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A MATHEMATICAL MODEL OF FOUR SPECIES SYN-ECOSYMBIOSIS COMPRISING OF PREY-PREDATION, MUTUALISM AND COMMENSALISMS-II (THREE OF THE FOUR SPECIES ARE WASHED OUT STATES)

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ABSTRACT:
This investigation deals with a mathematical model of a four species (S_1, S_2, S_3 and S_4) Syn-Ecological system (Three of the four species are washed out states). S_2 is a predator surviving on the prey S_1: the prey is a commensal to the host S_3 which itself is in mutualism with the fourth species S_4. Further S_2 and S_4 are neutral. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all, there are sixteen equilibrium points. Criteria for the asymptotic stability of four of the sixteen equilibrium points: Three of the four species are washed out states only are established in this paper. The linearized equations for the perturbations over the equilibrium points are analyzed to establish the criteria for stability and the trajectories illustrated.

Key words: Equilibrium state, stability, Mutualism, Commensalisms

1. INTRODUCTION:

Ecology relates to the study of living beings in relation to their living styles. Research in the area of theoretical ecology was initiated by Lotka [6] and by Volterra [12]. Since then many mathematicians and ecologists contributed to the growth of this area of knowledge as reported in the treatises of Meyer [7], Paul colinvaux [8], Freedman [2], Kapur [3, 4] etc. The ecological interactions can be broadly classified as prey-predation, competition, mutualism and so on. N.C. Srinivas [11] studied the competitive eco-systems of two species and three species with regard to limited and unlimited resources. Later, Lakshmi Narayan [5] has investigated the two species prey-predator models. Recently, stability analysis of competitive species was investigated by Archana Reddy [1]. Local stability analysis for a two-species ecological mutualism model has been investigated by B. Ravindra Reddy et. al [9,10].

2. BASIC EQUATIONS:

Notation Adopted:

$N_1(t)$	:	The Population of the Prey (S_1)
$N_2(t)$	:	The Population of the Predator (S_2)
$N_3(t)$	:	The Population of the Host (S_3) of the Prey (S_1) and mutual to S_4
$N_4(t)$	:	The Population of S_4 mutual to S_3
t	:	Time instant
a_1,a_2,a_3,a_4	:	Natural growth rates of S_1, S_2, S_3, S_4
$a_{11},a_{22},a_{33},a_{44}$	:	Self inhibition coefficients of S_1, S_2, S_3, S_4

a_{12},a_{21} : Interaction (Prey-Predator) coefficients of S_1 due to S_2 and S_2 due to S_1

a_{13} : Coefficient for commensal for S_1 due to the Host S_3

$a_{34,}$ a_{43} : Mutually interaction between S_3 and S_4

$\frac{a_1}{a_{11}}, \frac{a_2}{a_{22}}, \frac{a_3}{a_{33}}, \frac{a_4}{a_{44}}$: Carrying capacities of S_1, S_2, S_3, S_4

Further the variables N_1, N_2, N_3, N_4 are non-negative and the model parameters a_1, a_2, a_3, a_4; a_{11}, a_{22}, a_{33}, a_{44}; a_{12}, a_{21}, a_{13}, a_{24} are assumed to be non-negative constants.

The model equations for the growth rates of S_1, S_2, S_3, S_4 are

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^2 - a_{12}N_1N_2 + a_{13}N_1N_3 \quad \ldots. \quad (2.1)$$

$$\frac{dN_2}{dt} = a_2N_2 - a_{22}N_2^2 + a_{21}N_2N_1 \quad \ldots. \quad (2.2)$$

$$\frac{dN_3}{dt} = a_3N_3 - a_{33}N_3^2 + a_{34}N_3N_4 \quad \ldots. \quad (2.3)$$

$$\frac{dN_4}{dt} = a_4N_4 - a_{44}N_4^2 + a_{43}N_4N_3 \quad \ldots. \quad (2.4)$$

3. EQUILIBRIUM STATES:

The system under investigation has sixteen equilibrium states are given by

$$\frac{dN_i}{dt} = 0,\ i = 1,2,3,4 \quad \ldots\ldots \quad (3.1)$$

I. Fully washed out state:

(1) $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$

II. States in which three of the four species are washed out and fourth is surviving

(2) $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$ (3) $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$

(4) $\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = 0$ (5) $\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$

III. States in which two of the four species are washed out while the other two are surviving

(6) $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_4a_{34} + a_3a_{44}}{a_{33}a_{44} - a_{34}a_{43}}, \overline{N_4} = \frac{a_3a_{43} + a_4a_{33}}{a_{33}a_{44} - a_{34}a_{43}}$

(7) $\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$ (8) $\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$

(9) $\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$

(10) $\overline{N_1} = \frac{a_1 a_{33} + a_3 a_{13}}{a_{11} a_{33}}, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$

(11) $\overline{N_1} = \frac{a_1 a_{22} - a_2 a_{12}}{a_{11} a_{22} + a_{12} a_{21}}, \overline{N_2} = \frac{a_1 a_{21} + a_2 a_{11}}{a_{11} a_{22} + a_{12} a_{21}}, \overline{N_3} = 0, \overline{N_4} = 0$

IV. States in which one of the four species is washed out while the other three are surviving

(12) $\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = \frac{a_4 a_{34} + a_3 a_{44}}{a_{33} a_{44} - a_{34} a_{43}}, \overline{N_4} = \frac{a_4 a_{33} + a_3 a_{43}}{a_{33} a_{44} - a_{34} a_{43}}$

(13) $\overline{N_1} = \frac{\alpha_1}{\alpha_2}, \overline{N_2} = 0, \overline{N_3} = \frac{a_4 a_{34} + a_3 a_{44}}{a_{33} a_{44} - a_{34} a_{43}}, \overline{N_4} = \frac{a_4 a_{33} + a_3 a_{43}}{a_{33} a_{44} - a_{34} a_{43}}$

Where

$\alpha_1 = a_{13}(a_4 a_{34} + a_3 a_{44}) + a_1(a_{33} a_{44} - a_{34} a_{43}), \alpha_2 = a_{11}(a_{33} a_{44} - a_{34} a_{43})$

(14) $\overline{N_1} = \frac{a_1 a_{22} - a_2 a_{12}}{a_{11} a_{22} + a_{12} a_{21}}, \overline{N_2} = \frac{a_1 a_{21} + a_2 a_{11}}{a_{11} a_{22} + a_{12} a_{21}}, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$

(15) $\overline{N_1} = \frac{\beta_2}{\beta_1}, \overline{N_2} = \frac{\beta_3}{\beta_1}, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$

Where

$\beta_1 = a_{33}(a_{11} a_{22} + a_{12} a_{21}), \beta_2 = a_{22}(a_1 a_{33} + a_3 a_{13}) - a_2 a_{12} a_{33}$

$\beta_3 = a_{21}(a_1 a_{33} + a_3 a_{13}) + a_2 a_{11} a_{33}$

V. The co-existent state (or) Normal steady state

(16) $\overline{N_1} = \frac{\gamma_1 + a_{13} a_{22} \gamma_2}{\gamma_3}, \overline{N_2} = \frac{\gamma_4 + a_{13} a_{21} \gamma_2}{\gamma_3},$

$\overline{N_3} = \frac{a_4 a_{34} + a_3 a_{44}}{a_{33} a_{44} - a_{34} a_{43}}, \overline{N_4} = \frac{a_4 a_{33} + a_3 a_{43}}{a_{33} a_{44} - a_{34} a_{43}}$

Where

$\gamma_1 = (a_1 a_{22} + a_2 a_{12})(a_{33} a_{44} - a_{34} a_{43}), \gamma_2 = a_3 a_{44} + a_4 a_{34}$

$\gamma_3 = (a_{11} a_{22} + a_{12} a_{21})(a_{33} a_{44} - a_{34} a_{43}), \gamma_4 = (a_1 a_{21} - a_2 a_{11})(a_{33} a_{44} - a_{34} a_{43})$

The present paper deals with three of the four species are washed out states only. The stability of the other equilibrium states will be presented in the forth coming communications.

4. Stability of three of the four species washed out equilibrium states: (Sl. Nos 2,3,4,5 in the above Equilibrium States)

4.1 Stability of the Equilibrium State 2

To discuss the stability of equilibrium state $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$, we consider small deviations $u_1(t), u_2(t), u_3(t), u_4(t)$ from the steady state

i.e. $N_i(t) = \overline{N}_i + u_i(t), i = 1,2,3,4$ --- (4.1.1)

Substituting (4.1.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of u_1, u_2, u_3, u_4, we get

$$\frac{du_1}{dt} = a_1 u_1 \quad \ldots\ldots (4.1.2) \qquad \frac{du_2}{dt} = a_2 u_2 \quad \ldots (4.1.3)$$

$$\frac{du_3}{dt} = l_3 u_3 \ldots\ldots \quad (4.1.4) \qquad \frac{du_4}{dt} = -a_4 u_4 + \frac{a_{43} a_4}{a_{44}} u_3 \quad \ldots (4.1.5)$$

Here $l_3 = a_3 + \frac{a_{34} a_4}{a_{44}}$ ... (4.1.6)

The characteristic equation of which is

$$(\lambda - a_1)(\lambda - a_2)(\lambda - l_3)(\lambda + a_4) = 0 \quad \ldots (4.1.7)$$

The roots a_1, a_2, l_3 are positive and $-a_4$ is negative.

Hence the steady state is **unstable**.

The solutions of the equations (4.1.2), (4.1.3), (4.1.4), (4.1.5) are

$$u_1 = u_{10} e^{a_1 t} \quad \ldots (4.1.8)$$

$$u_2 = u_{20} e^{a_2 t} \quad \ldots (4.1.9)$$

$$u_3 = u_{30} e^{l_3 t} \quad \ldots (4.1.10)$$

$$u_4 = [u_{40} - \frac{a_{43} a_4 u_{30}}{a_{44}(l_3 + a_4)}] e^{-a_4 t} + \frac{a_{43} a_4 u_{30}}{a_{44}(l_3 + a_4)} e^{l_3 t} \quad \ldots (4.1.11)$$

where u_{10}, u_{20}, u_{30}, u_{40} are the initial values of u_1, u_2, u_3, u_4 respectively.

There would arise in all 576 cases depending upon the ordering of the magnitudes of the growth rates a_1, a_2, a_3, a_4 and the initial values of the perturbations $u_{10}(t)$, $u_{20}(t)$, $u_{30}(t)$, $u_{40}(t)$ of the species S_1, S_2, S_3, S_4. Of these 576 situations some typical variations are illustrated through respective solution curves that would facilitate to make some reasonable observations.

The solutions are illustrated in figures 1 & 2.

Case (i) : If $u_{10} < u_{20} < u_{30} < u_{40}$, $a_1 < a_2 < l_3 < a_4$

In this case initially S_4 dominates over the host (S_3) of S_1 till the time instant t_{34}^* and there after the dominance is reversed. Also, we notice that all the four species diverge away from the equilibrium point. Hence the equilibrium state is unstable.

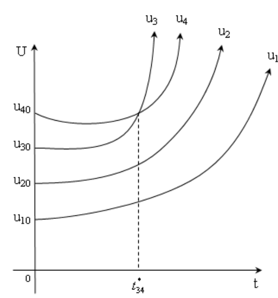

Fig. 1

Case (ii): If $u_{30} < u_{40} < u_{10} < u_{20}$, $l_3 < a_4 < a_2 < a_1$

In this case initially the predator (S_2) dominates over the prey (S_1) till the time instant t_{12}^* and there after the dominance is reversed. Also S_4 dominates over by the host (S_3) of S_1 till the time instant t_{34}^* and there after the dominance is reversed.

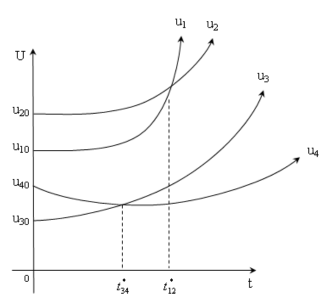

Fig. 2

Trajectories of Perturbations:

The trajectories in the u_1-u_2, u_1-u_3, u_2-u_3 planes are given by $(\frac{u_1}{u_{10}})^{a_2} = (\frac{u_2}{u_{20}})^{a_1}$, $(\frac{u_1}{u_{10}})^{l_3} = (\frac{u_3}{u_{30}})^{a_1}$ and $(\frac{u_2}{u_{20}})^{l_3} = (\frac{u_3}{u_{30}})^{a_2}$ respectively.

4.2 Stability of the Equilibrium State 3

$$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$$

Substituting (4.1.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of u_1, u_2, u_3, u_4, we get

$$\frac{du_1}{dt} = m_1 u_1 \quad \ldots (4.2.1) \qquad \frac{du_2}{dt} = a_2 u_2 \quad \ldots (4.2.2)$$

$$\frac{du_3}{dt} = -a_3 u_3 + \frac{a_{34} a_3}{a_{33}} u_4 \quad \ldots (4.2.3) \qquad \frac{du_4}{dt} = n_4 u_4 \quad \ldots (4.2.4)$$

Here $m_1 = a_1 + \frac{a_{13} a_3}{a_{33}}$, $n_4 = a_4 + \frac{a_{43} a_3}{a_{33}}$ ... (4.2.5)

The characteristic equation of which is

$$(\lambda - m_1)(\lambda - a_2)(\lambda + a_3)(\lambda - n_4) = 0 \quad \ldots (4.2.6)$$

The roots m_1, a_2, n_4 are positive and $-a_3$ is negative.

Hence the steady state is **unstable**.

The solutions of the equations (4.2.1), (4.2.2), (4.2.3), (4.2.4) are

$$u_1 = u_{10} e^{m_1 t} \quad \ldots (4.2.7) \qquad u_2 = u_{20} e^{a_2 t} \quad \ldots (4.2.8)$$

$$u_3 = [u_{30} - \frac{a_{34} a_3 u_{40}}{a_{33}(n_4 + a_3)}] e^{-a_3 t} + \frac{a_{34} a_3 u_{40}}{a_{33}(n_4 + a_3)} e^{n_4 t} \quad \ldots (4.2.9)$$

$$u_4 = u_{40} e^{n_4 t} \quad \ldots (4.2.10)$$

The solution curves are as shown in figures 3 & 4.

Case (i) : If $u_{20}<u_{30}<u_{10}<u_{40}$, $a_2<a_3<m_1<n_4$

In this case initially S_4 dominates the prey (S_1), host (S_3) of S_1 and the predator (S_2) in natural growth rate as well as in its initial population strength. It is evident that all the four species going away from the equilibrium point. Hence the equilibrium state is unstable.

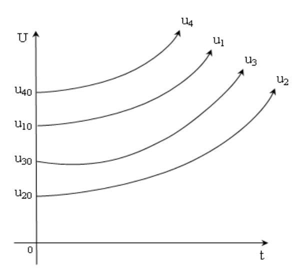

Fig. 3

Case (ii) : If $u_{10}<u_{30}<u_{20}<u_{40}$, $n_4<m_1<a_2<a_3$

In this case initially S_4 dominates the predator (S_2) and the prey (S_1) till the time instant t_{24}^*, t_{14}^* respectively and there after the dominance is reversed. Also the host (S_3) of S_1 dominates the prey (S_1) till the time instant t_{13}^* and there after the dominance is reversed.

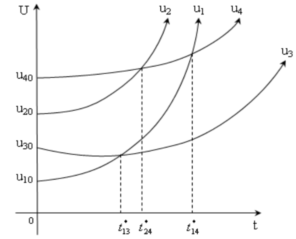

Fig. 4

Trajectories of Perturbations:

The trajectories in the u_1-u_2, u_1-u_4, u_2-u_4 and u_3-u_4 planes are given by $(\frac{u_1}{u_{10}})^{a_2} = (\frac{u_2}{u_{20}})^{m_1}$, $(\frac{u_1}{u_{10}})^{n_4} = (\frac{u_4}{u_{40}})^{m_1}$, $(\frac{u_2}{u_{20}})^{n_4} = (\frac{u_4}{u_{40}})^{a_2}$ and $\frac{u_3}{u_{30}} = (1-\frac{r_4}{u_{30}})(\frac{u_4}{u_{40}})^{\frac{-a_3}{n_4}} + (\frac{r_4}{u_{30}})(\frac{u_4}{u_{40}})$ respectively.

4.3 Stability of the Equilibrium State 4

$$\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = 0$$

Substituting (4.1.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of u_1, u_2, u_3, u_4, we get

$$\frac{du_1}{dt} = r_1 u_1 \quad \ldots (4.3.1) \qquad \frac{du_2}{dt} = -a_2 u_2 + \frac{a_{21}a_2}{a_{22}} u_1 \quad \ldots (4.3.2)$$

$$\frac{du_3}{dt} = a_3 u_3 \quad \ldots (4.3.3) \qquad \frac{du_4}{dt} = a_4 u_4 \quad \ldots (4.3.4)$$

Here $r_1 = a_1 - \frac{a_{12}a_2}{a_{22}}$ ... (4.3.5)

The characteristic equation of which is

$(\lambda - r_1)(\lambda + a_2)(\lambda - a_3)(\lambda - a_4) = 0$ ... (4.3.6)

Case (A): When $r_1 < 0$ (i.e., when $\frac{a_1}{a_2} < \frac{a_{12}}{a_{22}}$)

The roots $r_1, -a_2$ are negative and a_3, a_4 are positive. Hence the equilibrium state is **unstable.**

The solutions of the equations (4.3.1) (4.3.2), (4.3.3), (4.3.4) are

$$u_1 = u_{10} e^{r_1 t} \quad \ldots (4.3.7)$$

$$u_2 = [u_{20} - \frac{a_{21}a_2 u_{10}}{a_{22}(r_1 + a_2)}] e^{-a_2 t} + \frac{a_{21}a_2 u_{10}}{a_{22}(r_1 + a_2)} e^{r_1 t} \quad \ldots (4.3.8)$$

$$u_3 = u_{30} e^{a_3 t} \quad \ldots (4.3.9)$$

$$u_4 = u_{40} e^{a_4 t} \quad \ldots (4.3.10)$$

The solutions are illustrated in figures 5 & 6.

Case (i): If $u_{10} < u_{20} < u_{30} < u_{40}$ and $a_4 < a_3 < a_2 < r_1$ In this case initially S_4 dominates over the Host (S_3) of S_1 till the time instant t^*_{34} and there after the dominance is reversed.

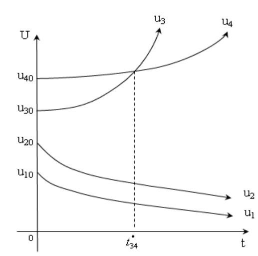

Fig. 5

Case (ii): If $u_{40} < u_{10} < u_{30} < u_{20}$ and $a_2 < r_1 < a_4 < a_3$

In this case initially the Prey (S_1) dominates over S_4 till the time instant t^*_{41} and there after the dominance is reversed. Also the Predator (S_2) dominates over the Host (S_3) of S_1 and S_4 till the time instant t^*_{32}, t^*_{42} respectively and the dominance is gets reversed there after.

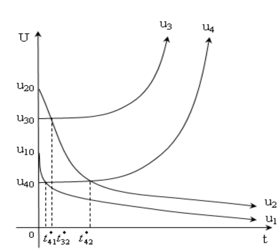

Fig. 6

Case (B): When $r_1 > 0$ (i.e., when $\frac{a_1}{a_2} > \frac{a_{12}}{a_{22}}$)

The roots r_1, a_3, a_4 are positive and $-a_2$ is negative. Hence the equilibrium state in **unstable**.
In this case the solutions are same as in case (A) and the solutions are illustrated in figures 7 & 8.

Case (i): If $u_{40} < u_{10} < u_{20} < u_{30}$ and $a_4 < r_1 < a_2 < a_3$

In this case initially the Predator (S_2) dominates over the Prey (S_1) and S_4 till the time instant t_{12}^*, t_{42}^* respectively and the dominance gets reversed there after.

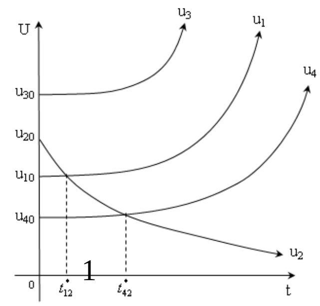

Fig. 7

Case (ii): If $u_{40} < u_{20} < u_{30} < u_{10}$ and $a_2 < r_1 < a_4 < a_3$

In this case the initially the Predator (S_2) dominates S_4 till the time instant t_{42}^* and there after the dominance is reversed. Also the Prey (S_1) dominates over the Host (S_3) of S_1 and S_4 till the time instant t_{31}^*, t_{41}^* respectively and the dominance gets reversed there after.

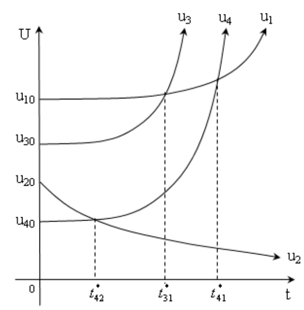

Fig. 8

Trajectories of perturbations:

The trajectories in the $u_1 - u_2, u_1 - u_3, u_1 - u_4, u_3 - u_4$ planes are given by

$$\left(\frac{u_2}{u_{20}}\right) = (1 - \frac{\psi_4}{u_{20}})\left(\frac{u_1}{u_{10}}\right)^{\frac{-a_2}{r_1}} + (\frac{\psi_4}{u_{20}})\left(\frac{u_1}{u_{10}}\right), \left(\frac{u_1}{u_{10}}\right)^{a_3} = \left(\frac{u_3}{u_{30}}\right)^{r_1}, \left(\frac{u_1}{u_{10}}\right)^{a_4} = \left(\frac{u_4}{u_{40}}\right)^{r_1} \text{ and}$$

$$\left(\frac{u_3}{u_{30}}\right)^{a_4} = \left(\frac{u_4}{u_{40}}\right)^{a_3}$$ respectively, where $\psi_4 = \frac{a_{21}a_2u_{10}}{a_{22}(r_1 + a_2)}$

4.4 Stability of the Equilibrium State 5

$$\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$$

Substituting (4.1.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of u_1, u_2, u_3, u_4, we get

$$\frac{du_1}{dt} = -a_1u_1 - \frac{a_{12}a_1}{a_{11}}u_2 + \frac{a_{13}a_1}{a_{11}}u_3 \quad \ldots (4.4.1) \qquad \frac{du_2}{dt} = q_2u_2 \quad \ldots (4.4.2)$$

$$\frac{du_3}{dt} = a_3u_3 \quad \ldots (4.4.3) \qquad \frac{du_4}{dt} = a_4u_4 \quad \ldots (4.4.4)$$

Here $q_2 = a_2 + \frac{a_{21} a_1}{a_{11}}$ ... (4.4.5)

The characteristic equation of which is

$$(\lambda + a_1)(\lambda - q_2)(\lambda - a_3)(\lambda - a_4) = 0 \quad \text{... (4.4.6)}$$

The roots q_2, a_3, a_4 are positive and $-a_1$ is negative.

Hence the equilibrium state in **unstable**.

The solutions of the equations (4.4.1) (4.4.2), (4.4.3), (4.4.4) are

$$u_1 = [u_{10} - \frac{a_1 a_{13} u_{30}}{a_{11}(a_3 + a_1)} + \frac{a_1 a_{12} u_{20}}{a_{11}(q_2 + a_1)}] e^{-a_1 t} + \frac{a_1 a_{13} u_{30}}{a_{11}(a_3 + a_1)} e^{a_3 t} - \frac{a_1 a_{12} u_{20}}{a_{11}(q_2 + a_1)} e^{q_2 t} \quad \text{… (4.4.7)}$$

$$u_2 = u_{20} e^{q_2 t} \quad \text{… (4.4.8)} \qquad u_3 = u_{30} e^{a_3 t} \quad \text{... (4.4.9)}$$

$$u_4 = u_{40} e^{a_4 t} \quad \text{... (4.4.10)}$$

The solution curves are as shown in figures 9 & 10.

Case (i): If $u_{40} < u_{30} < u_{10} < u_{20}$ and $a_3 < a_1 < a_4 < q_2$

In this case initially the Prey (S_1) dominates the Host (S_3) of S_1 and S_4 till the time instant t^*_{31}, t^*_{41} respectively and there after the dominance is reversed. Also the Host (S_3) of S_1 dominates over S_4 till the time instant t^*_{43} and there after the dominance is reversed.

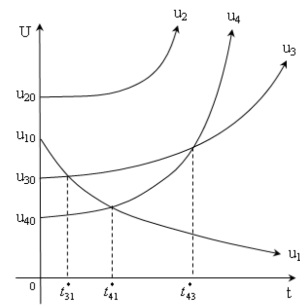

Fig. 9

Case (ii): If $u_{30} < u_{20} < u_{40} < u_{10}$ and $a_1 < a_4 < a_3 < q_2$

In this case the initially the Prey (S_1) dominates S_4, the Predator (S_2) and the Host (S_3) of S_1 till the time instant $t^*_{41}, t^*_{21}, t^*_{31}$ respectively and there after the dominance is reversed. Also S_4 dominates over the Predator (S_2) till the time instant t^*_{24} and the dominance gets reversed there after. Similarly S_4 dominates over the Host (S_3) of S_1 till the time instant t^*_{34} and the dominance gets reversed there after.

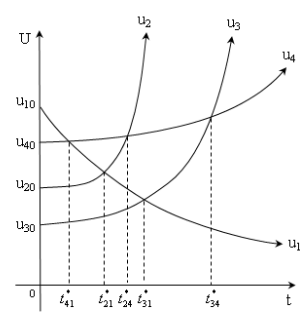

Fig. 10

Trajectories of Perturbations:

The trajectories in the $u_2 - u_3, u_2 - u_4, u_3 - u_4$ planes are given by $(\frac{u_2}{u_{20}})^{a_3} = (\frac{u_3}{u_{30}})^{q_2}, (\frac{u_2}{u_{20}})^{a_4} = (\frac{u_4}{u_{40}})^{q_2}, (\frac{u_3}{u_{30}})^{a_4} = (\frac{u_4}{u_{40}})^{a_3}$ respectively.

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