practice, an arbitrary disturbance distance is expressed as a superposition of certain basic modes and the stability of the system is examined with respect to each of these modes. Consider, for example, a system confined between two parallel. Plates. The physical variable are supposed to be the functions of one co-ordinate normal to the planes. In this we shall analyse an arbitrary disturbance in terms of two dimensional periodic waves.
Now, if $A(x, y, z, t)$ represents a typical amplitude describing the disturbance, we expand it in the from

$$A(x, y, x, t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_k(z,t) \exp i(K_x X + K_y Y) dK_x dK_y$$

Where K = $\surd(K^2_x + K^2_y)$

Is the wave number associated with the disturbance A (z,t). Because the perturbation equation is linear, the reaction of the system to a general disturbance can be determined, if we know the reaction of the system to disturbances of all assigned wave numbers. This is achieved by knowing the reaction of the system to the disturbances of all assigned wave numbers. In particular, the stability of the system will depend upon its stability to disturbances of all wave numbers and if the disturbance is unstable even for one wave number, the system will be unstable. The disturbance must be expanded in terms of suitable set of normal modes which must be complete for such an expansion to be possible. The equations governing the general perturbation can be specialized for the normal modes. We then eliminate the dependence on time by seeking solution of the form

$$A(r,t) = A(r) = e^{\sigma ct}$$

Where σ is a constant to be determined.

The subscript C has been attached to σ to emphasized that the value of this constant will be different for different normal modes distinguished by C.

In general, the characteristic value for σ_k will be complex and therefore.

$$\sigma_c = \sigma_c(r) + \sigma_c(i)$$

Where $\sigma_c^{(r)}$ and $\sigma_c^{(i)}$ will depend. Apart from c, upon the parameters $z_1, z_2..........z_n$ of basic flow. Now we have

(i) $\sigma_c^{(r)} < 0$ for all c = stability.

(ii) $\sigma_c^{(r)} > 0$ for at least one c = instability.

(iii) $\sigma_c^{(r)} = 0$ for all c = marginal state.

Condition (iii) defines a locus

$$\sigma_c(z_1, z_2..........z_n) = 0$$

In $(z_1, z_2..........z_n)$ space ; this locus separates states which are stable with respect to disturbance belonging to the particular mode c. also if $\sigma_c^{(r)} = 0 = \sigma_c^{(i)} = 0$ for every c, then the principal of exchange of stabilities is valid at the marginal state and the instability sets in though stationary cellular convection. But if $\sigma_c^{(r)} = 0 = \sigma_c^{(i)} = 0$ even at least for one c, then the principle of exchange of stabilities is not valid at the marginal state and we have the case of overstability.

Energy Method

This method consists merely in the calculation of the variation of energy of disturbances with time and conclusions depend upon whether the energy decreases or increases as time goes on. The theory admits an arbitrary form of the superimposed motion and demands only, that it should be compatible with the equation of continuity.

Since the pattern of disturbance changes in the course of time, the energy could momentarily increase and subsequently die away. Thus unless a flow is unstable or stable with all disturbances, no conclusive answer can be expected. In a viscous fluid, it can hardly be expected that the motions at very small length scales are not damped out. Consequently, the energy method may be expected to give only criteria of stability. For a better physical understanding of the instability of flow, it is still important to examine the mechanism of energy balance. The key mechanism is shifted in the shape of two components of velocity of oscillation by viscous forces at the solid boundary. This produces a Reynolds stress which converts energy from the basic flow to the disturbance.

This conversion mechanism can be described by an equation of the form

$$\frac{\partial E}{\partial t} = \rho M - \mu N$$

for small two-dimensional periodic disturbance between two parallel walls, where the rate of increase of kinetic energy $E = \iint \rho(u_1^{\,2} + v_1^{\,2})$ dxdy of the disturbance equals the conversion of energy from the basic flow into the disturbance by a Reynold shear $\rho_1 u_1 v_1$.

$\rho M = -\rho \iint u_1 v_1 \frac{du}{dy} dx\ dy$ is the viscous dissipation and

$\mu N = \mu \int \varepsilon' dx\ \ dy$ where $\varepsilon' = v_{1x} - u_{1y}$. Integration is taken over a fundamental rectangle width equal to one wave length. The relative magnitude of the terms on the R.H.S. of determines whether the energy of the disturbance increases or decreases. Their ratio is

$$\frac{\rho M}{\mu N} = R \frac{M'}{N'}$$

R is the Reynolds number and M', N' are dimensionless forms of M and N.

(i) The disturbance will die out if $\frac{RM'}{N} < 1$, that is if the Reynolds number is less than the ratio $\frac{N'}{M'}$.

(ii) The least value of $\frac{N'}{M'}$, therefore, gives a critical number Re below which the motion is completely stable.

Although there are certain modifications necessary, when finite disturbance are considered, the basic process is not different. Energy is converted from the basic flow to the disturbance by the action of the Reynolds stress and dissipated into heat by viscosity. To maintain a sustained oscillation, Reynolds stress, again, must have proper sign. Energy relations used every often for the study of fully developed turbulence, and the discussions obviously applicable to finite disturbances of fairly general type.

Conclusions: The approximations and assumptions should be even that not only the simplify mathematical formulation of the problem but should agree considerably with the physical requirement of the problem and in this way there assumptions and approximations shall help us in providing the best representation to the physical phenomena under investigation. Energy method will also helpful for calculate the variation for energy of disturbances with the time and conclusion depends upon whether the energy decreases or increases as time goes on.

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