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REFLECTION OF LONGITUDINAL MICRO-ROTATIONAL WAVE FROM A FIXED MICRO-ELASTIC SURFACE

E. Rama and K. Sambaiah*

Department of Humanities and Sciences, S.R. Engineering College, Ananthsagar, Warangal ,A.P., INDIA.E-Mail. rama_rkp@yahoo.in

*Department of Mathematics, Kakatiya University, Warangal ,A.P., INDIA.

ABSTRACT:

Reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic surface is studied and obtained the amplitude ratios of micro-rotation, transverse displacement and transverse micro-rotation waves. The result of micropolar theory is obtained as a particular case of it.

KEY WORDS: Reflection – micro-rotational wave -- micro-elastic surface.

INTRODUCTION:

The theory of micromorphic materials was developed by Eringen[1]. This theory is simplified by Koh[2] by extending the concept of coincidence of the principal directions of the stress and strain of classical elasticity to theory of micromorphic materials and called it as theory of micro-isotropic, micro-elastic solids. It consists of 12 deformation fields namely, 3 macro-displacements, 3 micro-rotations and 6 micro-strains. Further it has 10 elastic constants. If an element $\Delta V + \Delta S$ contains N discrete micro-material elements $\Delta V^{(\alpha)} + \Delta S^{(\alpha)}$, ($\alpha$=1,2,…., N), then the micro-rotational and micro – strains are the rotation and strain of micro elements $\Delta V^{(\alpha)} + \Delta S^{(\alpha)}$ about the centre of mass of $\Delta V + \Delta S$.

Eringen[3] have investigated the problem of plane waves from the flat boundary of a micropolar elastic half space and obtained the amplitude ratios of different reflected waves. Reflection and refraction of a longitudinal micro-rotational wave at an interface between two micropolar elastic media in welded contact is investigated by Tomar and Gonga[4]. In the present work the amplitude ratios of micro-rotation waves and transverse displacement waves at a fixed micro-isotropic, micro-elastic surface are obtained.

BASIC EQUATIONS:

The micro-displacement in the micro-elastic continuum is denoted by u_k and micro-deformation by ϕ_{mn}. Further the macro-strain $e_{km} = u_{(k,m)}$, the macro rotation vector $r_k = \frac{1}{2}\varepsilon_{kmn} u_{n,m}$, the micro-strain $\phi_{(mn)}$ and the micro-rotation vector $\phi_p = \frac{1}{2}\varepsilon_{pkm}\phi_{km}$, where ( ) denotes the symmetric part, comma indicates differentiation with respect to the coordinate x_k.

The stress measures are the asymmetric stress (macro stress) t_{km}, the relative stress (micro-stress) σ_{km} and stress moment t_{kmn}. The couple stress tensor m_{kl} is defined by $m_{kl} = \varepsilon_{pnm} t_{kmn}$.

The constitutive equations for micro-isotropic, micro-elastic medium are given by [2,5].

$$t_{(km)} = A_1 e_{pp}\delta_{km} + 2A_2 e_{km} \tag{1}$$

$$t_{[km]} = \sigma_{[km]} = 2A_3 \in_{pkm} \left(r_p + \phi_p\right) \tag{2}$$

$$\sigma_{[km]} = -A_4\phi_{pp}\delta_{km} - 2A_5\phi_{(km)} \tag{3}$$

$$t_{k(mn)} = B_1\phi_{pp,k}\delta_{mn} + 2B_2\phi_{(mn),k} \tag{4}$$

$$m_{kl} = -2\left(B_3\phi_{l,k} + B_4\phi_{k,l} + B_5\phi_{p,p}\delta_{kl}\right) \tag{5}$$

where [ ] denotes the anti-symmetric part and

$$\begin{aligned} A_1 &= \lambda + \sigma_1, & B_1 &= \tau_3 \\ A_2 &= \mu + \sigma_2, & 2B_2 &= \tau_7 + \tau_{10} \\ A_3 &= \sigma_5, & B_3 &= 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10} \\ A_4 &= -\sigma_1, & B_4 &= -2\tau_4 \\ A_5 &= -\sigma_2, & B_5 &= -2\tau_9 \end{aligned} \tag{6}$$

where $\lambda, \mu, \sigma_1, \sigma_2, \sigma_5, \tau_3, \tau_4, \tau_7, \tau_9$ and τ_{10} are elastic moduli.

The displacement equations of motion for micro-isotropic, micro-elastic body occupying a region R are given by

$$\left(A_1 + A_2 - A_3\right)u_{p,pm} + \left(A_2 + A_3\right)u_{m,pp} + 2A_3 \in_{pkm} \phi_{p,k} + \rho f_m = \rho\frac{\partial^2 u_m}{\partial t^2} \tag{7}$$

$$B_1\phi_{pp,kk}\delta_{ij} + 2B_2\phi_{(ij),kk} - A_4\phi_{pp}\delta_{ij} - 2A_5\phi_{(ij)} + \rho f_{(ij)} = \frac{1}{2}\rho j\frac{\partial^2\phi_{(ij)}}{\partial t^2} \quad (8)$$

$$2B_3\phi_{p,mm} + 2(B_4 + B_5)\phi_{m,mp} - 4A_3(r_p + \phi_p) - \rho l_p = \rho j\frac{\partial^2\phi_p}{\partial t^2} \quad (9)$$

where ρ is the average mass density, f_m is the body force per unit mass, $f_{(ij)}$ is the symmetric body moment per unit mass, j is the micro-inertia, l_p is the body couple per unit mass.

Reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic solid

K. Somaiah and K. Sambaiah[6] have shown that there are 12 waves propagate in an infinite micro-isotropic, micro-elastic solid with six different velocities.

a) A longitudinal displacement wave propagating with speed v_1 where

$$v_1^2 = \frac{A_1 + 2A_2}{\rho} \quad (10)$$

b) A longitudinal micro-rotation wave with speed v_2 where

$$v_2^2 = \frac{2(B_3 + B_4 + B_5)}{\rho j\left(1 - \frac{\omega_1^2}{\omega^2}\right)} \quad (11)$$

with a cut-off frequency

$$\omega_1^2 = \frac{4A_3}{\rho j} \quad (12)$$

c) Two sets of coupled transverse displacement and transverse micro-rotation waves with speeds v_3 and v_4 where

$$v_3^2 = \frac{1}{2\left(1 - \frac{2\omega_0^2}{\omega^2}\right)}\left[c_4^2 + c_2^2\left(1 - \frac{2\omega_0^2}{\omega^2}\right) + c_3^2\left(1 - \frac{3\omega_0^2}{\omega^2}\right)\right.$$

$$\left. + \left[c_4^2 - c_2^2 - c_3^2 + 2\left(c_2^2 + \frac{3}{2}c_3^2\right)\frac{\omega_0^2}{\omega^2} - 4c_3^2c_4^2\frac{\omega_0^2}{\omega^2}\right]^{1/2}\right] \quad (13)$$

$$v_4^2 = \frac{1}{2\left(1 - \frac{2\omega_0^2}{\omega^2}\right)}\left[c_4^2 + c_2^2\left(1 - \frac{2\omega_0^2}{\omega}\right) + c_3^2\left(1 - \frac{3\omega_0^2}{\omega^2}\right)\right.$$

$$-\left[c_4^2 - c_2^2 - c_3^2 + 2\left(c_2^2 + \frac{3}{2}c_3^2\right)\frac{\omega_0^2}{\omega^2} - 4c_3^2 c_4^2 \frac{\omega_0^2}{\omega^2}\right]^{1/2}\Bigg] \qquad (14)$$

with $\sqrt{2}$ □□$_0$ as cut-off frequency .

d) Six waves corresponding to micro-strains and they propagate with two distinct velocities.

We investigate the reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic surface. Eringen[3] shown that the incident and reflected waves propagate in the same plane. We select this plane to be the z =0 plane.

Thus we take

$$\vec{u} = (u,0,0) \qquad (15)$$

$$\vec{\phi} = (0,\ \phi_2,\ \phi_3) \qquad (16)$$

where u, ϕ_2, ϕ_3 are functions of y,z and t. The boundary conditions are u=0, ϕ_2=0, ϕ_3=0 at z=0.

We decompose the vector $\vec{u}$ and $\vec{\phi}$ in to scalar and vector potentials as

$$\vec{u} = \nabla G + \nabla \times \vec{H}, \qquad \nabla.\vec{H} = 0 \qquad (17)$$

$$\vec{\phi} = \nabla R + \nabla \times \vec{S}, \qquad \nabla.\vec{S} = 0 \qquad (18)$$

Substituting (17), (18) into (7) and (8), we get that these equations are satisfied if

$$\left(c_1^2 + c_3^2\right)\nabla^2 G = \ddot{G} \qquad (19)$$

$$\left(c_4^2 + c_5^2\right)\nabla^2 R - 2\omega_0^2 R = \ddot{R} \qquad (20)$$

$$\left(c_2^2 + c_3^2\right)\nabla^2 \vec{H} + c_3^2 \nabla \times \vec{S} = \ddot{\vec{H}} \qquad (21)$$

$$c_4^2 \nabla^2 \vec{S} - 2\omega_0^2 \vec{S} - \omega_0^2 \times \vec{H} = \ddot{\vec{S}} \qquad (22)$$

where

$$c_1^2 = \frac{A_1 + 2A_2 - 2A_3}{\rho},\ c_2^2 = \frac{A_2 - A_3}{\rho}$$

$$c_3^2 = \frac{2A_3}{\rho},\ c_4^2 = \frac{2B_3}{\rho j},\ c_5^2 = \frac{2\left(B_4 + B_5\right)}{\rho j},\ \omega_0^2 = \frac{2A_3}{\rho j} = \frac{c_3^2}{j} \qquad (23)$$

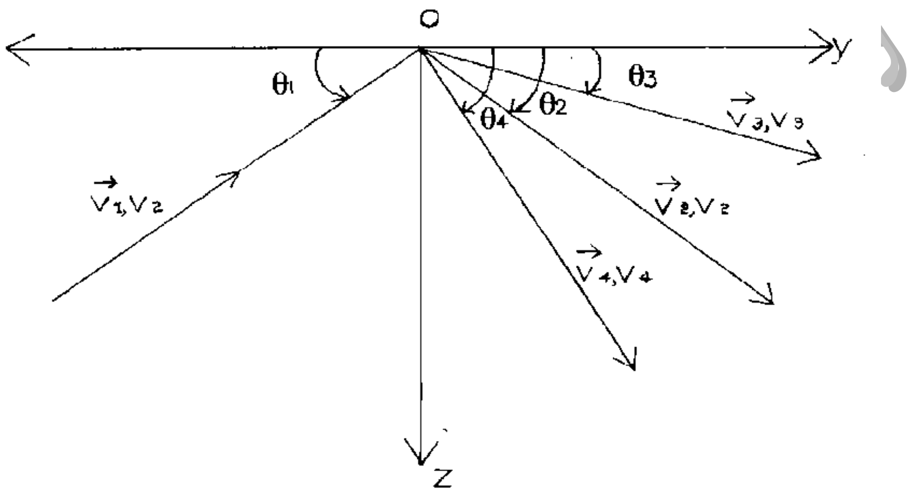
Consider an incident longitudinal micro-rotation wave travelling with speed v_2 in the direction $\vec{l}_1$ making an angle θ_1 with the plane boundary give rise to

a) A reflected longitudinal micro-rotation wave travelling with speed v_2 in the direction $\vec{l}_2$ and making an angle θ_2 with the fixed surface.

b) A set of reflected waves travelling with speed v_3 in the direction $\vec{l}_3$ and making an angle θ_3 with the surface.

c) A similar set of reflected waves travelling with speed v_4 in the direction $\vec{v}_4$ and making an angle θ_4 with the fixed surface.

The geometry of the problem is shown in fig.1



In thi

$R = a_1 \exp[ik$

$$H_p = \left[A_{py}\vec{j} - \left(\frac{\cos\theta_p}{\sin\theta_p} \right) A_{py}\vec{k} \right] \exp[ik\{\cos\theta_p y + \sin\theta_p z\} - i\omega_p t] \quad (24)$$

$$S_p = \vec{i}\, \frac{\beta_{Ap}}{\beta_{Bp}} \left(\frac{A_{py}}{\sin\theta_p} \right) \exp[ik_p\{\cos\theta_p y + \sin\theta_p z\} - i\omega_p t]$$

where $p = 3,4$; a_1, a_2 are the amplitudes of incident and reflected longitudinal micro- rotation waves respectively; A_{py} are the amplitudes of reflected coupled waves.

$$\beta_{Ap} = \left[\frac{i\omega_0^2}{k_p \left(v_p^2 - 2\frac{\omega_0^2}{k_p^2} - c_4^2 \right)} \right] \beta_{Bp} \quad (25)$$

$\omega_i = k_i v_i$, (i= 2,3,4)

ω_i is angular frequency and k_i is the wave number.

In view of (17) and (18), the boundary conditions become

$$u = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = 0$$

$$\phi_2 = \frac{\partial R}{\partial y} + \frac{\partial S}{\partial z} = 0 \quad (26)$$

$$\phi_3 = \frac{\partial R}{\partial y} - \frac{\partial S}{\partial z} = 0$$

Eringen[7] has shown that

$$\frac{\cos\theta_1}{v_2} = \frac{\cos\theta_3}{v_3} = \frac{\cos\theta_4}{v_4} \text{ and } \theta_2 = \theta_1 \tag{27}$$

$$\text{and } \omega_2 = \omega_3 = \omega_4 = \omega \text{ (say)} \tag{28}$$

Substituting (24) into (26) and using (27) and (28) we obtain

$$\sin\theta_4 z_2 + \sin\theta_3 z_3 = 0 \tag{29}$$

$$\cos\theta_2 z_1 + \frac{\beta_{A3}}{k_2\beta_{B3}} z_2 + \frac{\beta_{A4}}{k_2\beta_{B4}} z_3 = -\cos\theta_1 \tag{30}$$

$$\sin\theta_2 z_1 - \frac{\cos\theta_3}{\sin\theta_3}\frac{\beta_{A3}}{k_2\beta_{B3}} z_2 - \frac{\cos\theta_4}{\sin\theta_4}\frac{\beta_{A4}}{B_{B4}}\frac{1}{k_2} z_3 = \sin\theta_1 \tag{31}$$

where $z_1 = \frac{a_2}{a_1}$, $z_2 = \frac{k_3 A_{3y}}{a_1}$, $z_4 = \frac{k_4 A_{4y}}{a_1}$ (32)

Solving the equations (30) to (32) we obtain the amplitude ratios and they are given by

$$z_1 = \frac{1}{\Delta}\left[\frac{\beta_{A3}}{\beta_{B3}}\frac{1}{k_2}\cos(\theta_1+\theta_3) - \frac{\beta_{A4}}{\beta_{B4}}\frac{1}{k_2}\cos(\theta_1+\theta_4)\right] \tag{33}$$

$$z_2 = \frac{1}{\Delta}\sin\theta_3\sin(\theta_1+\theta_2) \tag{34}$$

$$z_3 = \frac{1}{\Delta}\sin\theta_4\sin(\theta_1+\theta_2) \tag{35}$$

where $\Delta = \frac{\beta_{A4}}{\beta_{B4}}\frac{1}{k_2}\cos(\theta_2-\theta_4) - \frac{\beta_{A3}}{\beta_{B3}}\frac{1}{k_2}\cos(\theta_2-\theta_3)$

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