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STEADY FLOW OF A VISCOUS FLUID THROUGH A POROUS MEDIUM OF FINITE THICKNESS THE BOTTOM OF WHICH IS IMPERMEABLE AND THERMALLY INSULATED AND THE OTHER SIDE IS STRESS FREE KEPT AT A CONSTANT TEMPERATURE WITH A CONSTANT HEAT SOURCE DISTRIBUTED UNIFORMLY IN THE FLOW REGION.

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Abstract:

This paper deals with a steady flow of a viscous fluid of finite depth in a porous medium over a fixed horizontal, impermeable and thermally insulated bottom where as the other side is stress free and kept at a constant temperature with a constant heat source distributed uniformly in the flow region. Exact solutions of Momentum and Energy equations are obtained when the temperatures on the fixed bottom and on the free surface are prescribed. Flow rate ,Mean velocity, Temperature , Mean Temperature , Mean Mixed Temperature in the flow region and the Nusselt number on the free surface have been obtained. The cases of large and small values of porosity coefficient have been obtained as limiting cases. Further the cases of small depth(shallow fluid) and large depth (deep fluid) are also discussed. The results are illustrated graphically.

Keywords: porous medium, heat source, velocity, temperature, porosity parameter.

Introduction:

Forced convective flows through porous and non porous channels for a variety of geometrics was examined by Raghavacharyulu N.Ch [1] in the year 1984 and G.V.Satyanarayana Raju [2] in the year 1989.Sharma Veena Kumari Mishra [3] examined thermo solute convection flow in a porous medium. Rajesh Yadav [4] in the year 2006 studied convective heat transfer through a porous medium in channels and pipes. Steady flow of a viscous fluid through a saturated porous medium of finite thickness, impermeable and thermally insulated bottom and the other side is stress free ,at a constant temperature is studied by Khaja Moinuddin and N.Ch.Pattabhi Ramacharyulu[5].

In this paper the steady flow of a viscous fluid of viscosity μ and of finite depth H through a porous medium of permeability coefficient 'k*' over a fixed impermeable, thermally insulated bottom and with a constant heat source 'F' distributed uniformly in the flow region is investigated. The flow is generated by a constant horizontal pressure gradient parallel to the fixed bottom. The momentum equation considered is the generalized darcy's

law proposed by Yama Moto and Iwamura[6] which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force.

The basic equations of momentum and energy are solved to get exact expressions for the velocity and temperature distributions. Employing these, the flow rate, mean velocity, mean temperature, mean mixed temperature and the Nusselt number on the free surface have been obtained and their variations are illustrated graphically.

The cases of A(i) High porosity(small α) (ii) Low porosity (large α) and

B(i) Large depths (large h)B(ii) Shallow depths(small h) are also discussed.

Mathematical formulation.

Consider the steady forced convective flow of a Newtonian viscous fluid of viscosity μ through a saturated porous medium of finite depth H over a fixed horizontal impermeable bottom. The flow is generated by a constant pressure gradient parallel to the plate. Further the bottom is thermally insulated. The free surface is exposed to the atmospheric temperature T_1 and a constant heat source F is distributed uniformly in the flow region.

With reference to a rectangular Cartesian coordinate system with the origin O on the bottom, X-axis in the flow direction (i.e. parallel to the applied pressure gradient) the Y-axis vertically upwards, the bottom is represented as Y=0 and the free surface as Y=H.



Flow Configuration

U(0) = 0

Let the convective flow be characterized by the velocity field $\upsilon = (\upsilon(Y), 0, 0)$ and the temperature T(Y). The choice of the velocity satisfies the continuity equation $\nabla U = 0$ -- (1)

The momentum equation:

and the energy equation :

In the above equations ρ is the fluid density, k^* the coefficient of porosity of the medium, 'c' is the specific heat, K the thermal conductivity of the fluid and P the fluid pressure and 'F' a constant heat source distributed uniformly in the flow region.

Boundary conditions:

The bottom is fixed
$$\therefore$$
 U (0) =0 ----(4a)

The free surface is shear stress free
$$\therefore \mu \frac{dU}{dY} = 0$$
 on Y=H. ----(4b)

The bottom is thermally insulated

$$\therefore \quad \frac{dT}{dY} | (Y=0) = 0 \qquad \qquad \text{---(5a)}$$

The free surface is exposed to the atmosphere

 $\therefore T(H) = T_1$ = temperature of the atmosphere. ---(5b)

In terms of the non-dimensional variables defined hereunder:

X=ax; Y=ay; H=ah; U =
$$\frac{\mu u}{\rho a}$$
; $P = \frac{\mu^2 p}{\rho a^2}$; $k^* = \frac{a^2}{\alpha^2}$
 $-\frac{\partial P}{\partial X} = \frac{\mu^2}{\rho a^3} c_1 \cdot \left(c_1 = \frac{-\partial p}{\partial x}\right)$; $T = T_0 + (T_1 - T_0)\theta$ $\frac{\partial T}{\partial X} = \frac{(T_1 - T_0)c_2}{a}$ where $c_2 = \frac{\partial \theta}{\partial x}$
 P_r (Prandtl number) = $\frac{\mu c}{K}$; E= $\frac{\mu^3}{\rho^2 a^2 K(T_1 - T_0)}$ $f = \frac{a^2 F}{K(T_1 - T_0)}$ ----(6)

(where a is some standard length and T_{0} the temperature at the bottom) the basic field equations

can be rewritten as :

Momentum equation:

$$\frac{d^{2}u}{dy^{2}} - \alpha^{2}u = -c_{1}$$
--(7)

and the Energy equation

$$\frac{d^2\theta}{dy^2} = P_r c_2 u - E \left(\frac{du}{dy}\right)^2 - f$$
 --(8)

together with the boundary conditions for velocity

$$u(0)=0 \text{ and } \frac{du}{dy} \Big|_{y=h} = 0$$
 --(9)

and for the temperature

$$\frac{d\theta}{dy}\Big|_{y=0} = 0$$
 and $\theta(h) = 1$ --(10)

The solution of these equations together with the related boundary conditions yield.

The velocity distribution:

$$u(y) = \frac{c_1}{\alpha^2} \left[1 - \frac{\cosh \alpha (h - y)}{\cosh(\alpha h)} \right]$$
--(11)

The flow rate in the non-dimensional form is

$$q = \int_{0}^{h} u dy = \frac{c_1}{\alpha^2} \left(h - \frac{\tanh(\alpha h)}{\alpha} \right)$$
--(12)

The mean velocity in non-dimensional form is

$$\overline{u} = \frac{1}{h} \int_{0}^{h} u dy = \frac{c_1}{\alpha^2} \left(h - \frac{\tanh(\alpha h)}{\alpha} \right)$$
--(13)

and the temperature distribution: $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

$$\theta(y) = 1 + \frac{P_r c_1 c_2}{\alpha^2} \left\{ \frac{(y^2 - h^2)}{2} + (h - y) \frac{\tanh \alpha h}{\alpha} + \frac{1}{\alpha^2 \cosh \alpha h} (1 - \cosh \alpha (h - y)) \right\} + \frac{E c_1^2}{2\alpha^2} \left\{ \frac{(y^2 - h^2)}{2\cosh^2 \alpha h} + \frac{(h - y) \tanh \alpha h}{\alpha} + \frac{(1 - \cosh 2\alpha (h - y))}{4\alpha^2 \cosh^2 (\alpha h)} \right\} + \frac{f(h^2 - y^2)}{2} - (14)$$

Further the mean temperature in the non-dimensional form is given by

$$\overline{\theta} = \frac{1}{h} \int_{0}^{h} \theta dy$$

$$= 1 + \frac{P_{r}c_{1}c_{2}}{\alpha^{2}} \left(\frac{-h^{2}}{3} + \frac{h \tanh \alpha h}{2\alpha} + \frac{1}{\alpha^{2} \cosh \alpha h} - \frac{\tanh(\alpha h)}{h\alpha^{3}} \right) - \frac{Ec_{1}^{2}}{2\alpha^{2}} \left(\frac{h^{2}}{3\cosh^{2} \alpha h} - \frac{h \tanh \alpha h}{2\alpha} - \frac{1}{4\alpha^{2} \cosh^{2} \alpha h} + \frac{\tanh \alpha h}{4h\alpha^{3}} \right) + \frac{fh^{2}}{3} - (15)$$

So the mean mixed temperature in the dimensionless form is

$$\begin{split} & h \\ & h \\ & \frac{1}{\alpha (h\alpha - \tanh \alpha h)} \left(\frac{-h^3}{3} + \frac{h^2 \tanh \alpha h}{\alpha} + \frac{2h}{\alpha^2 \cosh \alpha h} + \frac{h}{2\alpha^2 \cosh^2 \alpha h} - \right) - \\ & h \\ & \frac{h}{\alpha (h\alpha - \tanh \alpha h)} \left(\frac{\tanh \alpha h}{2\alpha^3} - \frac{2 \tanh \alpha h}{\alpha^3 \cosh \alpha h} - \frac{h \tanh^2 \alpha h}{\alpha^2} - \frac{h \tanh^2 \alpha h}{\alpha^2} - \right) - \\ & h \\ & \frac{Ec_1^2}{2\alpha (h\alpha - \tan \alpha h)} \left(\frac{-3 \tanh \alpha h}{4\alpha^3} + \frac{h^3}{3\cosh^2 \alpha h} - \frac{h^2 \tanh \alpha h}{2\alpha} - \frac{h^2 \tanh \alpha h}{\alpha^2 \cosh^3 \alpha h} - \frac{h}{\alpha^2 \cosh^3 \alpha h} - \frac{h}{\alpha^2 \cosh^3 \alpha h} - \frac{h}{\alpha^3 \cosh^2 \alpha$$

--(16)

Heat transfer coefficient Nusselt numbe On the free surface :

$$\frac{d\theta}{dy} \mid_{y=h} = \frac{P_r c_1 c_2}{\alpha^3} (h\alpha - \tanh \alpha h) - \frac{E c_1^2}{4\alpha^3 \cosh^2 \alpha h} (\sinh 2\alpha h - 2\alpha h) - fh$$
--(17)

Case 1: Fluid flow in a medium with high porosity that is flow for small values of α or large values of the porosity coefficient k*.

The flow parameters are presented here in terms of the non-dimensional variables neglecting powers of α higher than $O(\alpha^2)$.

$$u(y) = c_1 \left(\left(\frac{2hy - y^2}{2} \right) - \frac{\alpha^2}{24} \left(8h^3 y - 4hy^3 + y^4 \right) \right)$$
--(18)

Mean velocity:

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$$\overline{u} = \frac{1}{h} \int_{0}^{h} u dy = \frac{c_1 h^2}{15} (5 - 2\alpha^2 h^2)$$
--(19)

Temperature:

$$\theta(y) = 1 + \frac{p_r c_1 c_2}{720} \left\{ \left(120hy^3 - 30y^4 - 90h^4 \right) - \alpha^2 \left(y^6 - 6hy^5 + 40h^3 y^3 - 35h^6 \right) \right\} + \frac{Ec_1^2}{180} \left\{ \left(45h^4 - 90h^2 y^2 + 60hy^3 - 15y^4 \right) - \alpha^2 \left(35h^6 + 15h^4 y^2 + 20h^3 y^3 + 15h^2 y^4 - 12hy^5 + 2y^6 \right) \right\} + \frac{f(h^2 - y^2)}{2} --(20)$$

Mean temperature :

$$\overline{\theta} = \frac{1}{h} \int_{0}^{h} \theta dy$$

= $1 - \frac{p_r c_1 c_2}{50401} h^4 \left(462 + 449 \alpha^2 h^2 \right) + \frac{E c_1^2 h^4}{120960} \left(18144 - 10853 \alpha^2 h^2 \right) + \frac{f h^2}{3} --(21)$

Mean mixed temperature:

$$\frac{\int_{0}^{h} \theta u dy}{\int_{0}^{h} u dy} = 1 + \frac{prc_{1}c_{2}h^{4}}{1008(5 - 2\alpha^{2}h^{2})}(-396 + 491\alpha^{2}h^{2}) + \frac{Ec_{1}^{2}h^{4}}{24192(5 - 2\alpha^{2}h^{2})}(14256 - 28997\alpha^{2}h^{2}) + \frac{fh^{2}(462 - 139\alpha^{2}h^{2})}{336(5 - 2\alpha^{2}h^{2})}$$

--(22)

Nusselt number on the free surface:

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{p_r c_1 c_2}{15} h^3 (5 - 2\alpha^2 h^2) + \frac{E c_1^2 h^3}{15} \left(-5 + 4\alpha^2 h^2\right) - fh \qquad --(23)$$

Case 2: For large values of α i.e for low porosity the asymptotic flow characteristics are the following:

For large
$$\alpha$$
 sinh $\alpha h \approx \frac{e^{\alpha h}}{2}$; $\cosh \alpha h \approx \frac{e^{\alpha h}}{2}$ and $\tanh \alpha h \approx 1$ and neglecting terms of $O(\frac{1}{\alpha^3})$
we get

g

The velocity:
$$u(y) = \frac{c_1}{\alpha^2} \left(1 - e^{-\alpha y}\right)$$
 --(24)

Mean velocity:

$$\overline{u} = \frac{1}{\alpha^2}$$
--(25)

Temperature:

$$\theta(y) = 1 + \frac{P_r c_1 c_2}{2\alpha^2} (y^2 - h^2) + \frac{f(h^2 - y^2)}{2} - (26)$$

Mean temperature:

$$\overline{\theta} = 1 - \frac{P_r c_1 c_2}{3\alpha^2} h^2 + \frac{fh^2}{3} --(27)$$

Mean mixed temperature:
$$\frac{\int_{0}^{h} \theta u dy}{\int_{0}^{h} u dy} = 1 - \frac{P_r c_1 c_2}{3\alpha^2} h^2 + f(\frac{h^2}{3} - \frac{h}{6\alpha} - \frac{1}{6\alpha^2}) - -(28)$$

Nusselt number on the free surface: $\frac{d\theta}{dy}\Big|_{y=h} = \frac{P_r c_1 c_2}{\alpha^2} h - fh$

--(29)

Case 3: Flow for large depth i.e for large H:

For large h sinh $\alpha h \approx \frac{e^{\alpha h}}{2}$; $\cosh \alpha h \approx \frac{e^{\alpha h}}{2}$ and $\& \tanh \alpha h \approx 1$ and neglecting terms of $O(\frac{1}{h^3})$ we get The velocity: $u(y) = \frac{c_1}{\alpha^2}(1 - e^{-\alpha y})$ --(30) Mean Velocity: $\overline{u} = \frac{c_1}{h\alpha^2}(h - \frac{1}{\alpha})$ --(31) Temperature: $\theta(y) = 1 + \frac{P_r c_1 c_2}{\alpha^2} \left(\frac{y^2 - h^2}{2} + \frac{h - y}{\alpha} - \frac{e^{-\alpha y}}{\alpha^2}\right) + \frac{Ec_1^2}{\alpha^2} \left(\frac{e^{-2\alpha y}}{4\alpha^2} + \frac{h - y}{2\alpha}\right) + \frac{f(h^2 - y^2)}{2}$ (32)

Mean temperature:
$$\overline{\theta} = \frac{1}{h} \int_{0}^{h} \theta dy$$

= $1 + \frac{P_r c_1 c_2}{\alpha^2} \left(\frac{-h^2}{3} + \frac{h}{2\alpha} - \frac{1}{h\alpha^3} \right) + \frac{E c_1^2}{\alpha^2} \left(\frac{-1}{8h\alpha^3} + \frac{h}{4\alpha} \right) + \frac{fh^2}{3}$ -(33)
Mean mixed temperature: $\frac{\int_{0}^{h} \theta dy}{\int_{0}^{h} u dy}_{0}$

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$$=1+\left\{ \begin{aligned} \frac{P_{r}c_{1}c_{2}}{\alpha(h\alpha-1)} \left(\frac{-h^{3}}{3}+\frac{h^{2}}{\alpha}-\frac{1}{2\alpha^{3}}-\frac{h}{\alpha^{2}}\right) + \frac{Ec_{1}^{2}}{\alpha(h\alpha-1)} \left(\frac{3}{8\alpha^{3}}+\frac{h^{2}}{4\alpha}+\frac{1}{12\alpha^{3}}-\frac{h}{2\alpha^{2}}\right) \\ + \frac{f\alpha}{(h\alpha-1)} \left(\frac{h^{3}}{3}-\frac{h^{2}}{2\alpha}+\frac{1}{\alpha^{3}}\right) \\ --(34) \end{aligned} \right\}$$

Nusselt number on the free surface:

Case 4: Flow for shallow fluids that is 'h' small (retaining terms up to the $O(h^2)$).

The velocity:
$$u(y) = \frac{c_1}{24} \{ (24hy - 12y^2) - \alpha^2 (y^4 - 4hy^3) \}$$
 --(36)

Mean velocity:
$$\overline{u} = \frac{1}{h} \int_{0}^{h} u dy = \frac{c_1 h^2}{3}$$
 --(37)

Temperature:

$$\theta(y) = 1 + \frac{prc_1c_2}{720} \{ (120hy^3 - 30y^4) - \alpha^2 (y^6 - 6hy^5) \} + \frac{Ec_1^2}{180} \{ (60hy^3 - 15y^4 - 90h^2y^2) - \alpha^2 (15h^2y^4 - 12hy^5 + 2y^6) \} + \frac{f(h^2 - y^2)}{2} - (38)$$

Mean temperature:

$$\bar{\theta} = \frac{1}{h} \int_{0}^{h} \theta dy = 1 + \frac{fh^2}{3}$$
 --(39)

Mean mixed temperature:

$$\frac{\int_{0}^{h} \theta u dy}{\int_{0}^{h} u dy} = 1$$
--(40)

Nusselt number on the free surface :

$$\frac{d\theta}{dy}|(y=h) = -fh \qquad \qquad --(41)$$

Results and Discussions:

- 1. It is noticed that the velocity of the fluid decreases with the increase in the values of the porosity parameter α (Fig.1). A similar behavior is observed in the special case of small value of α (Fig.2) and becomes zero when α is very large (Fig.3). In the case of the large depth of the channel (i . e large h) the thickness of the boundary layer decreases with the increase in the value of the porosity parameter α (Fig.4). For shallow fluids (small h) contrary to the previous cases, the velocity of the fluid increases with the increasing values of the porosity parameter α (Fig.5).
- 2. It is observed that the mean velocity of the fluid increases with the increasing values of 'h ' and decreases with the increase in the values of the

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porosity parameter α (Fig 6). For the special cases of small α and large α it is clear that the mean velocity increases with the increasing pressure gradient c1 and decreases for the increasing values of α (Fig .7, Fig .8). In the case of large depths (that is large h) mean velocity decreases with the increasing values of the porosity parameter α (Fig .9) and for shallow depths it increases with the increasing pressure gradient c1 and the depth of channel 'h'(Fig .10).

- 3. Fig.11 and Fig.12 illustrates that the temperature slightly decreases with the increase in porosity parameter α and remains constant when the heat source f=100. It is evident from Fig.13 and Fig.14 that temperature of the fluid decreases with increasing porosity parameter α when f=10 and almost remains unaltered when f = 100, for different values of α . From Fig.15 it is noticed that temperature remains constant for f=100 and the large values of porosity parameter α . Temperature of the fluid flow increases with the increase in the porosity parameter α (Fig.16). It is observed that the temperature remains the same when f=10 and y<0.04 for different porosity parameter α , there after it decreases with the increases of α (Fig.17).
- 4. Mean temperature decreases with the increasing values of the prandtl number 'p'(Fig.18). In the case of small α mean temperature decreases with the increasing values of the prandtl number 'p' and the porosity parameter α (Fig.19).Fig.20 illustrates that the mean temperature also remains constant for f=100 when the porosity parameters α 's are large. For large depths mean temperature decreases with the increase in the prandtl number 'p' and has negligible variations for smaller values of p when f=100 (Fig.21).For the case of small h mean temperature remains constant from y=0.02 and increases for y<0.02 and theincreasing values of f(Fig.22).
- 5. Fig.23 illustrates that the mean mixed temperature increases with the increase in the porosity parameter α . In the case of small α mean mixed temperature decreases for the increasing prandtle number 'p' and reaches a constant value at $\alpha = 0.9$ when f=100(Fig.24). It is evident from Fig 25 that the mean mixed temperature increases with the increasing values of α in the case of large α . In the case of large h mean mixed temperature slightly decreases with the increase in smaller values of 'p' (Fig.26). For shallow depths mean mixed temperature becomes unity.
- 6. Heat transfer rate increases with the increasing values of the prandtl number 'p' when the constant heat source is f=10 (Fig 27). In the case of small α the rate of heat transfer increases with the increasing values of the prandtl number 'p' when f =10 (Fig28). The rate of heat transfer nusselt number in the case of large α on the free surface increases with the increase in the prandtl number 'p' (Fig.29). For f= 10 in the case of large h the rate of heat transfer nusselt number on the free surface increases with the increase of 'p' (Fig.30). In the case small h it is clear that the nusselt number decreases with the increase in f.







fig.10 mean velocity profile for small h



fig.11Temperature distribution for p=1 ,h=1,E=5,c1=1,c2=1,f=10



fig.12Temperature distribution for p=1 ,h=1,E=5,c1=1,c2=1,f=100







fig.14 Temperature distribution for p=1 ,h=1,E=5,c1=1,c2=1,f=100



fig.15 temperature distribution for p=1,h=1,c1=1,c2=1,f=100







fig.24 Mean mixed temperature for h=1,E=5,c1=1,c2=1,f=100







fig.27 Nusselt Number on free surface for h=1,E=5,c1=1,c2=1,f=10



fig.29 nusselt number on free surface for h=1,c1=1,c2=1,f=10



0.4 0.3 0.2 0.1 – -0.5 0.5 1.5 0 2 2.5 3 nusselt number -----

fig.30 nusselt number on free surface for large h=60, c1=1,c2=1,E=5,f=10

x 10⁴

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