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Transient MHD Flow Through Rectangular Channel With Conducting Walls

S. Raji Reddy

Department of Mathematics, MGIT, Gandipet, Hyderabad, A.P., India.
Email: jyothirajanna@yahoo.com

ABSTRACT

In the present paper transient Magneto Hydro-dynamic rectangular channel with conducting walls is investigated. The problem is discretized by using ADI scheme. An interesting observation is that magnetic field along any line changes with Hartman number. The magnetic field obtained steady in about 40 time steps. Initially the magnetic field is found to fluctuate very much until a steady state is obtained. An interesting observation of this investigation is that the velocity distribution at the central point is almost constant for all times and for all Hartman numbers. The occurrence of maximum velocity along any line is found to be independent of Hartman number. The steady state is reached for all Hartman numbers at the same time.

Key Words: ADI method, MHD, Rectangular channel.

INTRODUCTION

Y.Q.Hu and S.T.Wu have studied multidimensional transient magnetohydrodynamic flow. Using a Full Continuous Eulerian (FICE) scheme [1]. Lalita Jayaraman and G.Ramanaiah have studied the effect of couple stresses on transient magnetohydrodynamic coutte flow [2]. Bani Singh and Jialal have studied unsteady magnetohydrodynamic flow in rectangular channel [3-8]. Rao.P.S. and M.V.Murthy [9] have studied transient magnetohydrodynamic flow through a circular pipe. Rao.P.S. and J.Anand Rao [10] have studied transient MHD flow through elliptic channel. In view of the interest shown by above researchers in transient MHD flow in this paper transient MHD flow through rectangular channel with conducting walls is investigated.

MATHEMATICAL FORMULATION

The equations governing transient incompressible viscous electrically conducting fluid with conducting walls are given by

$$\nabla.\vec{V} = 0 \quad (1)$$

$$\rho[\frac{\partial\vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V}] = -\nabla P + \mu_f \nabla^2\vec{V} + \vec{J}\, X\, \vec{B} \quad (2)$$

$$\nabla . \vec{H} = 0 \tag{3}$$

$$\text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t} \tag{4}$$

$$\text{curl } \overrightarrow{H} = \vec{J} \tag{5}$$

$$\vec{J} = \sigma(\vec{E} + \vec{V} x \vec{B}) \tag{6}$$

Where $\vec{V}$ is velocity of the fluid particle

μ	is Magnetic permeability
ρ	is the density
P	is the pressure
μ_f	coefficient of viscosity
$\vec{H}$	is the magnetic field vector
$\vec{H}_0$	is applied magnetic field
V_0	is the magnetic induction vector
σ	is the electrical conductivity
$\vec{J}$	current density vector
$\vec{B}$	magnetic induction vector
$\vec{E}$	Electric field vector
μ_0	is the permeability of magnetic number

The incompressible electrically conducting fluid filling the rectangular channel which is at rest initially and put into motion by a pressure gradient at z = - ∞. The centre of rectangle is taken as origin under these assumptions

$$\vec{V} = U\vec{K}\ , \qquad \vec{H} = H_0 \vec{J} + H\vec{K} \tag{7}$$

the equations (1) and (5) reduces to

$$\rho \frac{\partial U}{\partial t} = -\frac{\partial p}{\partial z} + \mu_f \nabla^2 U + B_0 \frac{\partial H}{\partial x} \tag{8}$$

$$\nabla^2 H = \sigma \mu_0 \frac{\partial H}{\partial t} - \sigma B_0 \frac{\partial U}{\partial x} \tag{9}$$

The equations (8) and (9) have to be solved subject to the following boundary and initial conditions.

U(X,Y,0) = H(X,Y,0) = 0
U(X1,Y, t) = 0
U(X, Y1, t) = 0

Where x_1 and y_1 are points on the boundary. In addition we have on conducting walls.

$$\frac{\partial Hx}{\partial x} = 0, \qquad y = \pm y_1$$

$$\frac{\partial Hx}{\partial y} = 0, \qquad x = \pm y_1$$

Introducing the dimensionless quantities

$$X' = \frac{X}{a}, \quad Y' = \frac{Y}{a} \quad U' = \frac{U}{a^2/\mu \frac{\partial P}{\partial z}}$$

$$H' = \frac{H}{a^2\sqrt{\frac{\sigma}{U}\frac{\partial P}{\partial z}}}$$

$$P_m = \frac{\mu_f\, \sigma \mu_0}{\rho} = Magnetic\ prandtl\ number$$

$$M = aB_0\frac{\sqrt{\sigma}}{\mu_f} = Hartman\ number$$

$$t' = \frac{\frac{t}{a^2\rho}}{\mu_f}$$

Suppressing equations (8) and (9) reduces to

$$\frac{\partial U}{\partial t} = \nabla^2 U + M\frac{\partial H}{\partial x} - 1 \qquad (10)$$

$$P_m\frac{\partial H}{\partial t} = \nabla^2 H + M\frac{\partial U}{\partial X} \qquad (11)$$

The dimensionless boundary conditions are

$$U(x,y,0) = 0 \quad U(x,\pm1,t)=0 \quad U(\pm1,y,t)=0 \quad \frac{\partial H}{\partial X} = 0, y = \pm 1, \quad \frac{\partial H_x}{\partial y} = 0, x=\pm1$$

The flow is assumed to be symmetrical about lines passing through centre and parallel to the walls taking these line by X and Y axis, the flow pattern in the first quadrant is investigated. The region is discretized with h = .1 giving rise to 25 modal points.

Discretizing equations (10) and (11) by ADI scheme due to Peaceman and Rachford [22] we get

$$(p-\delta_x^2)U*(I,J) = \left(p+\delta_y^2\right)U^n(I,J) + 2S\mu_x\delta_x H^n(I,J) - h^2 \quad (12)$$

$$(q-\delta_x^2)H*(I,J) = \left(q+\delta_y^2\right)H^n(I,J) + 2S\mu_x\delta_x U^n(I,J) \quad (13)$$

$$(p-\delta_x^2)U^{n+1}(I,J) = \left(p+\delta_y^2\right)U^*(I,J) + 2S\mu_x\delta_x H^*(I,J) - h^2 \quad (14)$$

$$(q-\delta_x^2)H^{n+1}(I,J) = \left(q+\delta_y^2\right)H^*(I,J) + 2S\mu_x\delta_x U^{n+1}(I,J) \quad (15)$$

Where

$$p=\frac{2h^2}{\Delta t}, \qquad q=\frac{2h^2 P_m}{\Delta t}, \qquad s=\frac{Mh}{2} \ldots\ldots\ldots\ldots\ldots \quad (16)$$

and $\mu x, \delta x, \delta y$ have their usual meaning. The quantities with asterisks are the values of the fictious intermediate level. The boundary conditions are as follows

U[5,J] = 0   J = 0 to 5

U[I,5] = 0   I = 0 to 5

Using boundary and symmetric flow condition equations (12) to (15) give rise to the following set of equations.

$$\begin{bmatrix} (p+2) & -2 & 0 & 0 & 0 \\ -1 & (p+2) & -1 & 0 & 0 \\ 0 & -1 & (p+2) & -1 & 0 \\ 0 & 0 & -1 & (p+2) & -1 \\ 0 & 0 & 0 & -1 & (p+2) \end{bmatrix} \begin{bmatrix} U^*(0,J) \\ U^*(1,J) \\ U^*(2,J) \\ U^*(3,J) \\ U^*(4,J) \end{bmatrix} = B_1[I,J]$$

$$B_1[I,J] = U^n[I,J-1] + (p-2)U^n(I,J) + U^n(I,J+1) - SH^n(I-1,J)SH^n(I+1,J) - h^2 \quad (17)$$

$$\begin{bmatrix} (q+2) & -2 & 0 & 0 & 0 \\ -1 & (q+2) & -1 & 0 & 0 \\ 0 & -1 & (q+2) & -1 & 0 \\ 0 & 0 & -1 & (q+2) & -1 \\ 0 & 0 & 0 & -1 & (q+2) \end{bmatrix} \begin{bmatrix} H^*(0,J) \\ H^*(1,J) \\ H^*(2,J) \\ H^*(3,J) \\ H^*(4,J) \end{bmatrix} = B_2[I,J]$$

$$B_2[I,J] = H^n[I,J-1] + (q-2)H^n(I,J) + H^n(I,J+1) - SU^{n*}(I-1,J)SU^*(I+1,J)$$

(18)

$$\begin{bmatrix} (p+2) & -2 & 0 & 0 & 0 \\ -1 & (p+2) & -1 & 0 & 0 \\ 0 & -1 & (p+2) & -1 & 0 \\ 0 & 0 & -1 & (p+2) & -1 \\ 0 & 0 & 0 & -1 & (p+2) \end{bmatrix} \begin{bmatrix} U^{n+1}(0,I) \\ U^{n+1}(1,I) \\ U^{n+1}(2,I) \\ U^{n+1}(3,I) \\ U^{n+1}(4,I) \end{bmatrix} = B_3[I,J]$$

$$B_3[I,J] = U^*[I-1,J] + (p-2)U^*(I,J) + U^*(I+1,J) - SH^*(I-1,J)SH^*(I+1,J) - h^2$$

(19)

$$\begin{bmatrix} (q+2) & -2 & 0 & 0 & 0 \\ -1 & (q+2) & -1 & 0 & 0 \\ 0 & -1 & (q+2) & -1 & 0 \\ 0 & 0 & -1 & (q+2) & -1 \\ 0 & 0 & 0 & -1 & (q+2) \end{bmatrix} \begin{bmatrix} H^{n+1}(0,I) \\ H^{n+1}(1,I) \\ H^{n+1}(2,I) \\ H^{n+1}(3,I) \\ H^{n+1}(4,I) \end{bmatrix} = B_3[I,J]$$

$$B_4[I,J] = H^*[I-1,J] + (q-2)H^*(I,j) + H^*(I+1,J) - SU^{n+1}(I-1,J) + SU^{n+1}(I+1,J)$$

(20)

DISCUSSION:

The dimensionless region consist of ≤ 1 and ≤ 1, taking into account the symmetry of the flow of the region, the region $0 \leq x \leq 1$ and $0 \leq y \leq 1$ is discretised into 5x5 grid points. The velocity distribution is determined at each nodal point for a given Prandtl magnetic number (10^{-7}) and different values of Hartman number. Graphs have been drawn showing variation of velocity and magnetic field along some lines parallel to x and y axis.

Fig .1 shows variation of velocity along central line y = 0 at Hartman number M = 1 it is observed that the maximum velocity is obtained at the nodal point (1,0) the velocity is found to increase from (0,0) to (1,0) and decreases there afterwards. Hence the effect of magnetic field is to shift the point of occurrence of maximum velocity from (0,0) to 1,0) and steady state is obtained in 40 time steps. Fig .2 shows distribution of velocity on the line y = 2. It is observed that maximum velocity is obtained at the nodal point (1,2) and velocity is found to increase from (0,2) to (1,2) and decreases there after words. The velocity distribution is almost parabolic from (0,2) to (2,2) and becomes linear there afterwards with negative gradient along the line. Steady state is obtained in 40 time steps and velocity distribution along this line is less than that of central line. Fig .3 shows that the velocity distribution for Hartman number 5 along the central line. Maximum velocity is obtained at the nodal point (1,0) and velocity is found to increase from (0,0) to (1,0) and decreases there afterwards. Steady state is obtained in 40 time steps.

Fig .4 shows the velocity distribution along the line y = 2. The maximum velocity distribution along this line is less than obtained in 40 time steps. The velocity distribution along this line is less than that of central line. Fig .5 shows the distribution of velocity at the centre at different time steps. It is observed that there is no noticeable change in the velocity at central point for different values of time. An interesting observation of this investigation is that the velocity distribution at the central point is almost constant for all times and for all Hartman numbers. The occurance of maximum velocity along any line is found to be independent of Hartman number. The steady state is reached for all Hartman numbers at the same time.

Fig .6 shows distribution of magnetic field along the central line for Hartman number M = 1. It is observed that the magnetic field increases from (0,0) to (2,0) with time, obtain minimum at (2.5, 0). Fig .7 shows the distribution of magnetic field along the line y = 2 the magnetic field is found to obtained maximum at nodal point (1,2) and almost parabolic from (1,2) to (2,2) obtaining maximum at (3,2). Fig .8 shows distribution of magnetic field on the central line for M = 5. The magnetic field is found to fluctuate with time obtaining steady state in 40^{th} time step. Minimum magnetic field is found to occur at (3,0). Fig .9 shows distribution of magnetic field on the line y = 2 for M = 5. The magnetic field is found to increase from (0,2) to (1.5,2) and then decreases up to (2.2). Steady state is obtained in 40 time steps. An interesting observation is that magnetic field along any line changes with Hartman number. The magnetic field obtained steady state in about 40 time steps. Initially the magnetic field is found to fluctuate very much until a steady state is obtained.

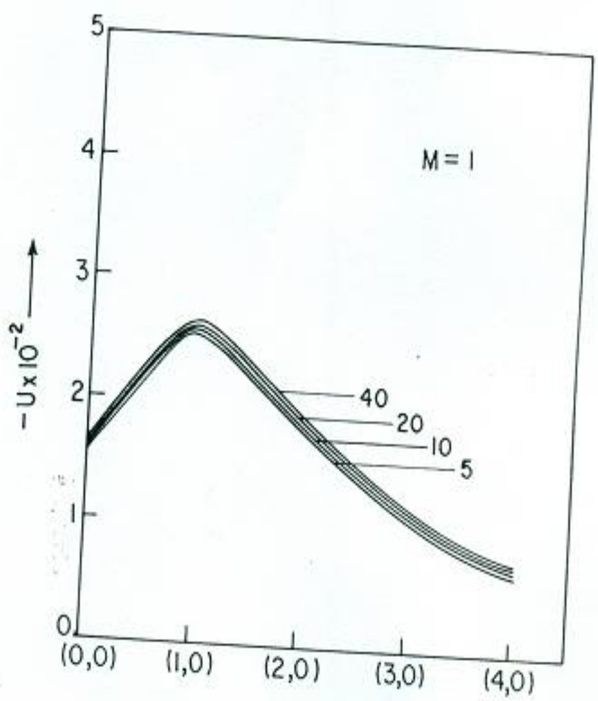

Fig.1

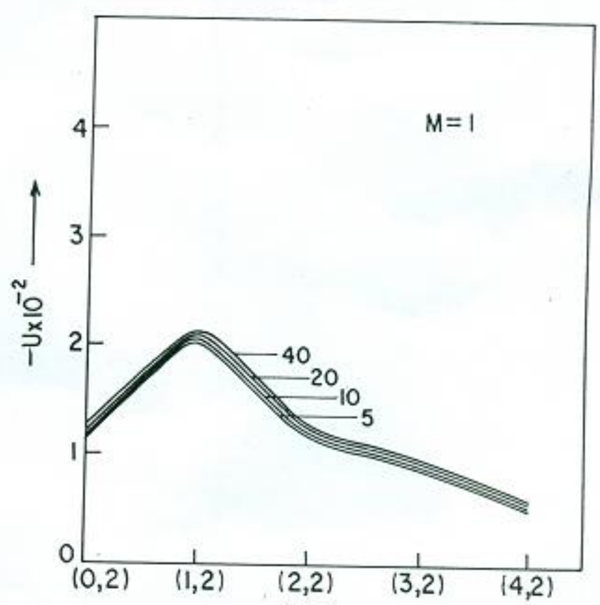

Fig.2

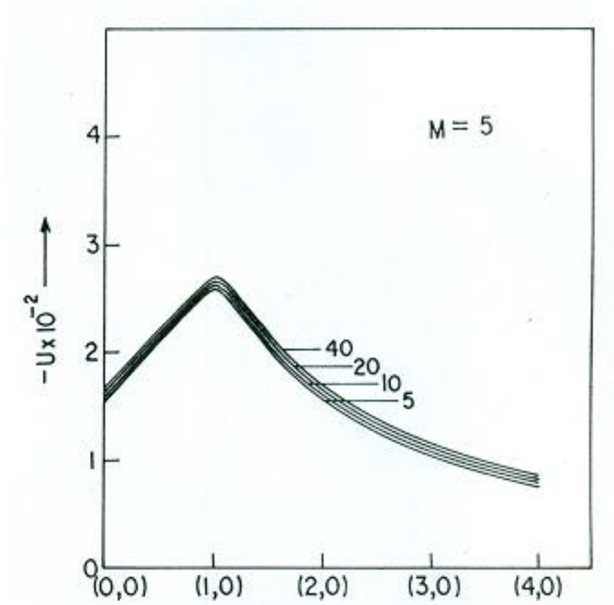

Fig.3

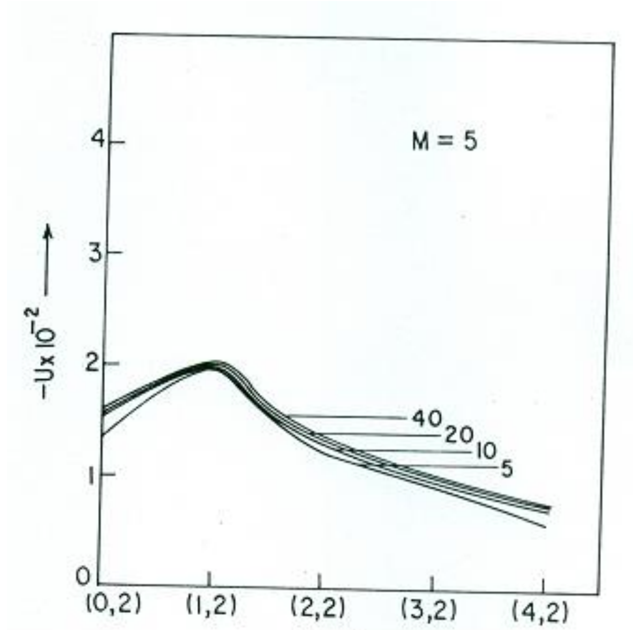

Fig.4

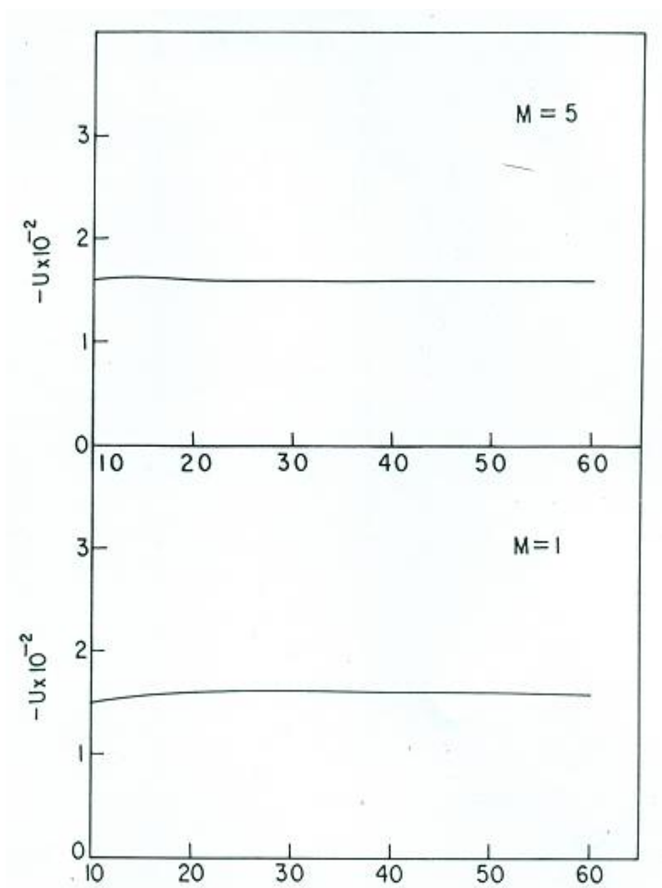

Fig.5

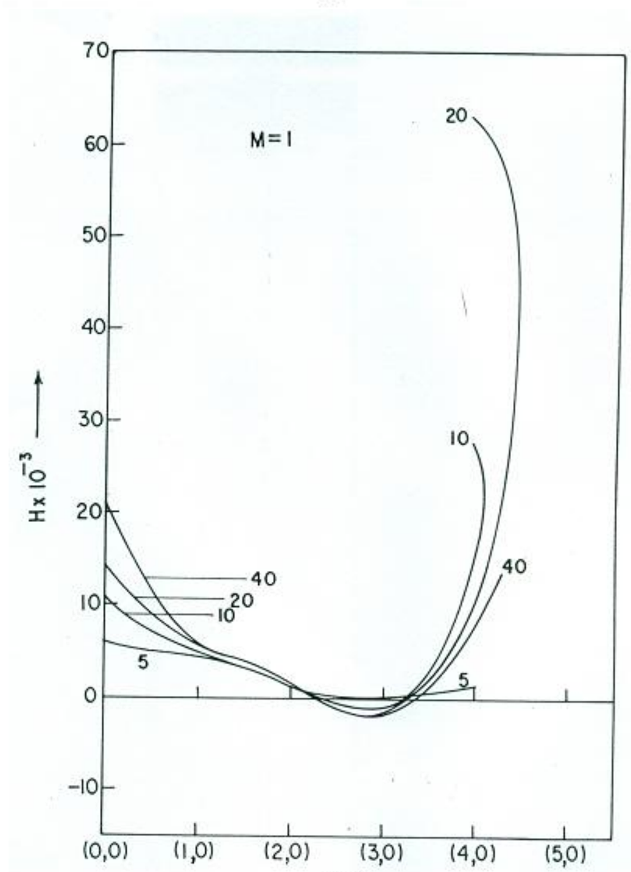

Fig.6

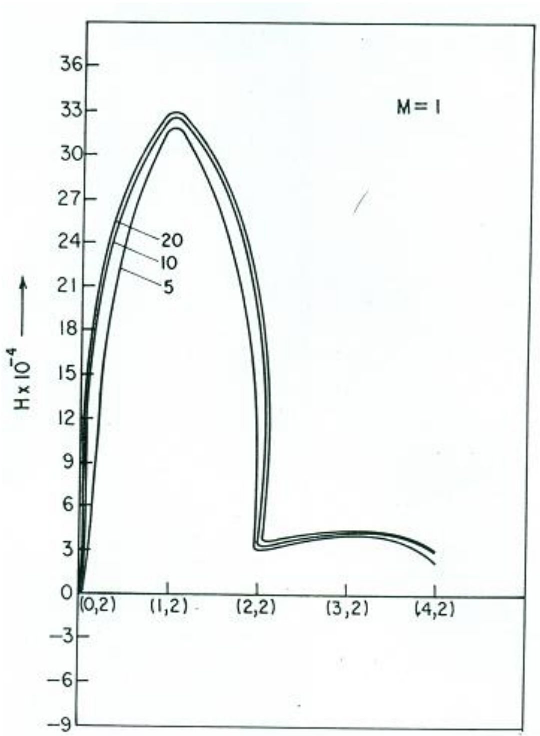

Fig.7

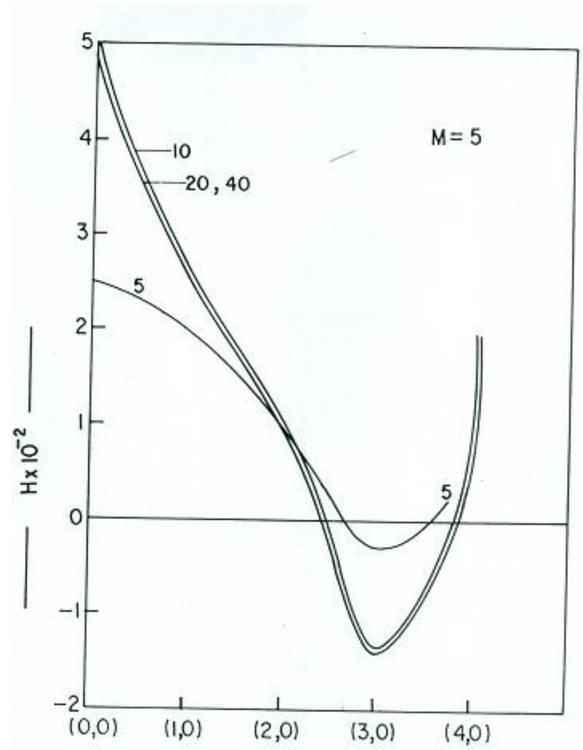

Fig.8

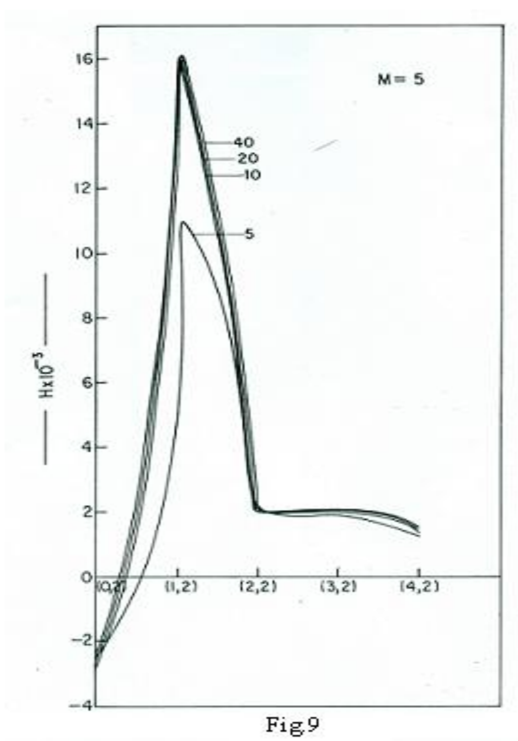

Fig.9

REFERENCES:

1. H.U, Y.Q., and S.T.Wu, A Full-Implicit continuous-Eulerian (FICE) scheme for multidimensional transit flow, Journal of Comp.Phys., 55, 33-64, (1984).
2. Lalitha Jayaraman, and G.Ramaniah, Indian journal of Technology, Vol.22, Dec.(1984).
3. Banisingh and jialal, Indian Journal of Technology Vol.12, May (1979) 184-189.
4. Banisingh and Jialala, Com.Meth.Appl.Mech.Engg., 40 (1983), 159-70.
5. Banisingh and Jialal, Indian Journal of Pure and Applied Mathematics, 14(1); Jan.(1983), 146-158.
6. Banisingh and Jialal, Indian Journal of Technology, Vol.2 Dec.(1984), 448-452.
7. Banisingh and Jialal, Int.Number.Meth.Fluids, 4(1984), 292-302.
8. Banisingh and Jialal, Indian Journal of Pure and Applied Mathematics, Vol.16(1), Nov.(1985).
9. Rao P.S., Steady motion of a sphere in a dusty gas, Def.Sci.Journal, vol.123, No.8 (1973).
10. Rao P.S., and J.Anand Rao, Proc. 15th National Conference on FM and FP, Dept. of Mechancial Engg., R.E.C., Srinagar, Kashmir, India, (1987).
11. Donald Greenspan., Introductory Numerical Analysis of Elliptic Boundary value problems.
12. Jain M.K., Numerical solution of Differential Equations, Willey Eastern Limited, New Delhi.
13. Smith, G.D., Numerical Solutions of partial Differential Equations, Oxford University Press (1965) London.
14. Samuel D.Conte/Carl de Boor, Elementary numerical analysis.
15. William F.Ames, Numerical methods for partial differential equations.